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Hubeyb Gurdogan, UC Berkeley CDAR
Nariman Maddah, Swiss Re Institute, Swiss Re Management Ltd
Reyhaneh Mohammadi, Northeastern University
Elena Pesce, Swiss Re Institute, Swiss Re Management Ltd
Alicia Montoya, Swiss Re Institute, Swiss Re Management Ltd
Jeffrey Bohn, UC Berkeley CDAR and One Concern
Katherine Dalis, One Concern

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University of California
Berkeley

Berkeley Consortium for Data Analytics in Risk

Swiss Re Institute

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Hubeyb Gurdogan[1], Nariman Maddah[2], Reyhaneh Mohammadi[3], Elena Pesce[2], Alicia Montoya[2], Jeffrey Bohn[1,4], and Katherine Dalis[4]

[1]Consortium for Data Analytics and Risk, University of California, Berkeley, USA
[2]Swiss Re Institute, Swiss Re Management Ltd, Mythenquai 50/60, 8022 Zurich, Switzerland
[3]Mechanical and Industrial Engineering Department, Northeastern University, 360 Huntington Ave, Boston, MA 02115, USA
[4]One Concern, Inc., 855 Oak Grove Ave, Menlo Park, CA 94025, USA

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Abstract

Today's supply chains are global, highly interconnected, and increasingly digital. These three attributes of supply chains compound the effects of disruptions in production. For a company comprised of many factories, a disruption in production at one site can impact production at other locations as well as production at other companies linked through the supply chain. Quantifying the financial impact of business interruption, such as production loss at a factory caused by a natural catastrophe (NatCat) such as an earthquake or hurricane, is challenging. The difficulty in quantification is due to complex risk propagation dynamics and complications related to the allocation of business profit to specific sites of production. Complex risk propagation dynamics reflect product and supplier dependencies and the inter-connectivity of related risks.

The aim of this research is to estimate production losses at company locations to enable the quantification of exposed business interruption values (i.e. potential gross profit/earnings losses) taking into account interdependencies among the company and the supplying partners within its supply chain network. This approach can provide insurers and reinsurers with the required financial metrics to better address these risks. In this paper, after defining the adapted stochastic fully decomposed supply chain network (FDSN), we propose a new methodology to model the production rate potential at each site of production as a stochastic process via a recursive procedure. Finally, we consider the HAZUS Earthquake Model (HAZUS-EM) to estimate downtime and to quantify the impact of business interruption. Business interruption is propagated through the FDSN given an interruption in production in the supply chain network.

KEYWORDS: supply chain, business interruption (BI), network model, risk propagation, contingent business interruption (CBI)

1 Introduction

Globalisation and the growing digitisation of supply chains are driving increased interdependencies which can result in catastrophic losses when an event occurs and propagates through the economy. These disruptions cost organisations around the world an average of 184 million U.S. dollars per company per year [1]. Recent events such as floods and droughts, as well as the Covid pandemic and geopolitical conflicts, have further highlighted the potential catastrophic nature of supply chain interruptions. The associated wide-spread losses from these events are pushing companies, insurers and reinsurers to strive to better understand and manage these risks and build in resilience measures to limit losses and reduce financial volatility.

In order to quantify companies' supply chain risks, we need to calculate the probability and intensity of business interruption and its potential propagation across trade networks, which can be complex in the case of products with a large number of parts produced at different locations. Moreover, expressing the business interruption risk in terms of financial impact assists in assessing the materiality of a risk and develop effective mitigation solutions.

To assess and price these risks, commercial insurance companies may consider allocating business performance values (gross profit or gross earnings) to a single location of an insured company. However, insurers have limited information regarding the supply chain of their insureds, which limits their ability to accurately quantify the risks stemming from interdependencies among the company's locations (so-called indirect or interdependent business interruption, BI) and/or suppliers' locations (so-called contingent business interruption, CBI). CBI focused on dependence on lifelines is not typically available from insurance companies despite increasing latent demand among large multinational corporations. The source of this supply-demand imbalance arises from lack of reliable data and related pricing and risk models. Another challenge is to quantify the redundancy of products, parts, and raw materials since an interruption's impact on a supply chain is highly influenced by these redundancy factors.

The more transparency insurers and reinsurers have regarding their individual client supply chain risk exposures, the better they can support managing and mitigating accumulation risks and expand insurability through better products and more comprehensive risk coverage. This said, the complexity of today's global supply chain networks cause corporations to struggle with understanding their own supply chain structures. Insurers can use growing open-source and commercially available datasets to plug the information gap on their clients' supply chain. Another challenge in pricing insurance policies is the random nature of hazards driving perils that cause loss. A wide range of perils ranging from shaking ground to fires can trigger a supply chain disruption.

Insurers need frequency estimates of a given hazard, the downstream cascade of generated perils, the expected impact to a given service level, and the estimated downtime arising from a given peril. Common commercial and open-source natural catastrophe (NatCat) models offer business interruption (BI) estimates, assuming production performance has been allocated to single locations without considering interdependencies among suppliers or dependence on lifeline networks (e.g., power grids, transportation networks, etc.). In practice, companies and re/insurers can consider distinguishing loss estimates in single targeted locations derived using (increasingly typical) NatCat models from BI losses subject to propagation risks arising from disruption to supply chain networks.

