Portfolio Optimization in an Uncertain World

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Act Think Impact

In this talk

- I point at a flaw in Modern Portfolio Theory (MPT).
- I propose a remedy.
- I connect with many studies that do so.

Modern Portfolio Theory

Mean-variance optimization*

Maximize the Utility of portfolio *x*:

$$U_x = x^T \mu \; - \; \lambda \cdot x^T \, V \, x$$

Hypothesis:

expected returns μ and covariance matrix V are known with <u>certainty</u>.

7. I.e., we assume that the investor does (and should) act as if he had probability beliefs concerning these variables.

- I focus on the <u>uncertainty</u> around V.

*Markowitz (1952)

Flaw in MPT

Taking matrix *V* for certain creates an imaginative world where

- prices obey to predefined laws
- there are no unforeseen price shocks

Such a world doesn't exist.

The more-often heard critic

"*Markowitz optimization is vulnerable to estimation error*"

is misleading, for it suggests there to be a true *V*.

The flaw is not to do with estimation error, it is the assertion of certainty.

Is there a world of certainty?

Babylonian times

- limited means of communication
- no Central Banks

High entropy

Post-war Singapore

- planned economy
- efficient governance

Low entropy

Degrees of uncertainty

utter darkness | today's world | MPT

- uniform distribution
- unforeseeable price shocks

- Maximise diversification

∞ entropy

- Gaussian distribution
- foreseeable shocks

- Minimise variance

0 entropy

Investment example

The equity-bond split

$\sigma_{EQ} = 15\%$

$\sigma_{BO} = 5\%$

$corr(_{EQ,BO}) = 0$

Utter darkness	Today's world	MPT
$X_{EQ} = 50\%$	$50\% > X_{EQ} > 10\%$	$X_{EQ} = 10\%$
$X_{BO} = 50\%$	$50\% < X_{BO} < 90\%$	$X_{BO} = 90\%$
- Maximize diversification		- Minimize variance
∞ entropy		**0 entropy**

Remedy: relax the certainty hypothesis

Introduce a parameter for entropy θ

Generalise the problem objective

protect against unforeseeable shocks

minimise foreseeable shocks

Maximise diversification

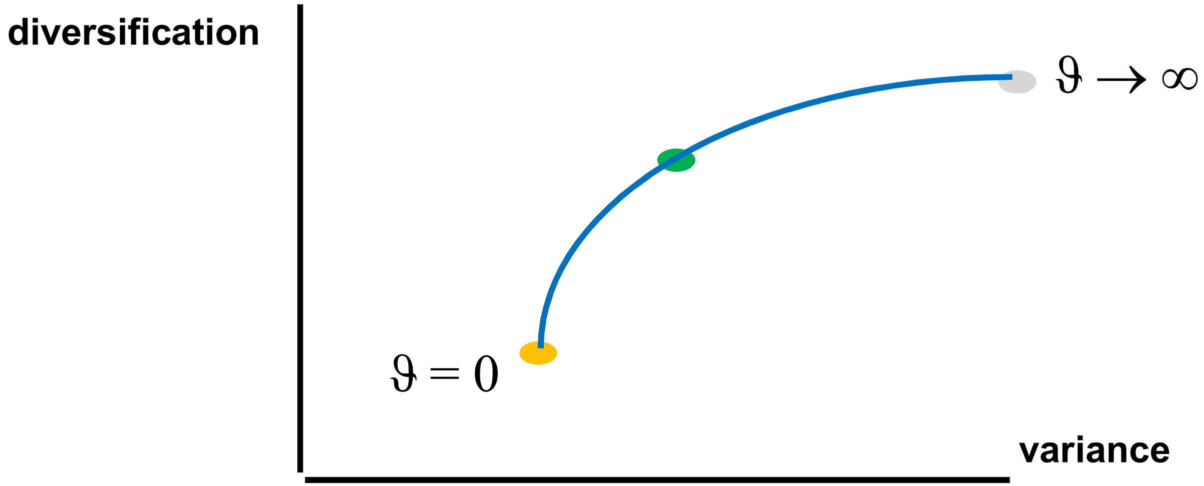
Minimise variance

$$\min \quad X^T X \quad + \theta \cdot \quad X^T V X$$

Generalized MPT

Diversification-variance efficient frontier

$$\min \qquad X^T X + \theta \cdot X^T V X$$



- high entropy 1 / N
- intermediate 1 / risk contribution
- low entropy minimum variance

Specify diversification

stake, footprint, span, exposure, ...

