Robo-Advising: Personalization and Goals-Based Investing

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CDAR Risk Seminar

University of California Berkeley

March 15, 2022

Introduction

What is Robo-Advising?

Robo-advising is algorithmic-driven financial planning

1. Tailored to clients' risk preferences and characteristics
2. Easy to implement – Automatic execution
3. Democratize financial professional advice

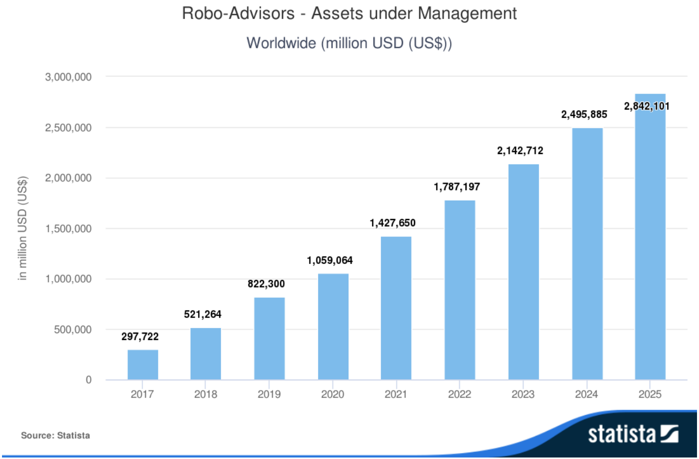
Robo-Advising Firms

- First robo-advisors launched in the wake of the 2008 financial crisis (Betterment, Wealthfront, Personal Capital, . . .)
- Current offerings include:
  - Affordable portfolio management (fees, account size, etc.)
  - Full automation (portfolio construction and rebalancing)
  - Tax-loss harvesting
- Becoming both day-to-day and long-term money managers

Robo-Advisor Strategies

- Robo-advisors differ in terms of investment strategies
- Wealthfront uses a long-term investing platform grounded on mean-variance portfolio maximization
- Betterment adopts a portfolio management approach that focuses directly on investors' financial goals

- Manage around $1.5 trillion in the United States (2021)



Pros of Robo-Advising

- Low entry-barriers (financial inclusion)
  - Low fees ($< 0.2\%$ vs. $> 1\%$)
  - Low or zero investment minimum (≈ 0 vs. $> \$100,000$)
- Unbiased financial advice
  - Any two customers treated the same by the algorithm
- Highly sophisticated algorithmically in executing a given task
  - Portfolio rebalancing, tax-loss harvesting, etc.
- Transparent financial advice
  - Decisions can be explained to investors and regulators

Cons of Robo-Advising

- Lack of "human touch" / algorithm aversion
  - Human advisors provide peace of mind (Rossi and Utkus (2019))
- Limited customization
- Not yet been tested (both operationally-wise and performance-wise) in diverse economic circumstances

Robo-Advising in the Asset Management Space (US)

ROBO-ADVISOR

BUSINESS-TO-CONSUMER (B2C)

WiseBanyan, wealthfront, Aspiration, alkanza, moneyfarm, WealthBar, nutmeg, 挖财 我的资产管家 wacai.com, FOLIO, WEALTHNAVI, scalable CAPITAL, indexa capital, Nest Wealth, Wealthsimple, sixpark, ELLEVEST, vaamo, Goalwise, M, invest.com, PERSONAL CAPITAL

BOTH B2B & B2C

Betterment, CASHBOARD, MESITIS, SIGFIG, EQIS, 懒投资 LANTOUZI.COM

BUSINESS-TO-BUSINESS (B2B)