In order to respond to these emerging requirements and better quantify supply chain risks, improve operational resilience, and profitably steer business in the context of complex supply chain dynamics, we discuss the following in this paper:

1. Network model to quantify interruptions within a given supply chain network, taking into account product dependencies and alternative suppliers (2.1)
2. Propagation model to quantify interruptions, conditional on probabilistic shocks at given nodes within a supply-chain network (2.2).
3. Methodology to model expected production losses within a reference time window, given downtime periods at each node (2.3).
4. Network resilience measures related to additional production capacities for products and materials in the supply-chain network model (2.4).
5. Use of the developed propagation model with probabilistic inputs from a NatCat model (3).

1.1 Literature Review

Among the papers on supply chain business interruption, we recommend a conceptual paper [2] on expected loss quantification. This paper stands out given its implementation of NatCat models. This said, these authors do not address interdependencies among suppliers, dependence on lifeline networks, or the network structure of the business operations within the company itself. (Note that nearly all published papers in this area do not address interdependencies and dependencies in a network context.) The author proposes the use of HAZUS NatCat models, which enable the estimation of business downtime conditional on given hazards, frequency of these hazards, and the impact level. The author proposes a case study on hurricanes and calculates the expected business losses in 100 years via multiplying the expected BI value by the ratio of the downtime and the length of the time window of interest at each hazard instance.

In contrast to the limited research conducted on BI and CBI quantification, there is extensive work on supply chain risk management (SCRM). This research is mainly driven by the need for companies to gain insights on supply chain disruption risks they face so as to manage, and where possible mitigate, vulnerabilities. Several approaches for supply chain disruption (SCDR) modeling have been proposed to measure the robustness and resilience of supply chains against NatCat and man-made risks. Desirable features of SCDR models would include the ability to model interdependencies and risk propagation, as well as quantify impact of hazardous events throughout a given network. Bayesian Belief graph-based models in [3, 4, 5, 6] are promising candidates for the development of these desirable properties, given their ability to reflect better the inherent interdependent network structure of supply chains.

The impact quantification or estimation of expected loss– particularly for manufacturers– requires the integration of business downtime into relevant models. Recent works by authors in [5, 6] include the time element into the modeling via the usage of Markov phase transition matrices on Bayesian Belief Networks. Nevertheless, the commonality of existing research in SCDR models, including Bayesian Belief Network models, is that they are not geared towards calculating disruption-driven expected losses for a manufacturer. Note that the main goal of SCRM research is not the estimation of business losses, but rather risk quantification at each node of a network via different measures in order to predict and mitigate risks.

To fill this research gap, the goal of this work is to develop a propagation model to understand better and measure the impact of hazardous events at each node of a given supply chain network, assuming the downtime and functionality probabilities as described in 2.1 are available.

2 Model Development

We focus on a final product manufacturer that is producing a single good for retail only and the scenario that a hazardous event H is inflicted on its supply chain. Our goal is to develop a model which allows us to calculate the expected loss ratio in total production of the final product conditional on the impact of H on the supply chain. We will begin with a simplifying assumption that the supply chain network for the production of the final end product is a Fully Decomposed Supply-Chain Network (FDSN) that is defined by the following conditions:

C1. Each plant in the network produces a single material/product and each plant in the network has a direct or indirect contribution to the final end product.

C2. No product requires itself for its production at any stage of the manufacturing process.

C3. There are no loops in the supply chain network. In other words, a supplier can never appear to be the buyer of the plants that sit at later stages of the production.

Remark. For the supply chain networks that do not satisfy condition *C1*, decomposing each multi producer plant at network into its copies that produces a single product and carries the identical risk characteristics is a potential remedy to make up for the separation that condition *C1* introduces between FDSN's and the general supply chain networks. The implementation of this idea is postponed to future research. Condition *C2* is expected to hold on a more general network than an FDSN as it is self-evident and domain justified. Conditions *C1* and *C2* together imply *C3*.

2.1 Network Setup

Definition 2.1. A directed acyclic graph $G = (V, E)$ is a FDSN skeleton if it satisfies the following:

1. The vertex set V models the plants in a FDSN for which each plant produces a single product. This set is partitioned into disjoint subsets,

$$V = \{v_0\} \cup \left(\bigcup_{j=1}^{k} V_j \right),$$

where each subset V_j consists of the vertices corresponding to the plants producing the same product (alternative suppliers of the same product) and the vertex v_0 corresponds to the only plant that produces the final end product. This set also determines k to be the number of products required to produce the final end product. .

2. The edge set E models the supplier/buyer relationships in a FDSN where a directed edge $e = (u, v) \in E$ from $u \in V$ to $v \in V$ is defined when the plant corresponding to v sources a product from the plant corresponding to u.

Definition 2.2. Let $G = (V, E)$ be an FDSN skeleton; $\mathcal{S}_v = \{u \in V \mid (u, v) \in E\}$ and $\mathcal{B}_v = \{u \in V \mid (v, u) \in E\}$ are defined to be, respectively, the supplier and buyer neighborhood of the plant corresponding to $v \in V$.

