A propagation model to quantify business interruption losses in supply chain networks

Hubeyb Gurdogan

UC Berkeley, CDAR

UC Berkeley, CDAR Seminar
April 5, 2022

Joint work with:

Nariman Maddah
Swiss Re Institute

Reyhaneh Mohammadi
Northeastern University

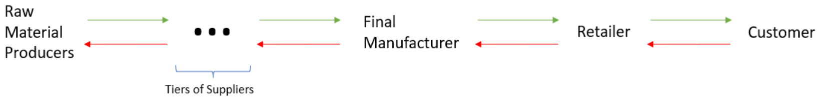
Elena Pesce
Swiss Re Institute

Alicia Montaya
Swiss Re Institute

Jeffrey Bohn
One Concern Inc.
UC Berkeley, CDAR

Katherine Dalis
One Concern Inc.

Supply Chain Networks



- A network of companies connected via supply and demand relationships.
- Material travel forward through supply links.
- Information travel backward through demand links.

So can the risk.

Business Interruption

- Reduction or complete stop of the business operations due to,
  - Shortage in demand
  - Shortage of supplies
  - Loss of functionality at the business location.
- Down time: duration of interruption.
- A local business interruption can travel both forward and backward, creating a cascading effect.

Reasons for Global Scale Disruptions

- Todays supply chains are,
  - Highly interconnected
  - Globalized
  - Increasingly digital

All together boosts the propagation of the disruption making it harder to quantify the effect of a local shock to the system.

Counter Measures for Companies on the Shelf

- Creating redundancies in their network.
- Increasing the visibility, transparency
- Increasing operational agility of the firms.
- Investing on storage capabilities and buffering strategies.

And then ideally hedge the remaining risk at the market:

Business interruption insurances(BI,CBI and NDBI) exist but...

Worries of the Insurer & Potential Solutions

- Low visibility of the client's supply chain

- Lack of propagation models that calculates the expected loss conditioned to a shock to the system(ex: an earthquake) while incorporating the network effects.

  - An essential component of this gap is the down time estimates of the effected locations that incorporates the lifelines.

Public/Private Data

&

Modeling Effort

&

One Concern inc. capabilities on down time estimates.

The Modeling Effort

Fully Decomposed Supply Chains

We focus on a reasonable subset of the supply chains, that we call Fully Decomposed Supply Chain(FDSN) satisfying the conditions:

C1. Each plant in the network produces a single material/product and each plant in the network has a direct or indirect contribution to the final end product.

C2. No product requires itself for its production at any stage of the manufacturing process.

C3. There are no loops in the supply chain network. In other words, a supplier can never appear to be the buyer of the plants that sit at later stages of the production.

Modeling the FDSN process - 1

We then model an instance of the FDSN by a triple (G, F, C) where,

- $G = (V, E)$ is a graph modeling the plants(V) and the trading relationships(E). The edge set E obeys analog of the conditions **C1**, **C2** and **C3**.
- The vertex set is made up as $V = \{v_0\} \cup \left(\bigcup_{j=1}^{k} V_j \right)$ where v_0 is modeling the final end node and V_k is modeling the set of plants producing the same product.
- $C : E \to [0, 1]$ records the instant trading amounts. These are normalized so that each product's output totals to 1 units. With this normalisation, 1 unit of final end product depends on 1 unit of each product type in the network
- $F : V \to [0, 1]$ models the functionality of each plant.

Modeling the FDSN process - 2

Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $(\mathcal{F}_t)_{t\in[0,T]}$ a filtration of the σ-algebras $\mathcal{F}_t$ we define an adapted stochastic FDSN process by $\mathcal{N}_t = (G, F_t, C)$ where,

- $F_t : \Omega \times V \to [0, 1]$ such that for each $v \in V$, $F_t(\_\_, v)$ is a stochastic process adapted to $(\mathcal{F}_t)_{t\in[0,T]}$.
- $\mathcal{N}_t(\omega) = (G, F_t(\omega, \_\_), C)$ is required to be a FDSN instance for every choice of $t \in [0, T]$ and $\omega \in \Omega$.

Rules of Diffusion & Production rate of the plants - 1

We require two rules for a node to produce;

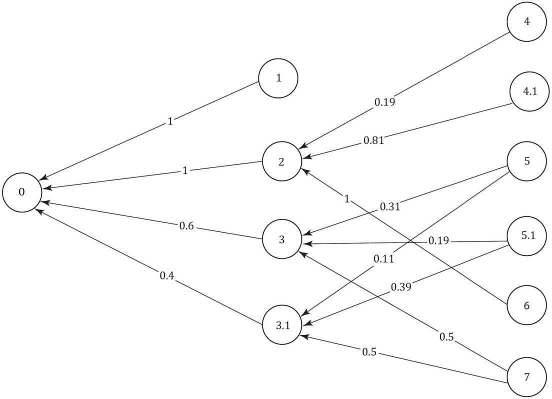
- They need to be functional
- They should have non-zero supplies for all its product dependencies.

