Tax-Rate Arbitrage

Realization of Long-Term Gains to Enable Short-Term Loss Harvesting

Lisa Goldberg, PhD
Head of Research, Aperio
Professor of the Practice of Economics and co-Director, Consortium of Data Analytics in Risk, University of California, Berkeley

Taotao Cai
Quantitative Researcher and Research Infrastructure Lead, Aperio

Pete Hand
Senior Advisor

Abstract

We look at an enhanced loss-harvesting strategy, tax-rate arbitrage, which exploits the differential between short- and long-term tax rates. Our study relies on ATBAT, an After-Tax Back-Testing Analysis Tool that lets us examine tax-managed strategies over numerous historical periods. For the ideal tax-rate arbitrage investor, one who is subject to federal only tax rates, who has a long horizon and a planned liquidation date, and who launches the strategy from all cash, tax-rate arbitrage generated an average of 0.78% in excess after-tax active return at a 10-year horizon relative to a standard loss-harvesting strategy. Other investors with different profiles may benefit from tax-rate arbitrage but typically to a lesser extent.

Keywords: tax-rate arbitrage, loss harvesting, tax-managed strategies, tax-rate differential, gains realization, ossification

US law generally requires that capital gains tax be paid only when securities are sold. This mandate has made the delaying of gains realization and methodical harvesting of losses into mainstays of tax-managed investing.[1] Diverse studies corroborate the benefits of these techniques in managing taxable public equity portfolios.[2] Studies that focus on investment horizon, however, such as Goldberg, Hand, and Cai 2019b, document a phenomenon that is not often discussed in print, even though it is well known to experienced advisors. Rising equity markets and loss harvesting itself, which lowers cost basis, diminish opportunities to harvest losses as a portfolio ages. This so-called *ossification* of a tax-managed portfolio has prompted the development of life-extending enhancements to standard loss harvesting.

In this article, we look at a particular enhanced loss-harvesting strategy, *tax-rate arbitrage*, which exploits the differential between long- and short-term tax rates.[3] Tax-rate arbitrage resets cost basis through targeted realization of long-term gains, which potentially increases

[1] Constantinides 1983 uses an option theoretic framework to quantify the value of timing loss and gain realization in tax-managed investing. A more recent and more comprehensive exposition of tax-managed investing is in Wilcox, Horvitz, and diBartolomeo 2006. Geddes, Goldberg, and Bianchi (2015) illustrate the value of an indexed loss-harvesting portfolio as part of an asset allocation that regularly generates capital gains.

[2] Jeffrey and Arnott 1993; Arnott, Berkin, and Ye 2000; and Arnott, Berkin, and Bouchey 2011 document the importance of tax management in equity portfolios of taxable investors. Berkin and Ye 2003 use a Monte Carlo simulation to quantify the benefits of highest in, first out (HIFO) accounting, as well as the incremental benefits of loss harvesting in a market with relatively high stock-specific risk, low average return and high dividend yield. Berkin and Luck 2010 uses a Monte Carlo simulation to demonstrate the benefits of tax management in an extended equity strategy. Using empirical data, Bergstresser and Pontiff 2013 documents the impact of taxes on value, growth, and size of portfolios, and shows that taxes exacerbate the equity premium puzzle. Israel and Moskowitz 2012 explores the impact of tax management on size, value, growth, and momentum. Geddes 2011 and Geddes and Tymoczko 2019 use Monte Carlo simulations to demonstrate the after-tax benefits of loss harvesting in a separately managed account relative to holding an exchange-traded fund (ETF). Sialm and Sosner 2018 quantifies after-tax returns in tax-managed 130-30 and long-short market-neutral strategies. Using historical back-tests, Santodomingo, Nemtchinov, and Li 2016 and Goldberg, Hand, and Cai 2019a examine after-tax return and risk profiles of popular factor tilts. Analogous profiles of index-tracking and carbon-free strategies are described in Goldberg, Hand, and Cai 2019b.

[3] Constantinides 1984; Dammon, Dunn, and Spatt 1989; and Stein, Vadlamudi, and Bouchey 2008 use Monte Carlo simulations to evaluate a tax-rate arbitrage strategy.

opportunities to realize short-term losses. This strategy may increase tax alpha for specific types of investors.

We carry out an empirical study of the incremental rewards and risks of tax-rate arbitrage relative to a standard loss-harvesting strategy, in which we do not deliberately realize gains. Results are generated with ATBAT, an After-Tax Back-Testing Analysis Tool, which measures performance of a tax-managed strategy over many different historical periods. By fixing the investment horizon and staggering start dates, we obtain a range of outcomes for tax-rate arbitrage and standard loss-harvesting strategies, thereby providing investors with a broad understanding of the different strategies' attributes.[4]

To be effective, tax-rate arbitrage must be customized for different types of investors. In our study, we vary investor type along three dimensions. The first dimension is the nature of the initial funding. We look at portfolios launched with cash and with ossified portfolios, where cost basis is (often substantially) below market price. The second dimension is the tax-rate scenario. We consider federal-only rates, which apply in states with no income tax such as Texas, and California rates, which are higher. The third dimension is disposition. We look at both the estate/donation disposition, in which taxes are never paid, and the liquidation disposition, in which taxes are paid at horizon end. As emphasized throughout this article, the benefits and risks of tax-rate arbitrage vary across investor types, even after customization. This variation

[4] The high path dependency of taxable strategies means that overlapping scenarios do not materially diminish independence of observations.

underscores the importance of investor-specific information, including attributes outside the scope of this article, in determining the appropriate strategy for a taxable investor.[5]

The viability of tax-rate arbitrage depends on an ample supply of realized short-term gains, external to the equity portfolio, that require offsetting. Also important is a horizon of at least three years, and likely longer for many types of investors, since the up-front cost of realizing long-term gains precedes the benefits that may accrue from the opportunity to harvest associated short-term losses. The strategy tends to work best for an investor who plans to liquidate, rather than donate or bequeath the securities in the portfolio. This advantage stems from the relatively high cost basis exhibited by tax arbitrage strategies at the investment horizon's end.[6]

In the United States, tax rates vary from state to state, and this affects the potential benefits of tax-rate arbitrage. Specifically, the expected value of tax-rate arbitrage increases with the Tax-Rate Differential, which is the ratio of short-term to long-term tax rates minus one. This means that the strategy tends to be more valuable in Texas, where the ratio is (40.8/23.8)–1 = 0.71, than in California, where the ratio is (54.1/37.1)–1 = 0.46, even though the rates are higher in California. Further, while applying the tax-rate arbitrage strategy to an ossified portfolio has often been beneficial, the incremental value of the strategy appears to be greater when we apply

---

[5] Consider, for example, an investor whose marginal tax rate changes over time. It may be possible to realize gains in a lower income year and book losses in a higher income year, thereby increasing the benefit of tax-rate arbitrage.