Measures for diversification

Rao's (1982) squared entropy	$-x^T x$
Shannon (information theory)	$-x^T ln(x)$
Roncalli (2013)	$-e^T ln(x)$

Measures depending on risk estimates **odd**

Choueifaty & Coignard (2008)	sum of asset-specific risks
Meucci (2009)	eigenvectors of covariance matrix

Coherent optimization methods

1. Covariance shrinkage
2. Risk parity
3. Bayesian optimization
4. Entropy-based optimization
5. Portfolio regularization

1. Covariance shrinkage*

Rao's measure

Augment the variance levels in the covariance matrix

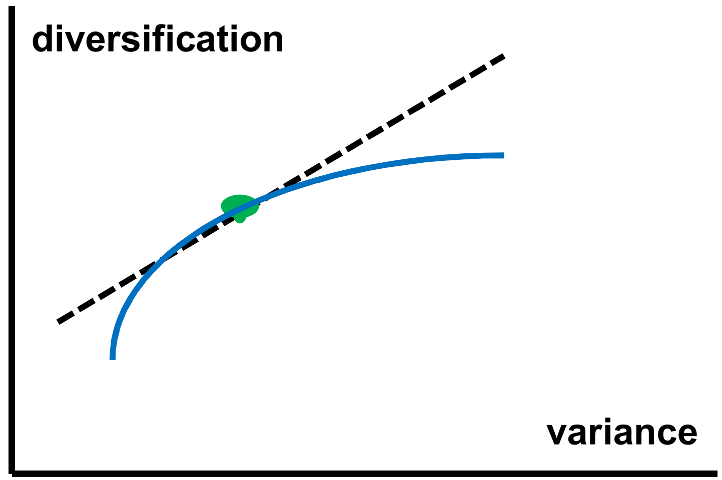
$$\text{Min. } x^T V\, x + \theta \cdot x^T x \quad \longleftrightarrow \quad \text{Min. } x^T (V + \theta \cdot I)\, x$$

* Ledoit, O., and M. Wolf. 2003. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection.
Journal of Empirical Finance 10(5)

2. Risk parity

Optimal variance/diversification ratio*

Min $e^T ln(x) + \theta \cdot x^T V\, x$



$$\frac{\delta x^T V x}{\delta x} = \frac{\delta e^T ln(x)}{\delta x} \leftrightarrow Vx = x^{-1} \leftrightarrow (Vx) \cdot (x) \text{ equal}$$

* Maillard, Roncalli, and Teiletche. 2010. The properties of equally weighted risk contribution portfolios. Journal of Portfolio Management 36(4)

3. Bayesian optimization

MIN. $\boldsymbol{x^T V_B\, x}$

Prior $\mathrm{R_0} \sim \mathrm{U}(0,1)$ uniform
Posterior $\mathrm{R_B} \sim \mathrm{n}(\mu, \mathrm{V_B})$ Gaussian

V_B is estimated such that the loss is minimized due to sub-optimality that would arise if the estimates turn out to be wrong.

Brown (1976) “Optimal portfolio choice under uncertainty” PhD thesis

4. Entropy-based optimization Shannon's measure

The Kullback and Leibler (1951) measure

$$\mathbf{x}^{\mathbf{T}} \ln(\mathbf{x}/\mathbf{q})$$

is equivalent to Shannon's,

if $q = \left(1/N, \ldots, 1/N\right)$.

Bera, A., and S. Park. 2008. Optimal portfolio diversification using maximum entropy principle. Econometric Reviews 27(4).

5. Portfolio regularization

Add a weight constraint

Min. $x^T V\, x + \theta \cdot x^T x$

LASSO, Ridge, elastic net

Bruder, Gaussel, Richard, Roncalli. 2013. Regularization of Portfolio Allocation.

Yet other coherent methods

1. Top-down approach* (not full optimisation).

2. Bridgewater® All Weather Funds.

3. Resampling by Michaud (1998)

4. Robust optimisation by Tutuncu and Koenig (2004)

5. Stability-adjusted optimisation by Kritzman (2016)

Odd specifications of diversification

Diversification ratio

$$\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{\sigma} \;/\; \boldsymbol{\sigma}_{\boldsymbol{p}}$$

Sum of specific risks on total risk.

In case of unforeseen shocks, diversification protection goes to pot.

Choueifaty and Coignard's (2008)

Investment example revisited

The equity-bond split

σ_{EQ} = 15%

σ_{BO} = 5%

corr($_{EQ,BO}$) = 0 **correlation parameter highly uncertain**

Utter darkness	Risk parity	MPT
X_{EQ} = 50%	X_{EQ} = 25%	X_{EQ} = 10%
X_{BO} = 50%	X_{BO} = 75%	X_{BO} = 90%

- Maximize diversification

∞ **entropy**

- Minimize variance

0 entropy

The entropy parameter

Interpretations

$$\boldsymbol{U_x = x^T\mu - \lambda \cdot x^T V\, x - \theta \cdot x^T x}$$

Risk aversion λ is a <u>subjective</u> parameter
Entropy θ is to a certain extent an <u>objective</u> parameter

Objective: degree of order in the economy
Subjective: degree of conviction

Q&A

THANK YOU FOR YOUR ATTENTION

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