Definition 2.3. Given the FDSN skeleton $G = (V, E)$ and the exogenous set of constants $\{\alpha_v \geq 0 \mid v \in V\}$ corresponding to excess capacity of each plant in the FDSN, functionality is defined via the function $F : V \to \mathbb{R}^+$ and consignment weights are defined via the function $C : E \to [0, 1]$ where the following are satisfied:

1. For every $j \in \{1, 2, .., k\}$

$$\sum_{u \in V_j} \sum_{v \in \mathcal{B}_u} C(u, v) = 1. \tag{1}$$

2. For each node $v \in V$,

$$F(v) \leq 1 + \alpha_v. \tag{2}$$

Given this set up, a triplet (G, F, C) is defined as an FDSN instance.

Remark. The functionality weights assigned to each node identify the extent to which the nodes are operational. For example, if $F(v) = 1$ the node is fully operational while if $F(v) = 0$ then the node is non-operational. Consignment weights, on the other hand, are assigned to the edges and identify agreed amounts of goods to be delivered by the supplier. Weights are normalised so that each product's output totals to 1 units. With this normalisation, 1 unit of final end product depends on 1 unit of each product type in the network.

While introducing the time element, we restrict our attention to the case in which the skeleton G and the pre-agreed consignment weights C are fixed whereas the functionality of nodes may vary. This is a conservative assumption given that actual companies are likely to establish new trade relationships if business interruptions occur.

Given that we include a time element, we consider the product flow as the **instant feed** of the materials between the nodes according to the amounts prescribed by the consignment weights. The consignments construct the supply network to produce 1 unit of the final end product for each unit of time when each plant is fully operational. We set the unit of time as 1 day but we are not restricting the model to discrete flow of time. The unit of final end product is selected so that the total production of the network in the time window of interest (e.g. a year) will correspond to the forecasted total production.

Definition 2.4. Let $G = (V, E)$ be a FDSN skeleton. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,T]}$ be a filtration of the σ-algebras $\mathcal{F}_t$. Furthermore, set $F_t : \Omega \times V \to [0, 1]$ such that for each $v \in V$, $F_t(\_\_, v)$ is a stochastic process adapted to the filtration $\mathbb{F}$. Now we define an adapted stochastic FDSN process $\mathcal{N} = (\mathcal{N}_t)_{t \in [0,T]}$ by $\mathcal{N}_t = (G, F_t, C)$ where we require $\mathcal{N}_t(\omega) = (G, F_t(\omega, \_\_), C)$ to be a FDSN instance for every choice of $t \in [0, T]$ and $\omega \in \Omega$.

2.2 Impact Propagation Model

As we have an adapted stochastic FDSN process $\mathcal{N}_t$, at each instance $\mathcal{N}_t(\omega)$ we have an instantaneous production rate of each plant that does not have to be equal to the functionality of the plant. At the end, the production rate should have two inputs; the functionality of the plant and the flow of supplies. We will provide a well suited recursive definition for the production rate potential of each plant in the FDSN skeleton G including the final end node v_0 responsible for producing the final end product.

Definition 2.5. Let $\mathcal{N}_t = (G, F_t, C)$ be a stochastic FDSN process adapted to the filtration $\mathbb{F}$. We define the production rate potential of each plant as a stochastic process $P_t : \Omega \times V \to \mathbb{R}^+$ via the following recursion,

$$P_t(\omega, v) = F_t(\omega, v) \quad \forall v \text{ s.t. } \mathcal{S}_v = \emptyset, \tag{3}$$

$$P_t(\omega, v) = \min\left(F_t(\omega, v), \left\{ \frac{\sum\limits_{\hat{v} \in S_v \cap V_i} P_t(\omega, \hat{v}) C(\hat{v}, v)}{\sum\limits_{\hat{v} \in S_v \cap V_i} C(\hat{v}, v)} \;\middle|\; i = 1, 2, .., k \right\}\right) \quad \forall v \text{ s.t. } S_v \neq \emptyset, \tag{4}$$

Remark. Note that the recursion calculates the potential of each plant, that can be truncated by its buyers in the recursion. For example, if the plant is able to produce 100% of its consignments but the buyers are only 50% operational, that would reduce the production of the plant accordingly through lack of demand. We assume that, prior the impact, the assembly plant is optimised to produce the retail demand and has no room to go beyond its capacity. In other words, $\alpha_{v_0} = 0$ and hence when the recursion reaches the final end node, the calculated production rate potential $P_t(\omega, v_0) \leq F_t(\omega, v_0) \leq 1 + \alpha_{v_0} = 1$ is realised since the demand from the final end node is fixed to 1 unit at each instance. Therefore under the assumption on the retail demand and the excess capacity of the final end producer, the forward recursion defined in the definition 2.5, able to express the realised production rate of the final end producer corresponding to vertex v_0.

Given a stochastic FDSN process we can now write down the loss ratio in total production of the final end node v_0 for a given time window $[0, T]$ as,

$$\mathcal{L}(\omega) = \frac{1}{T} \int_0^T (1 - P_t(\omega, v_0)) dt = 1 - \frac{1}{T} \int_0^T P_t(\omega, v_0) dt. \tag{5}$$

Proposition 2.1. *The quantity $P_t(\omega, v)$ given in definition 2.5 is well defined for every $v \in V$. In other words, the recursive procedure reaches every vertex of the graph.*

Proof. Fix $\omega \in \Omega$ and define a set $L \subset V$ to be the set of vertices where the quantity $P_t(\omega, v)$ is defined. By the initial step of the recursion given in definition 2.5, we know that set of leafs, $\{v \in V \mid S_v = \emptyset\}$ is a subset of L.