[6] Timing gives another perspective on the differential impact of tax-rate arbitrage on loss harvesting in the estate/donation and liquidation dispositions. In the estate/donation disposition, tax-rate arbitrage forces long-term gain realization that would not have otherwise occurred. In the liquidation disposition, tax-rate arbitrage accelerates long-term gain realization. Accordingly, the cost of tax-rate arbitrage is effectively lower in the liquidation disposition than in the estate/donation disposition.

it at inception to a cash-funded portfolio. It follows that an investor who anticipates short-term gains may want to apply tax arbitrage as early as possible.[7]

Tax-rate arbitrage contains a particular downside risk that a candidate investor should note. Consider a scenario where a tax-rate arbitrage investor realizes a long-term gain on a stock and reinvests the proceeds in the same stock in the hope of realizing a short-term loss. If the price of the stock stays flat or rises over the subsequent year, there is no payoff to the up-front cost, even if the investor has ample short-term gains that require offsetting. In practice, we realize long-term gains on multiple stocks in a tax-rate arbitrage strategy. Some of these realizations may lead to short-term loss-harvesting opportunities while others may not. A market move, however, can lead to correlated returns to tax-rate arbitrage across stocks at a point in time. While the up-front cost of tax arbitrage has typically been rewarded in the past, this outcome is not guaranteed, and tax-arbitrage investors need to be aware of the possibility of less favorable outcomes. This downside risk is not present in standard loss-harvesting strategies.

The Loss-Harvesting Life Cycle and Path Dependency

The return and risk profile of a tax-managed portfolio of public equities evolves over time, and dynamic can be illustrated with a cash-funded loss-harvesting portfolio that tracks a diversified index. Losses in a portfolio of this type are generally abundant at first, even in rising markets,[8]

---

[7] Investors with more complicated schedules of anticipated short-term gains may have some limited ability to time the gains-realization trades of tax-rate arbitrage accordingly.

[8] Historically, growth in the US market has been driven by a relatively small number of stocks, leading to median stock performance that has been well below index performance. More information is in Cembalest 2014 and Bessembinder 2018. At the same time, underperforming stocks have been abundant and substitutable, facilitating loss harvesting and tracking error minimization.

and tracking error (TE) has typically been low. Over time, however, losses become scarcer and tracking error drifts upward as loss-harvesting drives cost basis down. Rising prices amplify these effects.

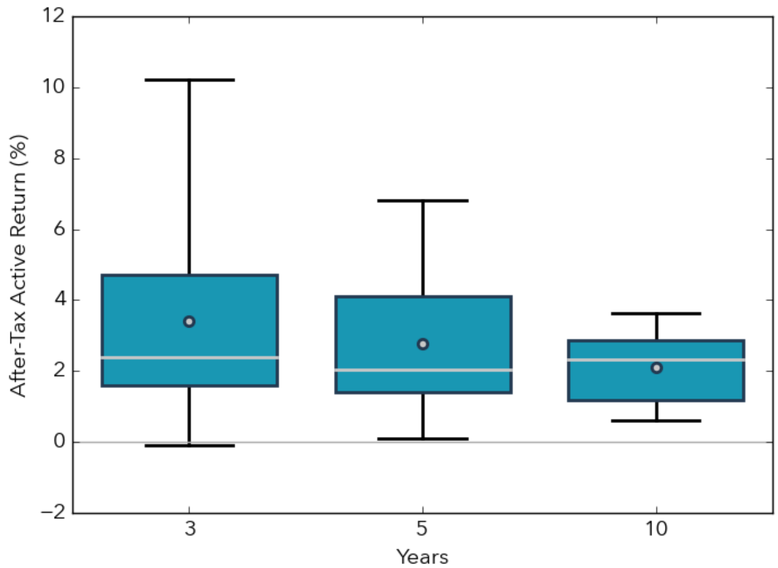
To see these effects, we use ATBAT to quantify the after-tax active return, which is the return difference between a portfolio and its benchmark after tax, of tax-managed indexing strategies at three-, five-, and 10-year horizons, benchmarked to the Russell 1000. Our analysis runs from June 1995 to March 2020. By staggering start dates on a quarterly basis, we obtain multiple outcomes for our strategies at each horizon of interest. Figure 1 shows outcomes of 88, 80, and 60 runs at three-, five-, and 10-year horizons. We display the distributions of outcomes in boxplots, which show the full range of outcomes at each horizon. The white line in each box is the median outcome, and the dot marks the average. The top and bottom of each box correspond to the 75th and 25th percentiles, respectively, and horizontal black lines mark the maximum and minimum observations.

The results show that annualized after-tax active return tended to decline as horizon length increased.[9] At the same time, outcomes became more compressed and also less skewed. This outcome is consistent with numerous studies that we have performed, varying strategy, benchmark, time frame, and horizon.[10] One of the main sources of the decline in after-tax active return with horizon is the tendency for cost basis on a typical tax lot to be considerably lower

[9] The decline in annualized after-tax return is typically accompanied by growth in the active dollar value of the portfolio.

[10] More information is found in Goldberg, Hand, and Cai 2019a and Goldberg, Hand, and Cai 2019b.

than market price in mature, tax-managed strategies. As we demonstrate below, tax-rate arbitrage mitigates this decline by strategically elevating cost basis (of selected tax lots).



	3 Years	5 Years	10 Years
Maximum	10.21	6.81	3.64
75th Percentile	4.71	4.10	2.86
Median	2.40	2.04	2.33
Average	3.41	2.78	2.10
25th Percentile	1.61	1.40	1.19
Minimum	–0.10	0.11	0.61

Figure 1: *Distributions of estate/donation annualized after-tax active return for hypothetical standard loss-harvesting portfolios launched from cash using federal-only tax rates in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

Implementing Tax-Rate Arbitrage

We sketch our implementation of a tax-rate arbitrage strategy, which proceeds in two steps. In the first step, we select suitable tax lots with unrealized long-term gains and sell them, thereby realizing long-term gains. In order to qualify as a long-term gain, a lot must be older than 365 days. The characteristics that make a lot suitable for realization are summarized in Table 1.

