A Term Structure Model for Dividends and Interest Rates

Damir Filipović
Joint work with Sander Willems

École Polytechnique Fédérale de Lausanne
Swiss Finance Institute

Consortium for Data Analytics in Risk Seminar
UC Berkeley, July 31, 2018

Overview

A new market for dividend derivatives

- How can we trade dividends?
  - Synthetic replication.
  - Dividend swaps (OTC) or dividend futures (on exchange).
  - Latest innovations: single names, options, dividend-rates hybrids, ...
- Dividend derivative pricing.
  - Buehler et al. (2010), Buehler (2015), Kragt et al. (2016), Tunaru (2017).
- Asset pricing: term structure of equity risk premium.
  - Lettau and Wachter (2007), Binsbergen et al. (2012), Binsbergen et al. (2013), Binsbergen and Koijen (2017).
- Interest rates: hybrid products, long maturity dividend claims.

Notional Outstanding Dividend Swaps and Futures

	Notional Amount Outstanding (U.S. Dollars Millions)		
Underlying Index	**Equity Index Future**	**Dividend Index Future**	**Dividend Swap**
EURO STOXX 50	137,717	9,854	1,035
S&P 500	320,964	N/A	4,512
Nikkei 225	23,924	854	452
FTSE 100*	59,616	759	101

Figure: Total notional outstanding as of June 2015. Source: Mixon and Onur (2016)

Notional Outstanding Dividend Swaps and Futures

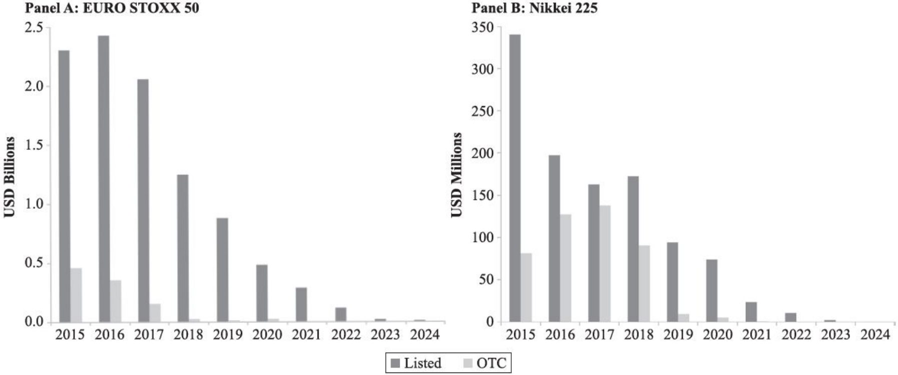


Figure: Notional outstanding per expiry as of June 2015. Source: Mixon and Onur (2016)

Contribution of this paper

Term-structure model for dividends and interest rates with

- Closed-form prices for dividend futures/swaps, bonds, and dividend paying stocks.
- Moment-based approximations for a broad class of exotic payoffs.
- Positive dividends and possible seasonal behaviour.
- Flexible correlation between dividends and interest rates.

Overview

Factor process

- Filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{Q})$, with $\mathbb{Q}$ risk-neutral pricing measure.
- Multivariate factor process X_t on $E \subseteq \mathbb{R}^d$

$$\mathrm{d}X_t = \kappa(\theta - X_t)\mathrm{d}t + \mathrm{d}M_t,$$

for $\kappa \in \mathbb{R}^{d \times d}$, $\theta \in \mathbb{R}^d$, and martingale M_t such that X_t is a polynomial jump-diffusion, cfr. Filipović and Larsson (2017).
- Generator $\mathcal{G}$ maps polynomials to polynomials:

$$\mathcal{G}\mathrm{Pol}_n(E) \subseteq \mathrm{Pol}_n(E), \quad \forall n \in \mathbb{N},$$

with $\mathrm{Pol}_n(E)$ space of polynomials on E of degree n or less.

PJD Moment Formula

- Fix polynomial basis for $\mathrm{Pol}_n(E)$:

$$H_n(x) = (h_1(x), \dots, h_{N_n}(x))^\top,$$

with $N_n = \dim(\mathrm{Pol}_n(E)) \le \binom{n+d}{n}$.

- $\mathcal{G}$ restricts to a linear operator on $\mathrm{Pol}_n(E)$

$$\mathcal{G} H_n(x) = G_n H_n(x).$$

- First order linear ODE for $s \mapsto \mathbb{E}_t[H_n(X_s)]$

$$\mathbb{E}_t[H_n(X_T)] = H_n(X_t) + G_n \int_t^T \mathbb{E}_t[H_n(X_s)] \,\mathrm{d}s$$

- Solving ODE gives for all $t \le T$

$$\mathbb{E}_t\left[H_n(X_T)\right] = \mathrm{e}^{G_n(T-t)} H_n(X_t).$$

Dividend Futures

- Instantaneous dividend rate:

$$D_t = p^\top H_1(X_t),$$

for $p \in \mathbb{R}^{d+1}$ such that $p^\top H_1(x) \geq 0$ for all $x \in E$.