Assume to the contrary that $\exists v \in V \backslash L$. Then $S_v \neq \emptyset$ and $\exists v_1 \in S_v$ such that $v_1 \notin L$. Similarly, that implies $S_{v_1} \neq \emptyset$ and $\exists v_2 \in S_{v_1}$ such that $v_2 \notin L$. Continuing in this fashion, we generate an infinite sequence of vertices $\{v_i\}_{i=0}^{\infty} \subset V \backslash L$ and the vertices form an infinite path. As $|V| < \infty$ dictates, the vertices in the sequence has to be repeated. This, in turn, creates a cycle that contradicts the acyclic assumption of G. As the vertex v is arbitrary, we conclude that $V \backslash L = \emptyset$ and hence $V = L$. This finalises the proof. □

The following corollary serves as a sanity check of the recursive definition in 2.5. When all plants are operational, the production rate potentials across the whole network are expected to be 1 and the recursive definition at 2.5 is in line with that.

Corollary. *Let $\mathcal{N}_t = (G, F_t, C)$ where $F_t(\omega, v) = 1$ for each $v \in V$ and $\omega \in \Omega$. Then $P_t(\omega, v) = 1$ for each $v \in V$ and $\omega \in \Omega$.*

Proof. Lets now define $K = \{v \in V \mid P_0(\omega, v) = 1 \ \forall \omega \in \Omega\}$. From the initial step of the recursion we get $\{v \in V \mid S_v = \emptyset\} \subset K$. Arguing similarly in the proof of the proposition 2.1 lets assume that there exist $v \in V \backslash K$. As we know recursion reaches to v by the proposition 2.1, it implies $S_v \neq \emptyset$ and $\exists v_1 \in S_v \backslash K$. Note that other wise if $S_v \subset K$ the recursion implies $v \in K$ as well. Continuing this fashion we construct an infinite sequence of vertices $\{v_i\}_{i=1}^{\infty}$ that also form an infinite path. As $|V| < \infty$ dictates, the vertices in the sequence has to be repeated. That creates a cycle that contradicts with the acyclic assumption of G. This shows $v \in K$. As the vertex v where arbitrary, we conclude that $V = K$. Hence we get $P_0(\omega, v) = 1$ for every $v \in V$ including the final end node v_0.

□

Now we provide an example to illustrate the definition 2.5.

Example 2.1. Assume the supply chain network of our end product v_0 (Node 0) shown in Figure 1. In this network, the nodes starting with same numbers produce the same product, e.g. plant v_4 and $v_{4.1}$ both produce product number 4 so that $V_4 = \{v_4, v_{4.1}\}$ and the numbers on the links show the consignment weights C. It exemplifies an FDSN instance taken at some time $t \in [0, T]$ for a fixed ω from an FDSN process. Moreover, we set all nodes except nodes $v_3, v_{4.1}$ and v_5 to be fully functional. By Equation (4) the production rate of nodes $v_{4.1}$ and v_5 would be equal to 0 and the production rate of nodes $v_1, v_4, v_{5.1}, v_6$ and v_7 is equal to 1. The production rate of other nodes is as follow:

$$P_t(v_2, \omega) = min\{F_t(v_2, \omega), 0.19 * P_t(v_4, \omega) + 0.81 * P_t(v_{4.1}, \omega), P_t(v_6, \omega)\} = 0.19$$

$$P_t(v_3, \omega) = min\big\{F_t(v_3, \omega), P_t(v_7, \omega), \frac{0.31 * P_t(v_5, \omega) + 0.19 * P_t(v_{5.1}, \omega)}{0.5}\big\} = 0$$

$$P_t(v_{3.1}, \omega) = min\big\{F_t(v_{3.1}, \omega), \frac{0.11 * P_t(v_5, \omega) + 0.39 * P_t(v_{5.1}, \omega)}{0.5}, P_t(v_7, \omega)\big\} = 0.78$$

$$P_t(v_0, \omega) = min\{F_t(v_0, \omega), P_t(v_1, \omega), P_t(v_2, \omega), 0.6 *_t (v_3, \omega) + 0.4 * P_t(v_{3.1}, \omega)\} = 0.19$$