THIRD FINANCIAL, trizic, LIQID, fount, QUANTIFEED, AdvisorEngine

ROBO-RETIREMENT

B2C

RobustWealth, FeeX, blooom

B2B

Guideline, Vestwell, forusall, VAULT RETHINKING RETIREMENT

B2B & B2C

nextcapital

MICRO-INVESTING

B2C

acorns Invest the Change, STASH

PORTFOLIO MANAGEMENT

B2C

RISBEEVIE

B2B

ADDEPAR, iQuantifi Life Solved., Quovo, Advizr, Jacobi

FINANCIAL SERVICES SOFTWARE

B2B

PLAID, STARBURST LABS, investcloud, riskalyze, scivantage

INVESTING TOOLS

B2C

nextmarkets, nerdwallet, huddlestock, openfolio, INSTAVEST

B2B & B2C

YCHARTS, TradingView, StockTwits, wikifolio, Alpaca, estimize

B2B

ALGOMI, LMRKTS, CANALYST, harvest, KENSHO, TRACKINSIGHT, OpenGamma

DIGITAL BROKERAGE

B2C

robinhood, YOMONI, 富途證券 WWW.FUTU5.COM, tastyworks., KAPITALL, eToro, raisin., rubicoin, SILICON INVESTING, SAVEDO, LOYAL3, STOCKPILE, DriveWealth, SOGOTRADE, 8

B2B

ARTIVEST, TRUMID, OPENDOOR, electronifie

B2B & B2C

tradier, motif INVESTOR

Modeling Robo-Advising: Human-Machine Interaction System

Model of Robo-Advising

- Investment process accounts for **dynamic** risk preferences and **repeated** interaction between client and robo-advisor
- Proposed framework captures features of the most prominent robo-advising firms
  - High portfolio personalization
  - Low and indirect client involvement

Human-Machine Interaction

- Client communicates life events, investment goals, etc.
- Robo-advisor processes input from the client and translates into the "client's utility function"
- Robo-advisor solves an optimal portfolio allocation problem on behalf of the client, and makes an investment recommendation
- Depending on the robo-advisor, the client may have the option of overriding the robo-advisor's portfolio recommendations

Robo-Advisor Under Mean-Variance Criterion

- Based on joint work with Thaleia Zariphopoulou and Sveinn Olafsson
- Communication between client and robo-advisor modeled as a human-machine interaction system
- Robo-Advisor solves a discrete-time **adaptive** mean-variance optimization problem for the client
- Robo-advisor updates the risk-reward trade-off coefficient based on information communicated from client

Beyond Classical Frameworks

- Stochastic and dynamic client's preferences introduce technical challenges
  - Uncertainty over the realization of a client's utility function
- Adaptive Control
  - System being controlled maintains the same dynamics, but optimality criterion changes in response to data generated by the system
  - Client's utility adapts to incoming information
  - Contrasts with classic adaptive control, where optimality criterion stays the same, but system dynamics adapts to incoming changes in the environment (Astrom and Wittenmark (1989))

Human-Machine Interaction

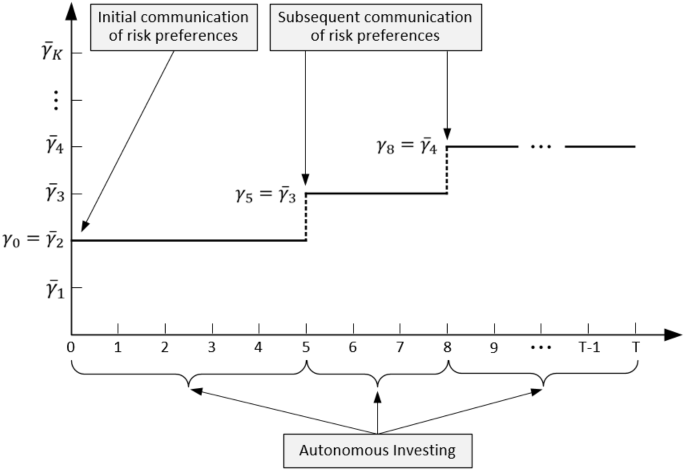
Client:

- Dynamic risk aversion process of client's intrinsic risk aversion: $(\gamma_n^C)_{n\geq 0}$
- Provides information to the robo-advisor at interaction times

Robo-Advisor:

- Constructs a *model* of the client's risk aversion: $(\gamma_n^R)_{n\geq 0}$
- Differs from $(\gamma_n^C)_{n\geq 0}$ due to imperfect human-machine interaction
  - Changes to client's demographics only observed at interaction times
  - Information γ communicated by client may be affected by behavioral biases
- Designs an optimal investment strategy