Lot Characteristic	Benefit
Shallow Long-Term Gain	Minimizes up-front tax cost
High Volatility	Elevates the likelihood that replacement securities will be volatile, which is favorable for future loss harvesting

Table 1: *Characteristics of lots that are suitable for long-term-gain realization.*

The lot selection process is specified in terms of:

$$\text{Lot Appreciation} = P/C - 1,$$

where P is price and C is cost basis. We harvest a long-term gain for every lot that satisfies:

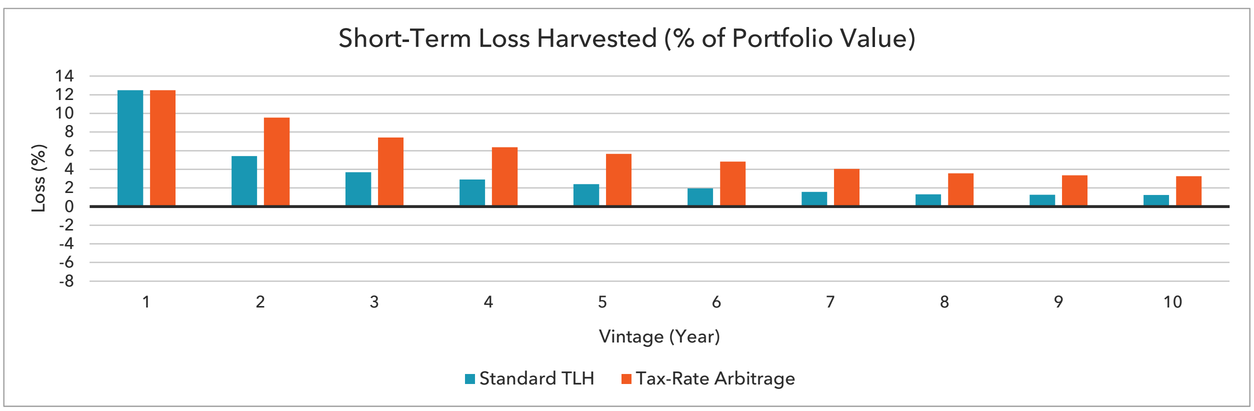
$$\text{Lot Appreciation} < \text{Security Scalar},$$

where the Security Scalar is a dynamic threshold that depends on market regime, lot volatility, investor type, and in some cases, age of strategy. The Security Scalar is central to effective tax-rate arbitrage, and it is precisely specified in Appendix C.

We set Security Scalar to be higher in more turbulent periods, when the value of aggregate short-term losses is likely to be higher than usual. With the same sort of reasoning, we set the Security Scalar to be higher for lots of more volatile securities. When there is a definite plan to liquidate the portfolio, we set the Security Scalar to be higher, since the elevated cost basis that results from tax arbitrage lowers liquidation impact. For an ossified portfolio, it is beneficial to begin with a lower Security Scalar and then increase, in order to avoid extensive gain realization at strategy inception. Finally, we set the Security Scalar to be higher when the Tax-Rate Differential is higher.

In the second step, we deposit the cash generated by selling the selected lots into the portfolio, which we then run through a standard factor-based, loss-harvesting optimization.[11] Further details are in Appendix B.

Figure 2 provides a first look into the difference in results between hypothetical tax-rate arbitrage and standard loss-harvesting portfolios in ATBAT. In the top panel, we show average net short-term losses harvested each year in the two strategies for cash-launched portfolios in the estate/donation disposition and the federal-only tax regime. In the bottom panel, we do the same for long-term losses. Consistent with the mechanism that drives tax-rate arbitrage, we observed that the value of short-term losses was, on average, materially greater than in standard loss harvesting after the first year, while the value of long-term losses was materially lower. Figure 2 rightly suggests that tax-rate arbitrage can be viewed as levered loss harvesting.



[11] Since there is no wash sale rule for the realization of gains, the optimizer may choose to buy back a security that was sold in the first step of tax-rate arbitrage.

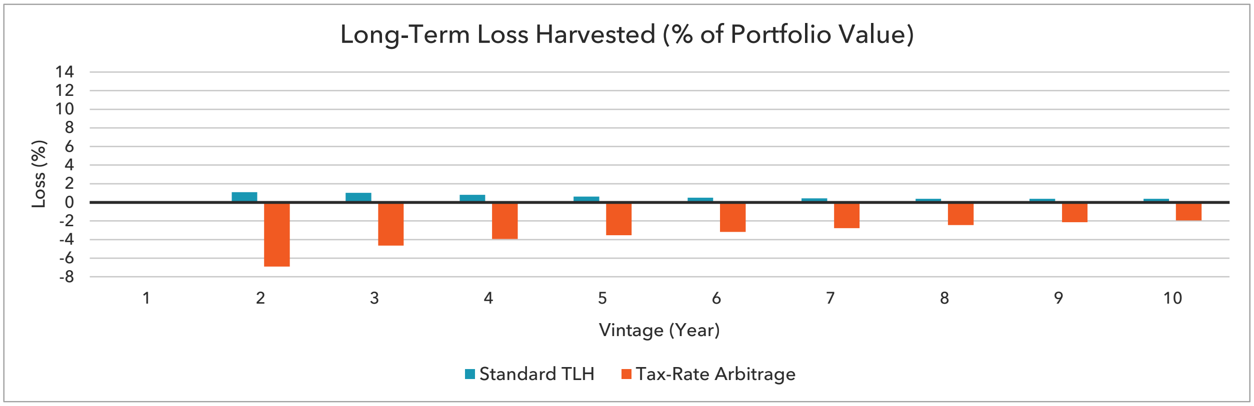


Figure 2: *Hypothetical standard loss-harvesting and tax-rate arbitrage portfolios in the estate/donation disposition and the federal-only tax regime launched from cash in ATBAT. Top panel: Average net short-term loss as a percentage of portfolio value. Bottom panel: Average net long-term loss as a percentage of portfolio value. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

Assessing the Effect of Tax-Rate Arbitrage

We measure excess return of a tax-rate arbitrage strategy relative to a standard loss-harvesting strategy by:

$$Tax\ Arbitrage\ Value\ Added = \ Return_{Tax\ Rate\ Arbitrage} - Return_{Standard\ Loss\ Harvesting}.$$

We calculate tax arbitrage value added (TAVA) for after-tax active return. We also calculate TAVA for tax alpha, which is the component of after-tax active return that comes from harvesting losses. Formulas for after-tax active return and tax alpha are in Appendix A.

We report realized tracking error for both standard and tax-rate arbitrage strategies, as well as incremental risk due to tax-rate arbitrage:

Incremental Risk = TE_tax-rate-arbitrage – TE_standard-loss-harvesting.

The risk profiles of standard loss-harvesting and tax-arbitrage strategies tend to diverge over time. In this study, we calibrate our strategies so that the median 10-year forecast tracking error of the tax-rate arbitrage enhanced strategy and the standard strategy match. Because of differences in turnover in the two strategies, our calibration virtually guarantees that median forecast tracking errors will not match at horizons that are shorter (or longer) than 10 years.

Empirical Study

As in the analysis of the loss-harvesting life cycle, we use ATBAT to quantify the impact of tax-rate arbitrage on loss harvesting at three-, five-, and 10-year horizons. In order to make the most of available data, we have more outcomes for shorter horizons. Further, we have more observations for portfolios launched with all cash than for ossified portfolios, as shown in Table 2. This is an artifact of our process for generating ossified portfolios, which requires five years of data. This means that our out-of-sample results for ossified portfolios are based on data from June 2000 to March 2020, in contrast to the longer period from June 1995 to March 2020 for cash-launched portfolios.

Count/Horizon	3 Years	5 Years	10 Years
Cash-Launched	88	80	60
Ossified	68	60	40

Table 2: *Numbers of observations for each strategy at each horizon.*

Our tax-rate regimes are specified in Table 3.