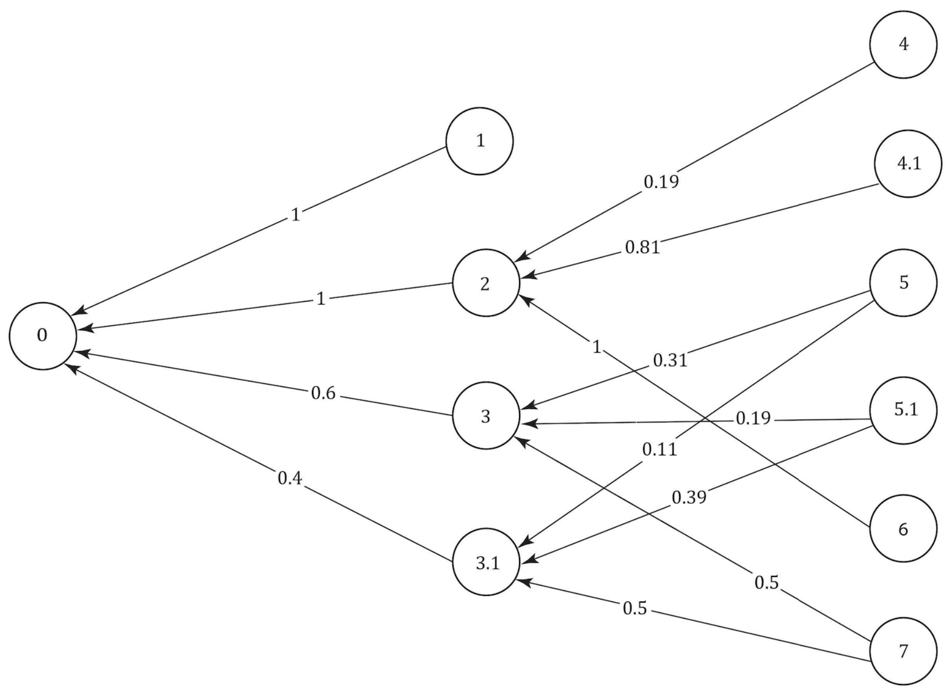

Figure 1: An example of a supply chain network dependencies

2.3 Modeling the Impact

For a given a supply chain network, we consider the impact of a hazardous event to each production plant using random variables determining the likelihood of a hit and the production plant downtime caused by the event. We consider the scenario that the impacted nodes recover fully after the downtime passes and during this phase no mitigating actions are taken from the non-impacted nodes.

Let $\mathcal{N}_t = (G, F_t, C)$ be a stochastic FDSN process adapted to the filtration $\mathbb{F} = \{\mathcal{F}_t\}$ that carries the growing information with time. Next, we consider the arrival of the hazard at time 0 and condition all the analysis on this event. Let $\delta_v \in \{0, 1\}$ and $d_v \in \mathbb{R}^+$ be random variables that determine impact and associated downtime, respectively. The values 0 and 1 for δ_v correspond to a hit and no hit, respectively.

For completeness, we set the downtime d_v to 0 when there is no hit. With this set up, we model the impact via setting the functionality as,

$$F_t(\omega, v) = \delta_v(\omega) 1_{\{t < d_v(\omega)\}} + 1_{\{d_v(\omega) \leq t\}}. \tag{6}$$

Each impacted plant will disrupt its own buyers and the impact will propagate downstream via the recursive definition of the production rate P_t of Equation (4).

Each $\omega \in \Omega$, will realise the random list of downtimes. Let $d_0, d_1, ..., d_{|V|-1}$ be the ordered statistics of the realised downtimes $\{d_v(\omega)\}_{v \in V}$ with $0 \leq d_0 \leq d_1 \leq ... \leq d_{|V|-1} \leq T$. We have $F_t(\omega, v)$ remain static on the intervals $t \in [d_{i-1}, d_i]$ by its definition above. The same feature is carried over to the production rate $P_t(\omega, v)$ through Equation (3) and Equation (4). Hence, following the formulation in Equation (5) we have for each $\omega \in \Omega$ the loss ratio in total production $\mathcal{L}$, which simplifies to

$$\mathcal{L}(\omega) = 1 - \frac{1}{T} \int_0^T P_t(\omega, v_0) dt = T - \sum_{i=1}^{|V|-1} (d_i - d_{i-1}) P_{d_{i-1}}(\omega, v_0) \tag{7}$$

We write an expected loss ratio conditioned on the arrival of the hazard at time 0, denoted by the event H, as follows:

$$\mathbb{E}[\mathcal{L}|H] = \mathbb{E}\big[T - \sum_{i=1}^{|V|-1} (d_i - d_{i-1}) P_{d_{i-1}}(\omega, v_0) \big| H\big] . \tag{8}$$

Example 2.2. Recall the FDSN instance in Example 2.1 and release the fixed time t to realise an FDSN process at the fixed random state ω. In this case, we set the length of the time window T to 12 and assume that the affected nodes have downtimes realised as $d_{v_{4.1}} = 5$, $d_{v_3}(\omega) = 8$ and $d_{v_5}(\omega) = 12$. We want to calculate the loss ratio in total production of node v_0 over time $[0, 12]$. Following the formula in Equation (7), we first need to calculate the production rate of node v_0 on times 0, 5 and 8. Using the example 2.1, we have $P_0(v_0, \omega) = 0.19$. At time 5 node $v_{4.1}$ will be fully functional and only nodes v_3 and v_5 remain non-functional. In this situation $P_5(v_2, \omega) = 1$ and $P_5(v_0, \omega) = 0.6 * P_5(v_3, \omega) + 0.4 * P_5(v_{3.1}, \omega) = 0.312$. At time 8 node v_3 will be functional and node v_5 would be the only non-functional node. Thus, $F_8(v_3, \omega) = 1$ and $P_8(v_3, \omega) = 0.38$. As a result, $P_8(v_0, \omega) = 0.6 * P_8(v_3, \omega) + 0.4 * P_8(v_{3.1}, \omega) = 0.6 * 0.38 + 0.4 * 0.78 = 0.54$. The loss ration in total production for a given ω is:

$$L(\omega) = \frac{1}{12}\big[12 - (5-0) * P_0(v_0, \omega) + (8-5) * P_5(v_0, \omega) + (12-8) * P_8(v_0, \omega)\big] = \frac{7.954}{12} \sim 0.66$$