The Interaction System



Market Dynamics

- Risky asset $(S_n)_{n\geq 0}$ and a risk-free asset $(B_n)_{n\geq 0}$,

$$S_{n+1} = (1 + Z_{n+1}(Y_n))S_n$$
$$B_{n+1} = (1 + r(Y_n))B_n$$

- Price dynamics modulated by a Markov regime switching model of economic conditions $(Y_n)_{n\geq 0}$ (Hamilton [1989])
- Given $Y_n = y$, the risk-free rate is $r(y) \geq 0$, and the risky asset's return has mean $\mu(y) > r(y)$, and variance $\sigma^2(y) > 0$
- Probability space $(\Omega, \mathcal{F}, \mathbb{P})$ also supports a sequence $(\epsilon_n)_{n\geq 1}$ of random variables, independent of $(Y_n)_{n\geq 0}$ and $(Z_n)_{n\geq 1}$
- Filtration $(\mathcal{F}_n)_{n\geq 0}$ defined by $\mathcal{F}_n = \sigma(Y_{(n)}, Z_{(n)}, \epsilon_{(n)})$

Human-Machine Interaction

- The interaction schedule $(T_k)_{k \geq 0}$ is an increasing sequence of **stopping times** with respect to the filtration $(\mathcal{F}_n)_{n \geq 0}$
- Interaction can be triggered by any combination of client-specific events, economic state changes, and market events
- Define $(\tau_n)_{n \geq 0}$ where $\tau_n := \sup\{T_k : T_k \leq n\}$ is the most recent interaction time occurring prior to or at time n
- Deterministic schedule: The sequence $(T_k)_{k \geq 0}$ is given by

$$T_k = k\phi, \quad k \geq 0$$

where $\phi \geq 1$ is the time between consecutive interactions

Robo-Advisor's Model of Client

- At each interaction time T_k, the client communicates her risk aversion coefficient γ_{T_k} to the robo-advisor
- Robo-advisor's filtration $(\mathcal{F}_n^R)_{n\geq 0}$ is generated by

$$D_n := \left(Y_{(n)}, Z_{(n)}, \tau_{(n)}, \gamma_{(n)}\right)$$

  which consists of market information $\left(Y_{(n)}, Z_{(n)}\right)$ and information from client-interaction $\left(\tau_{(n)}, \gamma_{(n)}\right)$
- Robo-advisor's model of the client's risk aversion $(\gamma_n^R)_{n\geq 0}$ is a process adapted to the robo-advisor filtration:

$$\gamma_n^R := \gamma_n^R(D_n)$$

- The filtration $(\mathcal{F}_n^R)_{n\geq 0}$ grows with the frequency of interaction

Adaptive Mean-Variance Criterion

- For a fixed horizon $T \geq 1$, the robo-advisor maximizes a **dynamic mean-variance** objective

$$J_n(x, d, \pi) := \mathbb{E}_{n,x,d}\left[\frac{X_T^\pi - X_n}{X_n}\right] - \frac{\gamma_n^R}{2} Var_{n,x,d}\left[\frac{X_T^\pi - X_n}{X_n}\right]$$

  where π is a self-financing strategy
- Initial condition fixes the robo-advisor's information set:
  - $\mathbb{P}_{n,x,d}(\cdot) := \mathbb{P}(\cdot | X_n = x, D_n = d)$
- Sequence of objective functions $(J_n)_{0 \leq n < T}$
  - Robo-advisor's model of client's risk preferences $\gamma_n^R > 0$ adapts to market, economic, and client-communicated information

Optimal Investment

Theorem

The optimal proportion of wealth invested in the risky asset at time n is

$$\pi_n^* = \frac{1}{\gamma_n^R} \frac{\mathbb{E}_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}{Var_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]} - R_{n+1} \frac{Cov_{n,d}(r_{n+1}^{\pi^*}, \widetilde{Z}_{n+1} r_{n+1}^{\pi^*})}{Var_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}$$

where $\widetilde{Z}_{n+1}$ is the excess market return at $n+1$, and

$$r_{n+1}^{\pi^*} = \frac{X_T^{\pi^*}}{X_{n+1}}$$

is the value of one dollar invested in the optimal portfolio between time $n+1$ and the terminal date T.