	Short-Term Rate	Long-Term Rate
Federal Only	40.8%	23.8%
California	54.1%	37.1%

Table 3: *Tax-rate scenarios.*

All of our strategies are constructed by minimizing forecast tracking error to the Russell 1000 benchmark while harvesting losses. The mathematical details are in Appendix B. In a tax-rate arbitrage strategy, the minimization step is preceded by gain realization, as described above.

Results

Below, we separately review the benefits of tax-rate arbitrage to after-tax active return outcomes for portfolios launched from cash and for ossified portfolios. We evaluate results across various time frames, tax regimes, and dispositions.

After-Tax Active Return: Portfolios Launched from Cash

We look first at the value that tax-rate arbitrage added to after-tax active return for portfolios launched from cash. At a 10-year horizon, tax arbitrage value added was positive, on average, in both the estate/donation and liquidation dispositions, and under both fed-only and California scenarios.

Relative to a standard loss-harvesting strategy, additional basis resetting from tax-rate arbitrage elevated cost basis and thus lowered liquidation impact, leading to a higher average TAVA in the liquidation disposition than in the estate/donation disposition. This effect is present in both tax-rate scenarios.

For both the estate/donation and liquidation dispositions, average TAVA was higher in the federal-only scenarios than in the California scenarios. This is because tax-rate arbitrage return depends on the Tax-Rate Differential, which is 0.71 in the fed-only scenario and 0.46 in the California scenario. An option-theoretic perspective on this point is in Appendix E.

Disposition	Tax Regime	Standard After-Tax Active Return	Average TAVA
Estate/Donation	Fed	+2.10%	+0.24%
Estate/Donation	CA	+2.88%	+0.18%
Liquidation	Fed	+1.21%	+0.78%
Liquidation	CA	+1.41%	+0.52%

Table 4: *Average TAVA, tax arbitrage value added, to after-tax active return for hypothetical portfolios launched from cash at a 10-year horizon in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

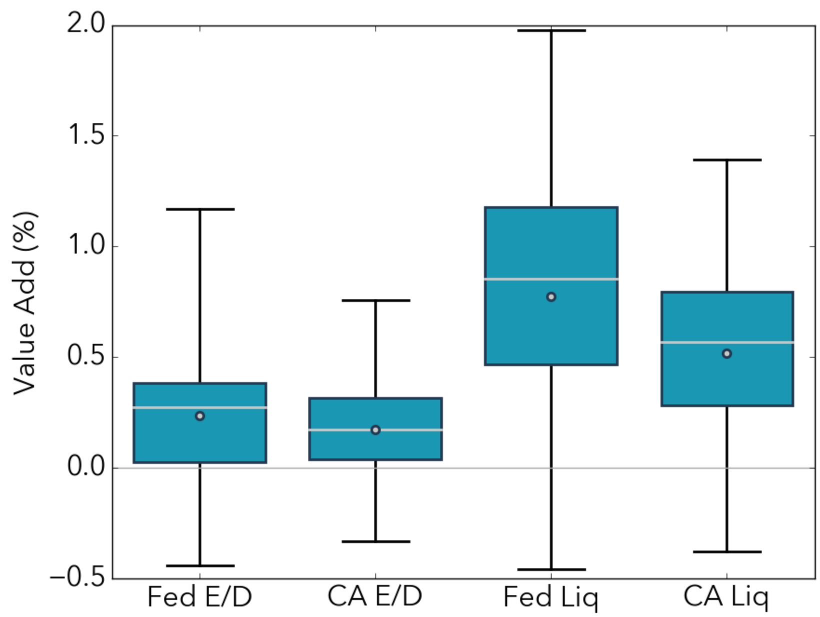
We expand on the results in Table 4 in two ways. First, we look at the time evolution of after-tax active return by adding TAVA averages at the three- and five-year horizons to the 10-year horizon results. Table 5 shows that average TAVA tended to increase with term. This makes sense, in light of the up-front costs associated with tax arbitrage. The most extreme manifestation of term dependence of TAVA was at the three-year horizon for estate/donation fed-only after-tax active return, which was negative. More generally, tax-rate arbitrage appears to be suitable only for a long-term investor, as the benefits of tax-rate arbitrage are less pronounced in the earlier years of the strategy.

Disposition	Tax Rate	3 Years	5 Years	10 Years
Estate/Donation	Fed	–0.12%	+0.07%	+0.24%
Estate/Donation	CA	0.00%	+0.10%	+0.18%

Liquidation	Fed	+0.15%	+0.50%	+0.78%
Liquidation	CA	+0.19%	+0.42%	+0.52%

Table 5: *Term structure of average TAVA, tax arbitrage value added, to after-tax active return for hypothetical portfolios launched from cash in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

Next, in Figure 3, we look at the distributions of 10-year TAVA in the estate/donation and liquidation dispositions for the fed-only and California tax regimes. TAVA was positive at the 25th percentile in all four cases. Scenarios launched just after the 2008–09 global financial crisis had the lowest TAVA, as a result of calm, upward-trending markets following that event. The highest-TAVA scenarios were launched just prior to the bursting of the dot-com bubble in 2000. The calm, upward trend of the market between the dot-com crisis and the global financial crisis facilitated abundant long-term gain realization, which elevated cost basis. The market turbulence in the summer of 2007 and in 2008 allowed loss-harvesting algorithms to transform the elevated cost basis into short-term losses.



	Tax Arbitrage Value Added

	Estate/Donation		Liquidation	
	Fed	**CA**	**Fed**	**CA**
Maximum	+1.17%	+0.76%	+1.98%	+1.39%
75th Percentile	+0.39%	+0.32%	+1.18%	+0.80%
Median	+0.27%	+0.17%	+0.86%	+0.57%
Average	+0.24%	+0.18%	+0.78%	+0.52%
25th Percentile	+0.02%	+0.04%	+0.47%	+0.28%
Minimum	–0.44%	–0.33%	–0.46%	–0.38%

Figure 3: *Distributions of TAVA, tax-arbitrage value added, to after-tax active return at a 10-year horizon for hypothetical portfolios launched from cash in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

After-Tax Active Return: Ossified Portfolios

Some investors fund a tax-rate arbitrage strategy with legacy securities. Others may embark on a standard loss-harvesting strategy and then switch to tax arbitrage. In either case, we use an ossified portfolio, for which cost basis is less than market price, to launch tax arbitrage. Ossified portfolios can be very different from one another, so a single analysis cannot give a complete picture of long-term performance. Still, we can begin to gauge the impact of legacy positions on tax-rate arbitrage strategies by considering some examples. In the analysis described below, with results shown in Table 6, we generate hypothetical ossified portfolios to launch our strategies with the process described in Appendix D. We describe the metrics we use to quantify ossification in Appendix F.

Average after-tax active return TAVA for ossified portfolios at a 10-year horizon was positive in all four cases we considered. As we can see, however, by comparing Tables 4 and 6, ossification cut average TAVA for after-tax active return at a 10-year horizon by at least 38% in the four cases considered.