- Linear dividend futures price:

$$\begin{aligned} D_{fut}(t, T_1, T_2) &= \mathbb{E}_t \left[\int_{T_1}^{T_2} D_s \, \mathrm{d}s \right] \\ &= p^\top \int_{T_1}^{T_2} \mathrm{e}^{G_1(s-t)} \, \mathrm{d}s \, H_1(X_t). \end{aligned}$$

- E.g., if $H_1(x) = (1, x^\top)^\top$, then

$$G_1 = \begin{bmatrix} 0 & 0 \\ \kappa\theta & -\kappa \end{bmatrix}.$$

Interest Rates

- Risk-neutral discount factor:

$$\zeta_t = \zeta_0 \mathrm{e}^{-\int_0^t r_s \, \mathrm{d}s}, \quad t \geq 0,$$

  where r_t denotes the short rate.
- Directly specify ζ_t:

$$\zeta_t := \mathrm{e}^{-\gamma t} q^\top H_1(X_t),$$

  for $\gamma \in \mathbb{R}$ and $q \in \mathbb{R}^{d+1}$ such that ζ_t is a positive and absolutely continuous process.
- Implied short rate:

$$r_t = \gamma - \frac{q^\top G_1 H_1(X_t)}{q^\top H_1(X_t)}.$$

- Cfr. Filipović et al. (2017), Ackerer and Filipović (2016).

Bond Prices

- Time-t price of zero-coupon bond maturing at $T \geq t$:

$$P(t,T) = \frac{1}{\zeta_t}\mathbb{E}_t[\zeta_T] = \mathrm{e}^{-\gamma(T-t)} \frac{q^\top \mathrm{e}^{G_1(T-t)} H_1(X_t)}{q^\top H_1(X_t)}.$$

- Linear discounted bond price $\zeta_t P(t,T)$.
- If $\Re(\mathrm{eig}(\kappa)) > 0$:

$$\lim_{T\to\infty} -\frac{\log(P(t,T))}{T-t} = \gamma.$$

Dividend Paying Stock

- Fundamental stock price

$$S_t^* = \frac{1}{\zeta_t}\mathbb{E}_t\left[\int_t^\infty \zeta_s D_s \,\mathrm{d}s\right].$$

- If $\Re(\mathrm{eig}(G_2)) < \gamma$, then

$$S_t^* = \frac{\vec{v}^\top \left(\gamma\,\mathrm{Id} - G_2\right)^{-1}\, H_2(X_t)}{q^\top H_1(X_t)} < \infty,$$

- Quadratic discounted fundamental stock price $\zeta_t S_t^*$.
- Define arbitrage-free stock price as

$$S_t = \frac{L_t}{\zeta_t} + S_t^*,$$

for some nonnegative (local) martingale L_t, cfr. Buehler (2015), Jarrow et al. (2007, 2010).

Overview

Maximum Entropy Moment Matching

- Pricing problem:

$$\pi_t = \mathbb{E}_t \left[F\Big(g(X_T) \Big) \right],$$

  with $g \in \mathrm{Pol}_n(E)$ and $F\colon \mathbb{R} \to \mathbb{R}$.
- Goal: Approximate density of $g(X_T)$ based on moments.
- Maximize Boltzmann-Shannon entropy:

$$\begin{aligned} \max_f \quad & -\int f(x) \ln f(x)\, \mathrm{d}x \\ \text{s.t.} \quad & \int x^n f(x)\, \mathrm{d}x = M_n, \qquad n = 0, \ldots, N \end{aligned}$$

- Unique solution:

$$f(x) = \exp\left(-\sum_{i=0}^{N} \lambda_i x^i \right)$$

Option Pricing

- Swaptions:

$$\begin{aligned}\pi_t^{swpt} &= \frac{1}{\zeta_t}\mathbb{E}_t\left[\left(\zeta_T \pi_T^{swap}\right)^+\right] \\ &= \frac{1}{\zeta_t}\mathbb{E}_t\left[\left(\zeta_T - \zeta_T P(T, T_n) - \delta K \sum_{k=1}^{n} \zeta_T P(T, T_k)\right)^+\right]\end{aligned}$$