The allocation π_n^* depends on both the **current** *risk-return tradeoff γ_n^R its* **future dynamics** *through the conditional expectations.*

Structure of Optimal Strategy

$$\pi_n^* = \frac{1}{\gamma_n^R} \frac{\mathbb{E}_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}{Var_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]} - R_{n+1} \frac{Cov_{n,d}(r_{n+1}^{\pi^*}, \widetilde{Z}_{n+1} r_{n+1}^{\pi^*})}{Var_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}$$

- $\frac{1}{\gamma_n^R} \frac{\mathbb{E}_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}{Var_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}$: Standard single-period Markowitz strategy
- $\frac{Cov_{n,d}(r_{n+1}^{\pi^*}, \widetilde{Z}_{n+1} r_{n+1}^{\pi^*})}{Var_{n,d}[\widetilde{Z}_{n+1} r_{n+1}^{\pi^*}]}$: **intertemporal** hedging component
  - Incorporates the effect of market returns and economic conditions on the client's risk aversion

$$\widetilde{Z}_{n+1} \uparrow \implies \gamma_{n+1}^R \downarrow \implies \pi_{n+1}^* \uparrow \implies r_{n+1}^{\pi^*} \uparrow \implies Cov_{n,d}(\dots) > 0$$

  - Negative hedging demand for clients whose risk aversion is negatively correlated to market returns (cf. Basak and Chabakauri [2010])

Risk Preference Model

Client's risk aversion $(\gamma_n^C)_{n\geq 0}$ changes because of:

- Passage of time
  - 1st generation robo-advisors $\approx$ TDFs
- Shocks to demographics $(\epsilon_n)_{n\geq 1}$
  - Barsky et al. [1997], Guiso and Paiella [2008], ...
- Market returns and economic conditions
  - Both risk aversion and market Sharpe ratio (λ) are **countercyclical**
    - Higher at business cycle troughs than at peaks
    - Lettau and Ludvigson [2010], Campbell and Cochrane [1999]:

$$Y \downarrow \implies \gamma^C \uparrow \implies \lambda \uparrow \implies S \downarrow$$

  - Client reduces market exposure when the market Sharpe ratio is high

Risk Preference Model

- At an interaction time n, the risk aversion value communicated by the client is

$$\gamma_n = \gamma_n^C \gamma_n^Z$$

where γ_n^C is the client's risk aversion and, for $\beta \geq 0$,

$$\gamma_n^Z = e^{-\beta\left(\frac{1}{\phi}\sum_{k=n-\phi}^{n-1}(Z_{k+1}-\mu_{k+1})\right)}$$

captures client's behavioral biases (e.g., trend-chasing)

- The **bias** γ_n^Z is based on recent stock market performance:
  - Market outperforming the client's expectations: $\gamma_n^Z < 1$
  - Market underperforming the client's expectations: $\gamma_n^Z > 1$

Risk Preference Model

- The robo-advisor's inference of client's risk aversion is

$$\gamma_n^R = \mathbb{E}[\gamma_n^C | \mathcal{F}_n^R] \gamma_{\tau_n}^Z$$

  where γ_n^C is the client's risk aversion and $\gamma_{\tau_n}^Z$ is the bias realized at the previous time of interaction
- Updated in real time based on the passage of time, realized market returns, and changes in economic conditions

Portfolio Personalization

- Relative difference between the robo-advisor's model and the client's risk aversion process:

$$\mathcal{R}(\phi, \beta) := \mathbb{E}\left[\frac{1}{T}\sum_{n=0}^{T-1}\left|\frac{\gamma_n^R - \gamma_n^C}{\gamma_n^C}\right|\right]$$