2.4 In-Network Resilience

In this subsection we will briefly discuss the scenario that allows plants in the network to increase their capacity to compensate for impacted nodes– but only through already established links. The plants that will compensate for the impacted plants will need to increase capacity and source more supplies. We will allow them to source more supplies from their established suppliers proportional to their previous trade volumes. If, for example, a supplier has an excess capacity of 10% and the buyer plant was sourcing 50% of its need from that supplier, then the supplier in our model will be able to send 55% of the buyer's need to produce one unit. This can be integrated into the baseline model by setting the functionality as,

$$F_t(v) = (1 + \alpha_v)\big[\delta_v 1_{\{t < d_v\}} + 1_{\{d_v \leq t\}}\big]. \tag{9}$$

This functional process choice will lead to the calculation of the maximum possible production rate for the final end node that can be reached via in-network resilience measures.

3 Implementation of the Propagation Model

In this section, we illustrate a simple network example. In this context, we perform numerical implementation of a model that calculates expected loss ratios conditional on hazardous events. This example focuses on earthquakes. For the hazard-model component, we use the Hazus Earthquake Model (HAZUS-EM [7]). The experiments will be performed on a fixed network example for a catalog of events. Given an FDSN skeleton $G = (V, E)$, we define a footprint of an event H as,

$$\mathcal{K}_H = \{(v, \theta_v) \mid v \in V,\ \theta_v \in [0, 0.5]\} \tag{10}$$

where $v \in V$ is the node and θ_v determines the Peak Ground Acceleration (PGA in fractions of ground acceleration, i.e. 9.81) magnitude that the earthquake delivers to the represented sites. We define the catalog of events $\mathcal{H}$ as a collection of events H.

3.1 HAZUS Earthquake Model

The HAZUS Earthquake Model (HAZUS-EM) models the fragility curve of each building type as a probability of receiving or exceeding a damage state conditioned on the PGA magnitude delivered to the site,

$$\mathbb{P}(X \geq s|\theta) = \phi\Big(\frac{1}{\beta}_s ln\Big(\frac{\theta}{\bar{\theta}_s}\Big)\Big) \tag{11}$$

where

- X is a discrete random variable that determines the damage state of the building. It has the event space as the damage states ordered as

  none < slight < moderate < extensive < complete

  and s stands for a fixed damage state from the above list.
- θ is the PGA in unit of g that impact buildings.
- $\bar{\theta}_s$ is the median value of PGA at which the building reaches the threshold of the damage state s. In other words, when PGA value θ is the same as $\bar{\theta}_s$ then the probability of the building being in the damage state greater or equal than s is 0.5
- β_s is the standard deviation of the natural logarithm of the damage state s.
- ϕ is the standard normal cumulative distribution function.

The constants $\bar{\theta}_s$ and β_s for each damage state s are estimated a priori and provided by HAZUS-EM in Table 1. Conditional on the random variable X, it provides estimates of the median downtime of each business type in Table 2.

3.2 Network construction and input selection

We consider an FSDN on a time interval of T consecutive days where an earthquake is assumed to have struck on day 0. This is represented by an FDSN process $\mathcal{N}_t = (G, F_t, C)$ with the skeleton G and the consignment weights C as displayed in Figure 2. In order to provide the random variables δ_v and d_v for each node, HAZUS-EM requires the input of the business type and the building type corresponding to the plants presented by the nodes. We accommodate this state as below:

- For the nodes in our network, we select several arbitrary industrial categories, i.e. IND1 (Heavy Industrial), IND2 (Light Industrial), IND4 (Metals/Minerals Processing) and IND5 (High Technology) which are color coded in the graph of Figure 2.
- For simplicity, all nodes in our network have the same building type– steel frame building with up to 3 stories and moderate-code seismic design level (S2L according to HAZUS-EM).

3.3 Monte Carlo Simulation

For a time horizon of T days and an event catalog $\mathcal{H} = \{H_1, H_2, ..., H_{10}\}$ of size 10 with the corresponding footprints as presented in Table 3. For each arrival of an event $H \in \mathcal{H}$ at beginning of the time window, we perform a Monte Carlo simulation to calculate the expected conditional loss ratio $\mathbb{E}[\mathcal{L}|H]$. We outline the steps of the simulation for an arbitrary event $H \in \mathcal{H}$ as below.

1. We populate a sample of damage states of each node in the skeleton G,

$$\{x_v^1\}_{v\in V}, \{x_v^2\}_{v\in V}, ..., \{x_v^n\}_{v\in V}$$

   from the collection of random variables $\{X_v\}_{v\in V}$ where each X_v has the cumulative probability distribution adopted from Equation (11),

$$\mathbb{P}(X_v > s) = \phi\left(\frac{1}{\beta_s} ln\left(\frac{\theta_v}{\bar{\theta}_s}\right)\right) \tag{12}$$

   where θ_v is retrieved from the footprint $\mathcal{K}_H$ and the constants β_s and $\bar{\theta}_s$ are looked up from Table 1 for each damage state. Note that they remain the same for each node as we assumed a single building type, S2L.