Disposition	Tax Regime	Standard After-Tax Active Return	Average TAVA
Estate/Donation	Fed	0.93%	+0.08%
Estate/Donation	CA	1.27%	+0.09%
Liquidation	Fed	0.15%	+0.48%
Liquidation	CA	–0.06%	+0.24%

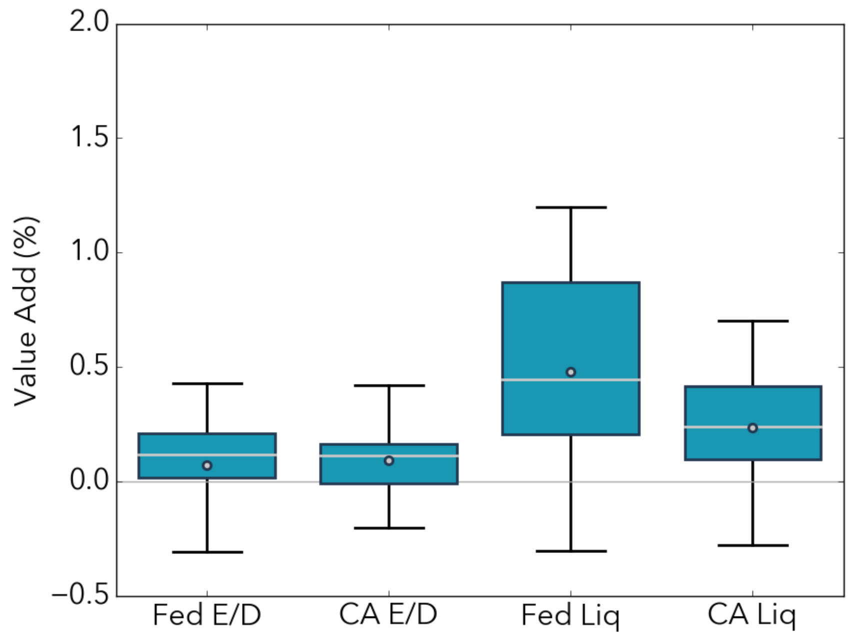
Table 6: *Average TAVA, tax-arbitrage value added, to after-tax active return for hypothetical ossified portfolios at a 10-year horizon in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 2000—March 2020.*

At shorter horizons of three and five years, average TAVA for after-tax active returns was inconsequential in the estate/donation disposition but substantial in the liquidation disposition. The impact of ossification on after-tax active return TAVA was less pronounced at shorter horizons, as shown by comparing Tables 5 and 7.

Disposition	Tax Rate	3 Years	5 Years	10 Years
Estate/Donation	Fed	–0.08%	–0.01%	+0.08%
Estate/Donation	CA	–0.03%	+0.04%	+0.09%
Liquidation	Fed	+0.22%	+0.32%	+0.48%
Liquidation	CA	+0.19%	+0.22%	+0.24%

Table 7: *Term structure of average TAVA, tax-arbitrage value added, to after-tax active return for hypothetical ossified portfolios in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 2000—March 2020.*

The box plots shown in Figure 4 indicate that at a 10-year horizon, roughly three-quarters of our runs led to positive TAVA for after-tax active return when strategies were launched from ossified portfolios.



	Tax Arbitrage Value Added			
	Estate/Donation		Liquidation	
	Fed	CA	Fed	CA
Maximum	+0.43%	+0.42%	+1.20%	+0.71%
75th Percentile	+0.21%	+0.17%	+0.87%	+0.42%
Median	+0.12%	+0.12%	+0.45%	+0.24%
Average	+0.08%	+0.09%	+0.48%	+0.24%
25th Percentile	+0.02%	–0.01%	+0.21%	+0.10%
Minimum	–0.30%	–0.20%	–0.30%	–0.28%

Figure 4: *Distributions of TAVA, tax-arbitrage value added, to after-tax active return at a 10-year horizon return for hypothetical ossified portfolios in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 2000—March 2020.*

The results in this section show that tax-rate arbitrage tended to offer superior returns relative to standard loss harvesting in 10-year ossified strategies, although the benefits were lower than in cash-only strategies. This suggests that an all-cash investor interested in the extra loss-harvesting capacity that tax arbitrage can provide may want to begin the process at inception.

Decomposing After-Tax Active Return

We can express after-tax active return (ATAR) as a sum of pre-tax active return (PTAR) and tax alpha (TA):

$$\text{ATAR} = \text{PTAR} + \text{TA}.$$

We look at the impact of tax arbitrage on these components of tax alpha in the simplest case: a cash-launched portfolio in the estate/donation disposition in the fed-only tax regime.

	After-Tax Active Return	Pre-Tax Active Return	Tax Alpha
Standard TLH	2.10%	–0.12%	2.22%
Tax-Rate Arbitrage	2.34%	–0.23%	2.57%
TAVA	+0.24%	–0.11%	+0.35%

Table 8: *Decomposition of average after-tax active return of hypothetical cash-launched portfolios in the estate/donation disposition and federal-only tax regime at a 10-year horizon in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

Table 8 suggests that tax-rate arbitrage created a pre-tax drag, on average. Roughly one-third of the additional 0.11% in pre-tax drag in the tax-rate arbitrage strategy was due to trading costs.

At the same time, tax-rate arbitrage added value in our hypothetical strategy, on average, to tax alpha by making it easier to harvest short-term losses. A measure of the enhancement is *short-term basis*, which is the dollar value of the basis of short-term positions divided by the market value of the portfolio.[12]

[12] The formula for short-term basis is precisely specified in Appendix F.

Table 9 shows the increase in short-term basis due to tax-rate arbitrage at three-, five-, and 10-year horizons. Relative to the estate/donation disposition, the higher Security Scalar in the liquidation disposition appears to lead to more trading and a higher short-term basis.

Short-Term Basis (%)	**3 Years**	**5 Years**	**10 Years**
Standard TLH	16.5	10.4	5.7
Tax-Rate Arbitrage: Estate/Donation	45.8	34.5	18.9
Tax Rate Arbitrage: Liquidation	74.3	69.5	59.1

Table 9: *Average short-term basis for hypothetical cash-launched portfolios in the estate/donation disposition and fed-only tax regime at three-, five-, and 10-year horizons in ATBAT. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

To conclude this section, we look year by year at the average value of harvested losses in the hypothetical standard loss-harvesting and tax-rate arbitrage strategies. By construction, the values are the same in the first year. In the second year, the standard strategy outperformed, on average, since long-term gains were deliberately realized in the tax-rate arbitrage strategy. As we show in Figure 5, the difference between average tax alpha in the tax-rate arbitrage and standard loss-harvesting strategies was positive beginning in year three; it grew until year five and then leveled off. This observation underscores the preference need for a tax-rate arbitrage investor to have a relatively long horizon in order to potentially benefit from the strategy.