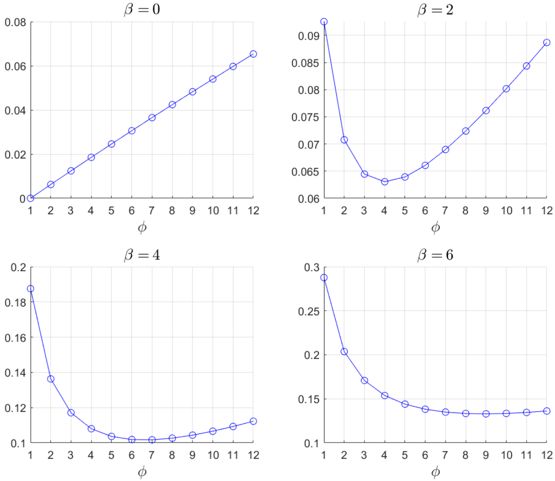
  - $\phi \geq 1$: time between consecutive interaction times
  - $\beta \geq 0$: strength of client's behavioral bias

- Robo-advisor faces a tradeoff between **information acquisition rate** and **accuracy of acquired information**

Proposition

1. *There exists a unique value of ϕ that minimizes $\mathcal{R}(\phi, \beta)$*
2. *Optimal value of ϕ is increasing in β*

Optimal Interaction Frequency



- Magnitude of behavioral bias increases with the interaction frequency

Modeling Robo-Advising: Goal-Based Investment

Goal-Based Investing

Quoting Robert Merton:

"Goal-based investing will be very important in the next decade. For example, if you have a goal of funding retirement or a benefit plan, you set the goal and manage it through a process called LDI (liability-driven investing). **If you follow a liability-driven goal, then regardless of whether your Sharpe ratio exceeds those of your competitors, you can outperform competitors who lose their focus on the goal.**

We will be driven to the idea of greater service by knowing the client better, understanding what the client really needs, getting the client to identify what the actual goal is, and then designing dynamic strategies that achieve that goal."

- *Q Group Panel Discussion: Looking to the Future.* Moderator: Martin Leibowitz. Panelists: Andrew Lo, Robert Merton, Stephen Ross, and Jeremy Siege

Beyond Portfolio Volatility

- Most academic research associates risk with the volatility of an investor's portfolio
- However, most investors associate risk with the probability of not attaining their goals
- **Important distinction between goal risk and volatility**:
  - Decreasing standard deviation in an underfunded investor's portfolio increases likelihood of not attaining goals

Goal-Based Wealth Management

Different tiers of investment goals

Goal-Based Wealth Management

Realize	Avoid	Success Probability
Dreams	Concerns	50%
		55%
		60%
Wishes	Worries	65%
		70%
		75%
Wants	Fears	80%
		85%
Needs	Nightmares	90%
		95%

Four tiers of goals (Brunel [2015])

Goal-Based Wealth Management

Investment goals (ranked among top three)

Response	
Retirement	46%
Planning a vacation	43%
Emergencies	40%
Purchase a new home	35%
Invest in/start a business	29%
Major renovation	23%
School/university fees	19%
Inheritance for my family	16%
Luxury purchases	13%
Wedding	8%
Philanthropy	4%
Other investment goals	24%

Source: Franklin Templeton

Handling Multiple Goals

- How to prioritize between **multiple goals**?
- Should an investor forgo an immediate goal in order to increase the chances of attaining future goals?
  - Relative importance of goals
  - Partial fulfillment of goals (e.g., buy a cheaper car)
- Efficiency gains from optimizing all goals in a single portfolio
  - Enables offsets between underfunded and overfunded goals

Asset-Liability Management

- Think of goals as the building blocks of a financial plan
- Based on **asset and liability management**, a technique widely used by institutional investors
  - Manage assets to meet future liabilities
- Within the client account, every goal has a target amount and a target date at which the client desires to meet the goal

Stochastic Control Framework

- Based on joint work with Yuchong Zhang
- Multiple goals for a single account
- Fungibility across goals
- Tradeoff between immediate goal consumption and savings to meet future goals

Client's Input

Client provides the following input:

- Initial wealth x_0
- Contribution rate l
  - E.g., fraction of each paycheck deposited to robo-advising account
- Goals and desired success rate (G_k, T_k, α_k), $k = 1, 2, \ldots, M$
  - G_k is the amount of k-th goal
  - T_k is the deadline of k-th goal, $T_1 < T_2 \cdots < T_M$
  - α_k is the client's specified (relative) probability that the k-th goal is achieved at T_k, $\sum_{k=1}^{M} \alpha_k = 1$

Fundedness and Wealth Process

- Client's wealth $X(t)$ allocated to N risky assets
  - Log-asset dynamics driven by an N-dimensional Brownian motion
- $\pi(t) = (\pi_0(t), \ldots, \pi_N(t))$ are the fractions of wealth invested in each of the $N+1$ assets
- At time T_k, the amount $G_k\theta_k$ is withdrawn by the client, where $\theta_k \in [0,1]$ is $\mathcal{F}(T_k)$-measurable:

$$X(T_k) = X(T_k^-) - G_k\theta_k$$

- θ_k is the **funding ratio** for the k-th goal
  - $\theta_k = 1$: the k-th goal is fully funded
  - $\theta_k < 1$: the k-th goal is partially funded with funding ratio θ_k

Goal-Based Optimization Problem

- Choose asset allocation π and funding ratio strategy θ to maximize the expected goal fundedness, weighted by their importance:

$$\mathbb{E}\Big[\langle\alpha,\theta\rangle\Big] = \mathbb{E}\Big[\sum_{k=1}^{M}\alpha_k\theta_k\Big]$$

- Require (π,θ) such that $X(t) \geq 0$
- No borrowing constraint: $\pi \geq 0$
  - Robo-advisors do not go short, and just invest client's money
- Can be extended to incorporate other user-specified constraints:
  - Maximal fraction of wealth allocated to risky assets (Betterment)

Value Function and Safety Level

- Solve stochastic control problem backward in time
- Value function

$$V(t,x) := \sup_{\pi,\theta} \mathbb{E}\Big[\sum_{k:T_k>t} \alpha_k \theta_k \Big| X(t^-) = x \Big]$$

- Safety level at time t denoted by $\mathfrak{s}(t)$
  - Smallest wealth level $X(t^-)$ such that all remaining goals can be fully funded with probability one
- V and $\mathfrak{s}$ can be obtained recursively

Dynamic Programming Approach I

- Final goal maturity:
  - $V(T_M, x) = \sup\{\alpha_M \theta_M : \theta_M \in [0,1],\ G_M \theta_M \le x\}$
  - All wealth used to fund the goal

$$\theta_M^*(x) = \frac{x \wedge G_M}{G_M}, \qquad V(T_M, x) = \alpha_M \theta_M^*(x), \qquad \mathfrak{s}(T_M) = G_M$$

- At intermediate goal maturities, solve a static optimization problem:

$$V(T_k, x) = \sup_{\theta_k} \left\{\alpha_k \theta_k + V\left(T_k^+, x - G_k \theta_k\right)\right\}$$

  such that $\theta_k \in [0,1]$, and $G_k \theta_k \le x$

- The optimal fundedness θ_k^* quantifies the decision between fulfilling an immediate goal versus saving for future liabilities

Dynamic Programming Approach III

- Between intermediate goal maturities, find $V(t,x)$ and $\mathfrak{s}(t)$ for $T_{k-1} < t < T_k$:
  (i) The safety level satisfies an ODE

$$\mathfrak{s}'(t) = I + r\mathfrak{s}(t)$$

  on $(T_{k-1}, T_k]$ with terminal condition $\mathfrak{s}(T_k)$

  (ii) The allocation π satisfies an **HJB** equation

$$V_t + \sup_{\pi} \left\{ V_x(I + rx + x\pi(\mu - r1_N)) + \frac{1}{2} V_{xx} x^2 \|\pi\Sigma\|^2 \right\} = 0$$

  on $(T_{k-1}, T_k]$ with terminal condition $V(T_k, x)$ and boundary cond.