2. For each sampled $\{x_v^i\}_v$ we derive $\delta_v^i = \mathbb{1}_{\{x_v^i = none\}}$ and look up the downtime d_v^i from the Table 2 through the corresponding damage state x_v^i and the business type of the node v. We build up the functionality processes for each node.

3. Feeding the functionality processes into the recursive definition 4, and using the equation 7, we calculate the loss ratio in total production $\mathcal{L}_i$ associated with each sampled $\{x_v^i\}_v$.

4. The Monte Carlo estimate of the expected loss ratio $\mathbb{E}[\mathcal{L}|H]$ conditioned on the hazardous event H is

$$\frac{1}{n}\sum_{i=1}^{n} L_i.$$

We set $T = 360$ and we consider two scenarios on the functionality processes. First one is, we assume no extra capacity in the function as in Equation (6). In the other scenario, we allow the extra capacity of each node to enter in the functionality as in Equation (9) where we assume a homogeneous extra capacity of %10 to each node. For each of the above scenarios and for each hazard H in the event catalog $\mathcal{H}$, we perform the above implementation 4 times, once for each quarter of the time interval $[0, T]$. We present the results in Table 4.

4 Conclusions

We have developed a framework that may be considered in the context of calculating expected losses in total production for final end manufacturers in an FDSN conditional to probabilistic shocks at given nodes within a specific supply-chain network.

Key benefits of our propagation model include:

1. Accounts for alternative suppliers and additional production capacity within a network.

2. Integrates the time element through an instant feed structure.

3. Distinguishes between the functionality and production rate potential of a plant as the latter is subject to network effects.

4. Compatible with NatCat models and can be used as a complementary module to better estimate business interruption (BI) losses with interdependency effects. The former capability is illustrated on simple network data where our model is paired with HAZUS-EM.

5. Developed in a way that the functionality process introduced in Equation (6) in Section 2.3 actually works with any functionality/downtime making it flexible enough to use with non-NatCat models.

4.1 Future expansions and extensions

Our propagation model is based on the assumption that the FDSN remains static, meaning companies do not establish new trade relationship after events. The current propagation mechanism flows from suppliers to buyers, following the production materials (forward propagation). The model is not able to calculate production rates in case of reduced demand from customers (backward propagation). The model assumes instant feed and ignores the geographical distances between nodes, time-depending variations in production, and any buffer in the network. Moreover, the recursion process to calculate production rates is deterministic.

Future research could explore:

- Introducing other resilience features to allow for the possibility that companies can establish new trade relationships, in order to relax the assumption of the static FDSN.
- Creating a mechanism for backward propagation.
- Extending the model with less restrictive assumptions on the lags in material flow due to distance and additional buffers.
- Extending the recursion process to take into account probabilistic variables.

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Table 1: Equivalent-PGA Structural Fragility for Building Type S2L (adopted from Table 5-28 in [7])

Damage State(s)	Median Equivalent PGA(θ_s)	Log Standard Deviation(β_s)
Slight	0.20	0.64
Moderate	0.26	0.64
Extensive	0.46	0.64
Complete	0.84	0.64

Table 2: Building Median Downtime Based on Business Type and Damage State (adopted from Table 11.8 and 11.9 in [7])

		Damage State			
		Slight	Moderate	Extensive	Complete
Business Type	Heavy Industrial	5	90	240	360
	Light Industrial	1	18	72	144
	Metals/Minerals Processing	2	18	72	144
	High Technology	4	27	108	216

Table 3: The Event Catalog $\mathcal{H}$

H_1	Affected Node (v)	2.1	4.1	7.1	10	13.2	15.1
	PGA (θ_v)	0.220	0.190	0.250	0.140	0.140	0.100
H_2	Affected Node (v)	4.2	8	11	12	16	17.2
	PGA (θ_v)	0.4796	0.4513	0.3775	0.1690	0.0491	0.3610
H_3	Affected Node (v)	4	4.1	7	9	10.1	15.1
	PGA (θ_v)	0.3037	0.194	0.1976	0.4222	0.1752	0.2417
H_4	Affected Node (v)	3	4.2	6	7.1	13.2	16
	PGA (θ_v)	0.3902	0.0378	0.3911	0.3072	0.1086	0.3748
H_5	Affected Node (v)	3.1	4.1	6	10	14	-
	PGA (θ_v)	0.3262	0.4776	0.0779	0.1537	0.4075	-
H_6	Affected Node (v)	3	6	12	15.2	16	-
	PGA (θ_v)	0.0224	0.1418	0.214	0.3414	0.0838	-
H_7	Affected Node (v)	2	4.2	7	9	11	12
	PGA (θ_v)	0.261	0.3514	0.1639	0.1152	0.4986	0.3995
H_8	Affected Node (v)	4.2	6	9	13	13.1	-
	PGA (θ_v)	0.0483	0.2452,	0.1542	0.2241	0.2607	-
H_9	Affected Node (v)	6.2	7	8	10	11.1	17.1
	PGA (θ_v)	0.1035	0.4941	0.4135	0.0442	0.3857	0.3164
H_{10}	Affected Node (v)	2.1	6	8	11.1	-	-
	PGA (θ_v)	0.3530	0.2060	0.4570	0.4120	-	-