Figure 5: *Top panel: Average value of harvested losses per year in hypothetical standard loss-harvesting and tax-rate arbitrage strategies in ATBAT. Bottom panel: Difference between average value of losses in the standard loss-harvesting strategy and tax-rate arbitrage. Results are gross of fees and assume a round-trip trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*

Risk

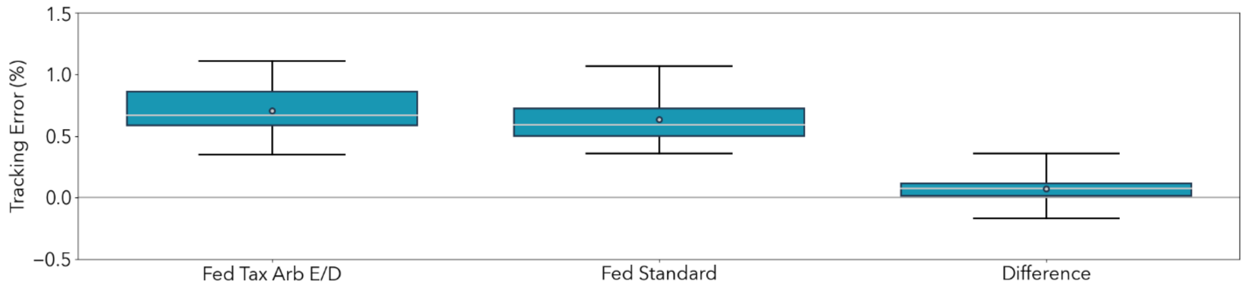
In the previous sections, we looked at how tax-rate arbitrage affects after-tax return in a loss-harvesting strategy. Here, we focus on risk, as measured by realized tracking error between our strategies and their benchmarks.

To begin, we mention two observations that affect strategy calibration. All else equal, tax-rate arbitrage tends to increase turnover, which leads to lower tracking error. Consequently, calibrating a standard loss-harvesting strategy and a tax-rate arbitrage strategy to have the same average forecast tracking error at one horizon necessarily means that the average forecast tracking errors of the two strategies must differ at other horizons. Our second consideration is that the forecast tracking error of any optimized portfolio is biased downward. This means that over many periods, realized tracking error tends to be higher than forecast tracking error. In this study, we calibrate our cash-launched tax-rate arbitrage strategy to have an average forecast tracking error at a 10-year horizon of 0.65%, which matches the average forecast tracking error at a 10-year horizon for our standard strategy. We use the same calibration for our ossified runs.

Distributions of realized tracking errors of our strategies for the estate/donation fed-only runs at a 10-year horizon are shown in Figure 6. The results shown are for strategies launched from cash. Tax arbitrage tended to slightly increase both the level and dispersion of realized tracking error of loss-harvesting strategies, which can be explained by our calibration scheme.[13] Results are qualitatively similar for California and liquidation runs. The results presented in Figure 7 are for ossified strategies, where tax-rate arbitrage lowered both the level and dispersion of realized tracking error. As for the comparison in the cash-launched study, this outcome is a result of our calibration.[14] Again, the results are qualitatively similar for California and liquidation runs.

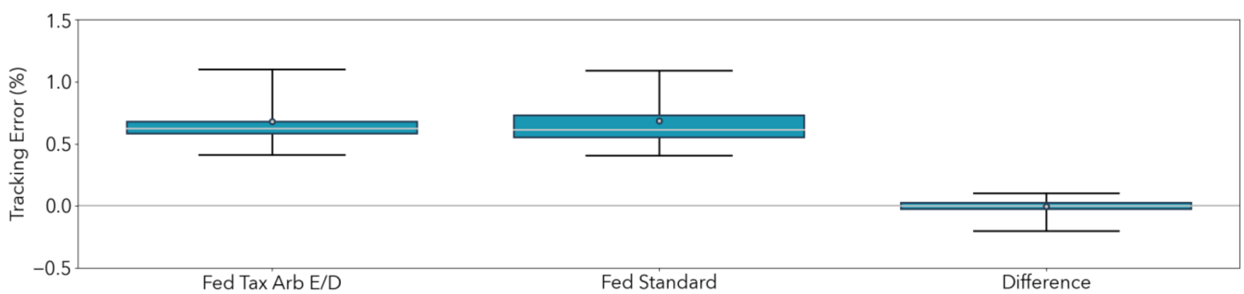
[13] In order to match average forecast tracking error at a 10-year horizon, we need to set forecast tracking error higher for the tax-rate arbitrage strategy in the early years. Doing so amounts to lowering risk aversion.

[14] The strategy parameters for the ossified study were taken from the calibration for cash-launched strategies. A dedicated calibration for ossified portfolios would likely lead to raising the forecast tracking error for tax arbitrage, or equivalent, to lowering risk aversion.



	Fed Tax Arb	Fed Standard	Difference
Maximum	1.11	1.07	0.36
75th Percentile	0.86	0.73	0.12
Median	0.67	0.60	0.08
Average	0.70	0.64	0.07
25th Percentile	0.59	0.51	0.02
Minimum	0.35	0.36	-0.17

Figure 6: *Distributions of realized tracking error for hypothetical standard estate/donation fed-only loss-harvesting and tax-rate arbitrage strategies at a 10-year horizon for cash-launched portfolios in ATBAT. Results are gross of fees and assume a round-trip trading cost of trading cost of 0.08%. Benchmark: Russell 1000 Index. June 1995—March 2020.*



	Fed Tax Arb	Fed Standard	Difference
Maximum	1.10	1.09	0.10
75th Percentile	0.68	0.73	0.03
Median	0.62	0.61	0.00
Average	0.68	0.68	-0.00
25th Percentile	0.59	0.55	-0.02
Minimum	0.41	0.41	-0.20

Figure 7: *Distributions of realized tracking error for hypothetical standard estate/donation fed-only loss-harvesting and tax arbitrage strategies at a 10-year horizon for ossified portfolios in ATBAT. Results are gross of fees and assume a round-trip trading cost of trading cost of 0.08%. Benchmark: Russell 1000 Index. June 2000—March 2020.*

Summary

We have explored the incremental rewards and risks of tax-rate arbitrage, the targeted realization of long-term gains in a loss-harvesting strategy. Our results rely on an After-Tax Back-Testing Analysis Tool, which generates historical outcomes over numerous periods, thereby providing a broad perspective on how tax-managed strategies have performed, and allowing us to tailor our analysis to different types of investors.

We found that while many long-horizon investors with an ample supply of short-term gains may benefit from tax-rate arbitrage, the ideal candidate for this strategy is an investor who plans to liquidate at horizon end, has a high Tax-Rate Differential, and implements the strategy at inception in a cash-funded portfolio. At the same time, it is important to keep in mind that the up-front cost of tax-rate arbitrage leads to risks that are not present in standard loss-harvesting strategies. The impact of tax-rate arbitrage depends crucially on both market forces and the way in which the strategy is calibrated. For tax-rate arbitrage, as for virtually every other tax-management tool, one size does not fit all.