$$(V_t + IV_x)(t,0) = 0 \quad \text{and} \quad V(t, \mathfrak{s}(t)) = \sum_{k:T_k \geq t} \alpha_k$$

- Value function can be characterized as the unique viscosity solution of the HJB **nonlinear** equation

Numerical Setting

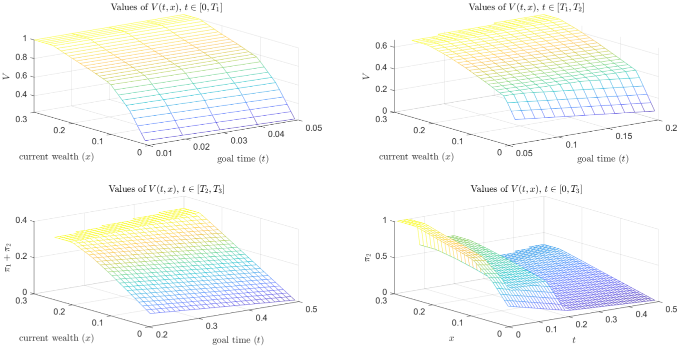
Consider two different profiles of clients:

- Generation Z (1997 - 2012)
  - Top investment goals are short term: buy a car, finance a vacation, pay off debt
- Millennials (1981 - 1994/6)
  - Top financial priorities are saving for retirement and saving for "life milestones and future goals"

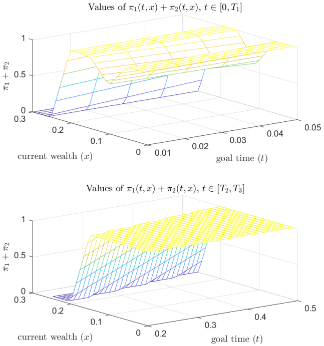
Specification of Client Parameters

- Generation Z: equal priority goals
  - Buy car: amount = \$30,000, Time = 5 months
  - Finance a vacation: amount = \$10,000, Time = 2 years
  - Pay off debt: amount = \$250,000, Time = 5 years
- Millennials:
  - Buy house: priority \$73%, amount = \$250,000, Time = 5 years
  - Retirement: priority \$75%, amount = \$80,000 x 20 = \$1,600,000, Time = 35 years

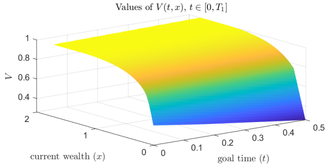
Value Function: Generation Z

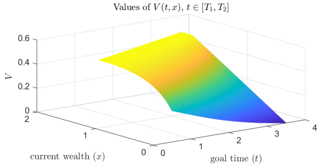


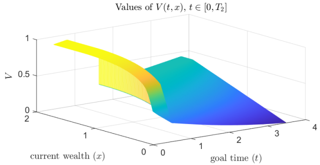
Investment Strategy: Generation Z



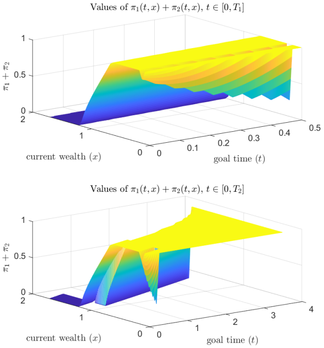
Value Function: Millennials







Investment Strategy: Millennials



Dialogue with Client

- Expected fundedness of remaining goals:

$$F_t := \mathbb{E}\Big[\sum_{k:T_k>t} \alpha_k \theta_k^* \Big| \mathcal{F}_t\Big]$$

  may deviate from the initial target $F_t^{(0)} := \mathbb{E}\big[\sum_{k:T_k>t} \alpha_k \theta_k^* \big| \mathcal{F}_0\big]$

- $F_t < F_t^{(0)}$: goals are underfunded relative to initial target
  - Suggest to client to increase contribution I or lower goal amounts
- $F_t \approx F_t^{(0)}$: goal funding is "on track"
- $F_t > F_t^{(0)}$: goals are overfunded relative to initial target
  - Suggest to client to increase goal amounts or divert part of the contribution I to other consumption

Thank you!