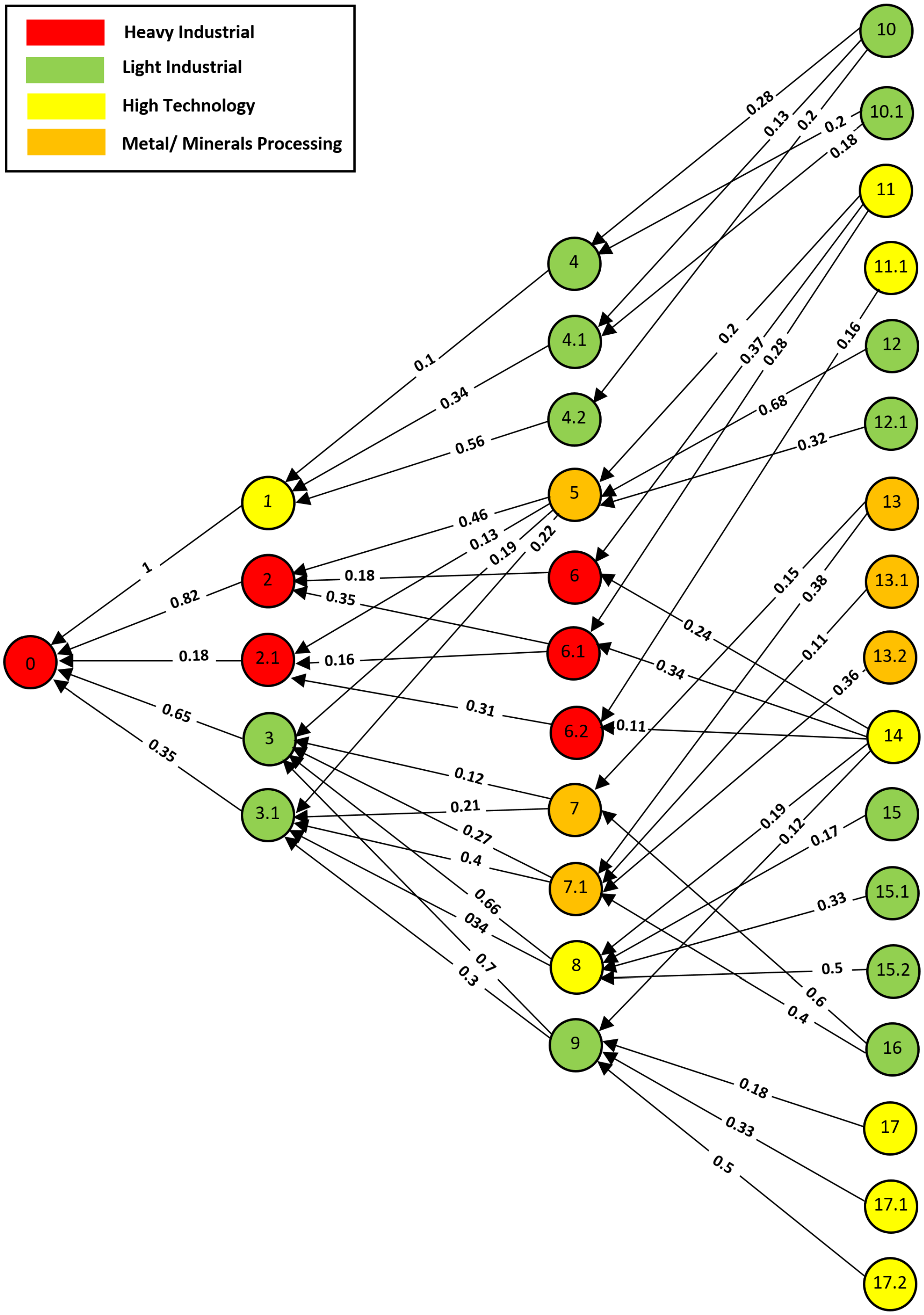

Figure 2: supply chain network used in model implementation

Table 4: Quarterly Expected Loss Ratio Corresponding to Events in Event Catalog

Event	Capacity	Q1	Q2	Q3	Q4
H_1	Normal	19.4	3.2	1.6	0.0
	Extra (10%)	17.2	2.3	0.9	0.2
H_2	Normal	60.1	23.6	6.6	0.0
	Extra (10%)	60.0	23.5	6.6	0.0
H_3	Normal	45.4	8.4	0.0	0.0
	Extra (10%)	45.4	8.4	0.0	0.0
H_4	Normal	49.8	15.9	8.5	3.2
	Extra (10%)	47.3	13.5	6.3	2.4
H_5	Normal	55.1	19.7	5.2	0.0
	Extra (10%)	54.8	19.5	5.2	0.0
H_6	Normal	20.8	3.0	0.6	0.0
	Extra (10%)	19.2	2.6	0.5	0.0
H_7	Normal	64.3	27.6	13.3	2.8
	Extra (10%)	64.3	27.6	13.3	2.8
H_8	Normal	22.6	5.1	3.3	0.8
	Extra (10%)	20.0	4.0	2.4	0.6
H_9	Normal	56.1	20.3	5.4	0.0
	Extra (10%)	55.6	20.1	5.4	0.0
H_{10}	Normal	61.9	26.0	11.1	2.0
	Extra (10%)	60.8	24.9	9.7	1.3