References

Arnott, Robert D., Andrew L. Berkin, and Paul Bouchey. 2011. "Is Your Alpha Big Enough to Cover Its Taxes? Revisited." *Investments & Wealth Monitor* (January/February): 6–20.

Arnott, Robert D., Andrew L. Berkin, and Jia Ye. 2000. "How Well Have Taxable Investors Been Served in the 1980s and 1990s?" *Journal of Portfolio Management* 26, no. 4 (Summer): 84–93.

Bergstresser, Daniel, and Jeffrey Pontiff. 2013. "Investment Taxation and Portfolio Performance." *Journal of Public Economics* 97: 245–57.

Berkin, Andrew L., and Christopher G. Luck. 2010. "Having Your Cake and Eating It Too: The Before- and After-Tax Efficiencies of an Extended Equity Mandate." *Financial Analysts Journal* 66, no. 4: 33–45.

Berkin, Andrew L., and Jia Ye. 2003. "Tax Management, Loss Harvesting and HIFO Accounting." *Financial Analysts Journal* 59, no. 4: 91–102.

Bessembinder, Hendrik. 2018. "Do Stocks Outperform Treasury Bills?" *Journal of Financial Economics* 129, no. 3: 440–57.

Cembalest, Michael. 2014. "The Agony and the Ecstasy: The Risks and Rewards of a Concentrated Stock Position." J.P. Morgan Eye on the Market.

Constantinides, George M. 1983. "Capital Market Equilibrium with Personal Tax." *Econometrica* 51, no. 3 (May): 611–36.

Constantinides, George M. 1984. "Optimal Stock Trading with Personal Taxes: Implications for Prices and the Abnormal January Returns." *Journal of Financial Economics* 13, no. 1 (March): 65–89.

Dammon, Robert M., Kenneth B. Dunn, and Chester S. Spatt. 1989. "A Reexamination of the Value of Tax Options." *Review of Financial Studies* 2, no. 3: 341–72.

Geddes, Patrick. 2011. "The Pursuit of After-Tax Returns: Indexed Exchange-Traded Funds vs. Indexed Separately Managed Accounts." *Investments & Wealth Monitor* (September/October): 38–43.

Geddes, Patrick, Lisa R. Goldberg, and Stephen W. Bianchi. 2015. "What Would Yale Do If It Were Taxable?" *Financial Analysts Journal* 71, no. 4: 10–23.

Geddes, Patrick, and Robert Tymoczko. 2019. "Indexed ETFs vs. Indexed Separately Managed Accounts: A User's Guide." Aperio Group.

Goldberg, Lisa R., Pete Hand, and Taotao Cai. 2019a. "Tax-Managed Factor Strategies." *Financial Analysts Journal* 75, no. 2: 79–90.

Goldberg, Lisa R., Pete Hand, and Taotao Cai. 2019b. "Rewards and Risks of Loss-Harvesting Strategies." *Investments & Wealth Monitor* (May/June): 36–43.

Israel, Ronen, and Tobias J. Moskowitz. 2012. "How Tax Efficient Are Equity Styles?" Fama-Miller Working Paper No. 77, Chicago Booth Paper No. 12-20.

Jeffrey, Robert H., and Robert D. Arnott. 1993. "Is Your Alpha Big Enough to Cover Its Taxes?" *Journal of Portfolio Management* 19, no. 3 (Spring): 15–25.

Santodomingo, Rey, Vassilii Nemtchinov, and Tianchuan Li. 2016. "Tax Management of Factor-Based Portfolios." *Journal of Index Investing* 7, no. 2: 78–86.

Sialm, Clemens, and Nathan Sosner. 2018. "Taxes, Shorting, and Active Management." *Financial Analysts Journal* 74, no. 1: 88–107.

Stein, David M., Hemambara Vadlamudi, and Paul W. Bouchey. 2008. "Enhancing Active Tax Management through the Realization of Capital Gains." *Journal of Wealth Management* 10, no. 4: 9–16.

Wilcox, Jarrod, Jeffrey E. Horvitz, and Dan diBartolomeo. 2006. *Investment Management for Taxable Private Investors*. The Research Foundation of CFA Institute.

Appendix A: Decomposition of After-Tax Active Return

After-tax active return:

ATAR = P(after tax) – B(after tax)

Pre-tax active return:

PTAR = P(pre-tax) – B(pre-tax)

Tax alpha:

TA = ATAR – PTAR

Appendix B: Objective for Standard Loss Harvesting

Our standard loss-harvesting strategy minimizes:

$$f(h) = (h - h_B)^T(\lambda_D D + \lambda_F X F X^T)(h - h_B) + \lambda_{TC} TC(h, h_0) + \lambda_T T(h, h_0, r_{ST}, r_{LT})$$

where,

λ_F	Common factor risk aversion
λ_D	Specific risk aversion
λ_{TC}	Transaction cost multiplier
λ_T	Tax multiplier
h	Portfolio holding weights
h_0	Initial portfolio weights
h_B	Benchmark holding weights
D	Specific covariance matrix

F	Factor covariance matrix
r_{ST}	Short-term tax rate
r_{LT}	Long-term tax rate
$TC(h, h_0)$	Transaction cost function
$T(h, h_0, r_{ST}, r_{LT})$	Tax liability function

Appendix C: Security Scalar

In any period, the selection of lots for long-term gains for realization is defined in terms of the Security Scalar, a dynamic threshold that depends on market regime, lot volatility, investor type, and in some cases, age of strategy. An important component of the Security Scalar is the Tax-Rate Differential (TRD), defined:

$$TRD = \frac{ST\ Rate}{LT\ Rate} - 1.$$

In mathematical terms,

$$Security\ Scalar = disposition\ constant \times \frac{tax\ rate\ differential}{default\ differential} \times \frac{security\ vol}{Long\ term\ market\ vol}$$

where:

$$default\ differential = fed\ only\ tax\ rate\ differential$$

In our empirical study, we set the disposition constant to be 0.25 for estate/donation portfolios and 0.50 for liquidation investors. This choice leads to a higher Security Scalar, and hence greater gain realization, for liquidation investors, for whom the impact of cost basis at the end of the investment horizon is salient. These values can be tuned to individual investor needs. A higher Tax-Rate Differential also leads to a higher Security Scalar. The positive relationship between the value of tax-arbitrage and Tax-Rate Differential is derived in Appendix E. It makes sense, therefore, to allow more gain harvesting (by raising the Security Scalar) when the Tax-Rate Differential is higher. Finally, its positive dependence on individual security volatility elevates the Security Scalar in more volatile markets and for more volatile securities. We set the long-term market volatility to be 22% per year, which was the average month-end forecast volatility of the S&P 500 Index of the past 10 years.