We define the production rate potential of each plant as a stochastic process $P_t : \Omega \times V \to \mathbb{R}^+$ via the following recursion,

$$P_t(\omega, v) = F_t(\omega, v) \;\; \forall v \text{ s.t. } \mathcal{S}_v = \emptyset,$$

$$P_t(\omega, v) = \min\left(F_t(\omega, v), \left\{ \sum_{\hat{v} \in S_v \cap V_i} \left(P_t(\omega, \hat{v}) \Big(\frac{C(\hat{v}, v)}{\sum_{\hat{v} \in S_v \cap V_i} C(\hat{v}, v)} \Big) \right) \;\middle|\; i = 1, 2, .., k \right\} \right)$$

Rules of Diffusion & Production rate of the plants - 2



This figure exemplifies an FDSN instance taken at a time $t \in [0, T]$. Lets make an exercise of calculating the production rate of the final node for the case where the nodes 3 and 5 are fully disrupted and all of the remaining nodes are functional.

Modeling the Impact - NatCat friendly way

- We consider the arrival of a hazardous event H at time 0 of $[0, T]$ and condition all the analysis on this event.
- Let $d_v \in \mathbb{R}^+ \cup \{0\}$ be a random variable that is the down time associated with each node v. If the node is not impacted the down time is set to 0.
- The impact is then built in the functionality process as,

$$F_t(\omega, v) = 1_{\{t \geq d_v(\omega)\}}$$

Loss ratio in production - 1

- Each $\omega \in \Omega$ will realize a random list of downtimes. Let $d_0, d_1, ..., d_{|V|-1}$ be the ordered statistics of the realized downtimes $\{d_v(\omega)\}_{v \in V}$ with $0 \leq d_0 \leq d_1 \leq ... \leq d_{|V|-1} \leq T$
- For each $v \in V$, the functionality $F_t(\omega, v)$ remain static on the intervals $[d_{i-1}, d_i)$
- The same feature is carried over to the production rate $P_t(\omega, v)$ through the recursion.

Loss ratio in production - 2

Hence the loss ratio in total production can be defined and simplified as,

$$\mathcal{L}(\omega) = \frac{1}{T}\int_0^T (1 - P_t(\omega, v_0))dt = T - \sum_{i=1}^{|V|-1} (d_i - d_{i-1}) P_{d_{i-1}}(\omega, v_0).$$

The expected loss ratio in total production conditioned on the hazard H would be then,

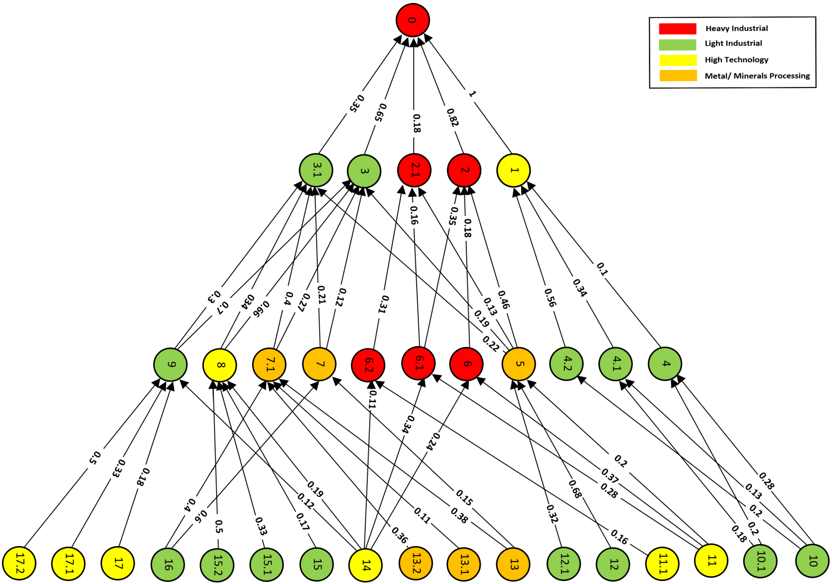
$$\mathbb{E}[\mathcal{L}|H] = \mathbb{E}\Big[T - \sum_{i=1}^{|V|-1} (d_i - d_{i-1}) P_{d_{i-1}}(\omega, v_0)\Big| H\Big].$$

Implementation of the Propagation Model

Summary of the Implementation

- The numerical implementation will focus on earthquakes.
- We fix a synthetically generated FDSN process and use HAZUS earthquake model for the hazard component.
- For each event H from a catalog of events $\mathcal{H}$ and for each building type, business type HAZUS eartquake model provides a conditional distribution of the down times.
- We populate a sample of down times and calculate the associated loss ratio L for each and perform Monte-Carlo simulation to obtain $\mathbb{E}[L|H]$ for each $H \in \mathcal{H}$

A Synthetic Network



Event Catalog & *QuarterlyExpectedLossRatios*

Table: The Event Catalog $\mathcal{H}$

H_1	Affected Node	2.1	4.1	7.1	10	13.2	15.1
	PGA	0.220	0.190	0.250	0.140	0.140	0.100
H_3	Affected Node	4	4.1	7	9	10.1	15.1
	PGA	0.3037	0.194	0.1976	0.4222	0.1752	0.2417
H_7	Affected Node	2	4.2	7	9	11	12
	PGA	0.261	0.3514	0.1639	0.1152	0.4986	0.3995
H_{10}	Affected Node	2.1	6	8	11.1	-	-
	PGA	0.3530	0.2060	0.4570	0.4120	-	-

Table: Quarterly Expected Loss Ratio Corresponding to Events in Event Catalog

Event	Capacity	Q1	Q2	Q3	Q4
H_1	Normal	19.4	3.2	1.6	0.0
H_3	Normal	45.4	8.4	0.0	0.0
H_7	Normal	64.3	27.6	13.3	2.8
H_{10}	Normal	61.9	26.0	11.1	2.0

Questions ?