We illustrate the effect of the Security Scalar with a hypothetical example based on California tax rates. We set the disposition constant to 0.25 and the long-term market volatility to 22%. Our example focuses on six long-term lots with unrealized gains in a hypothetical estate/donation portfolio. Lots 1 and 2 belong to low-volatility security ABC. We set ABC's volatility close to long-term market volatility, as is typical in financial markets.[15] Lots 3 and 4 correspond to medium-volatility security DEF. Lots 5 and 6 correspond to security XYZ, which is highly volatile.

The Security Scalar for ABC Lots 1 and 2 is relatively low, primarily due to ABC's low volatility. A lower Security Scalar discourages gain realization, and that is desirable for lower-volatility lots since they are less likely to generate short-term losses. For DEF Lots 3 and 4, the Security Scalar is lower than the disposition constant, despite DEF's volatility being higher than

[15] As a result of diversification, the volatility of the lowest-risk security of an index is close to (and often higher than) index volatility.

long-term market volatility. This is because California rates are less favorable than federal only for tax-rate arbitrage. The reason for this dynamic is discussed in Appendix E. The Security Scalar for XYZ Lots 5 and 6 is large due to XYZ's high volatility, encouraging the realization of long-term gains.

As shown in Table 10, the long-term gains in Lots 1, 3, and 5 were too large to be realized in a tax-rate arbitrage strategy, since their Lot Appreciations exceeded their Security Scalars. In contrast, long-term gains for Lots 2, 4 and 6 were realized in the hope of harvesting short-term losses.

Disposition Constant	0.25		
Long-Term Market Vol.	22%		
Example ST Rate	54.1%	Fed ST Rate	40.8%
Example LT Rate	37.1%	Fed LT Rate	23.8%

Lot Number	Security	Age of Lot (Days)	Long-Term Market Vol.	Security Volatility	**Security Scalar**	Basis	Price	**Lot Appreciation**	**Liquidate Lot to Take Gains?**
1	ABC	400	22%	21%	15%	$10	$15	50%	**No**
2	ABC	500	22%	21%	15%	$14	$15	7%	**Yes**
3	DEF	400	22%	31%	23%	$39	$50	28%	**No**
4	DEF	500	22%	31%	23%	$45	$50	11%	**Yes**
5	XYZ	400	22%	75%	55%	$105	$180	71%	**No**
6	XYZ	500	22%	75%	55%	$125	$180	44%	**Yes**

Table 10: *Top panel: Settings for our hypothetical example. Bottom panel: Security Scalars, Lot Appreciations, and their components for Lots 1—6.*

Appendix D: Generating Hypothetical Ossified Portfolios

In this study, we construct an ossified portfolio to launch a strategy turning the clock back five years from our target start date. We launch a standard, loss-harvesting strategy on this earlier date, and the result after five years is the starting portfolio for an ossified portfolio run. Because we dedicate some of our data to generating ossified portfolios, the time period for the empirical study of ossified portfolios begins later, in June 2000, than for the study of our cash-launched portfolios, which begins in June 1995.

Appendix E: Valuing Tax-Rate Arbitrage and Its Relationship to the Tax-Rate Differential

Suppose an investor realizes a long-term capital gain at tax rate L and immediately reinvests the proceeds of the sale in a stock with the hope of realizing a short-term capital loss at tax rate S. If P denotes stock price, then the transaction is effectively an American put option with payoff $S(P – P0)^+$. A lower bound for the value of the transaction is the Black-Scholes value V of a European put option with a term of one year times the S. Since the cost of the transaction is the difference between today's price and cost basis times the long-term rate, a lower bound for the return on the transaction is:

$$SV/(L(P0 - C)) - 1 = (TRD + 1)*V/(P0 - C),$$

where the Tax-Rate Differential (TRD) is defined above.

Appendix F: Portfolio Value Decomposition and Measurement of Ossification

The market value of a portfolio can be decomposed into two components: basis and unrealized net gains. Basis represents the capital used to establish all the positions in a portfolio, and unrealized net gains represents the total profit or loss in the event of full liquidation.

Unrealized net gains can be further decomposed into unrealized gains and unrealized losses, with the former tallying gains from all profitable positions and the latter tallying losses from all losing positions.

Finally, basis, unrealized gains and unrealized losses can each be broken down into short-term and long-term components. A lot is a short-term position if it has been held in a portfolio for, at most, 365 days; otherwise, the lot is a long-term position. This leads to the tax-aware decomposition of portfolio market value:

- Short-Term Basis
- Long-Term Basis
- Unrealized Short-Term Gain
- Unrealized Long-Term Gain
- Unrealized Short-Term Loss
- Unrealized Long-Term Loss

<table>
<tr><th colspan="6">**Portfolio Value**</th></tr>
<tr><td colspan="2" rowspan="2">Basis</td><td colspan="4">Unrealized Net Gain</td></tr>
<tr><td colspan="2">Unrealized Gain</td><td colspan="2">Unrealized Loss</td></tr>
<tr><td>ST Basis</td><td>LT Basis</td><td>Unrealized ST Gain</td><td>Unrealized LT Gain</td><td>Unrealized ST Loss</td><td>Unrealized LT Loss</td></tr>
</table>

These six components sum to portfolio market value, as illustrated in the numerical example in Table 11.

Lot Number	Age (Days)	Short or Long Term	Number of Shares	Current Price	Basis	Unrealized Net Gain per Share	Current Market Value	Total Basis	Unrealized Loss	Unrealized Gain
1	1	ST	10	$10	$25	–$15	$100	$250	–$150	
2	10	ST	10	$20	$10	$10	$200	$100		$100
3	400	LT	10	$50	$75	–$25	$500	$750	–$250	
4	500	LT	10	$250	$100	$150	$2,500	$1,000		$1,500
Portfolio Level							$3,300	$2,100	–$400	$1,600

Portfolio Level	Basis	Unrealized Loss	Unrealized Gain Loss
Short-Term	$350	–$150	$100
Long-Term	$1,750	–$250	$1,500
Total	$2,100	–$400	$1,600

Table 11: *Tax-aware decomposition of the market value of a hypothetical portfolio into six components.*

The tax-aware decomposition of portfolio market value leads to two measurements of portfolio ossification.

Short-term basis percentage (featured in Table 9) is a portfolio's short-term basis divided by its current value. A larger short-term basis percentage indicates a higher potential for short-term loss realization, and thus a lower ossification level.

Total basis percentage is a portfolio's (short plus long) basis divided by its current value. An alternative to short-term basis percentage for measuring ossification, total basis percentage gauges the likelihood that lots in a portfolio will become losses. A lower total basis percentage indicates a lower likelihood of loss harvesting. This metric is commonly used to set tax alpha expectations. For the hypothetical ossified portfolios used to launch strategies in our empirical study, average total basis percentage was approximately 55%, on average, at strategy inception.

Appendix G: Glossary

ATAR: after-tax active return

ATBAT: After-Tax Back-Testing Analysis Tool

PTAR: pre-tax active return

TA: tax alpha

TAVA: tax arbitrage value added

TE: tracking error

TLH: tax-loss harvesting

TRD: Tax-Rate Differential