- Stock options

$$\begin{aligned}\pi_t^{stock} &= \frac{1}{\zeta_t}\mathbb{E}_t\left[(\zeta_T S_T - \zeta_T K)^+\right] \\ &= \frac{1}{\zeta_t}\mathbb{E}_t\left[(L_T + \zeta_T S_T^* - \zeta_T K)^+\right]\end{aligned}$$

Option Pricing

- Dividend options

$$\pi_t^{div} = \mathbb{E}_t\left[\left(\int_{T_0}^{T_1} D_s\,\mathrm{d}s - K\right)^+\right]$$
$$= \mathbb{E}_t\left[(I_{T_1} - I_{T_0} - K)^+\right],$$

  with $I_T = \int_0^T D_s\,\mathrm{d}s$.
- Augment factor process: (I_t, X_t) is PJD.
- Compute moments $\mathbb{E}_t\left[(I_{T_1} - I_{T_0})^n\right]$ using law of iterated expectations.

Overview

Linear Jump-Diffusion

- Specify martingale part $\mathrm{d}M_t$ as

$$\mathrm{d}X_t = \kappa(\theta - X_t)\,\mathrm{d}t + \mathrm{diag}(X_{t-})\,(\Sigma\,\mathrm{d}B_t + \mathrm{d}J_t)$$

- B_t: standard d-dimensional Brownian motion, $\Sigma \in \mathbb{R}^{d\times d}$ lower triangular with $\Sigma_{ii} > 0$
- J_t: compensated compound Poisson process, jump intensity ξ and i.i.d. jump amplitudes $\mathrm{e}^Z - 1$, $Z \sim \mathcal{N}(\mu_J, \Sigma_J)$.
- Unique positive solution if $\kappa\theta \geq 0$ and if $\kappa_{ij} \leq 0$ for $i \neq j$.
- Allows for flexible instant. correlation between factors through Σ.
- Moments in closed-form (PJD).

Overview

Model Specification

- Five factor model $X_t = (X^I_{0t}, X^I_{1t}, X^I_{2t}, X^D_{1t}, X^D_{2t})^\top$
- Rate factors $X^I_t = (X^I_{0t}, X^I_{1t}, X^I_{2t})^\top$:

$$\mathrm{d}X^I_t = \begin{bmatrix} \kappa^I_0 & -\kappa^I_0 & 0 \\ 0 & \kappa^I_1 & -\kappa^I_1 \\ 0 & 0 & \kappa^I_2 \end{bmatrix} \begin{bmatrix} \theta^I - X^I_{0t} \\ \theta^I - X^I_{1t} \\ \theta^I - X^I_{2t} \end{bmatrix} \mathrm{d}t + \mathrm{diag}(X^I_t) \begin{bmatrix} 0 & 0 \\ \Sigma^I_{11} & 0 \\ \Sigma^I_{21} & \Sigma^I_{22} \end{bmatrix} \begin{bmatrix} \mathrm{d}B_{1t} \\ \mathrm{d}B_{2t} \end{bmatrix},$$

  with $\zeta_t = e^{-\gamma t} X^I_{0t}$, $\theta^I = 1$, and $\gamma = 4.2\%$.

- Dividend factors $X^D_t = (X^D_{1t}, X^D_{2t})^\top$:

$$\mathrm{d}X^D_t = \begin{bmatrix} \kappa^D_1 & -\kappa^D_1 \\ 0 & \kappa^D_2 \end{bmatrix} \begin{bmatrix} \theta^D - X^D_{1t} \\ \theta^D - X^D_{2t} \end{bmatrix} \mathrm{d}t + \mathrm{diag}(X^D_{t-}) \left(\begin{bmatrix} \Sigma^D_{11} & 0 \\ \Sigma^D_{21} & \Sigma^D_{22} \end{bmatrix} \begin{bmatrix} \mathrm{d}B_{3t} \\ \mathrm{d}B_{4t} \end{bmatrix} + \begin{bmatrix} \mathrm{d}J_{1t} \\ \mathrm{d}J_{2t} \end{bmatrix} \right),$$

  with $D_t = X^D_{1t}$.

- Explicit restrictions on parameters s.t. $S^*_t < \infty$.

Model Specification

- Jump-diffusive stock price bubble:

$$\mathrm{d}L_t = L_{t-}(\sigma^L \mathrm{d}B_t^L + \mathrm{d}J_t^L)$$

- Assume L_t independent of X_t

$$\begin{aligned}\pi_t^{stock} &= \frac{1}{\zeta_t}\mathbb{E}_t\left[(L_T + \zeta_T S_T^* - \zeta_T K)^+\right] \\ &= \frac{1}{\zeta_t}\mathbb{E}_t\left[C_t^M(L_t, \tilde{K}(X_T))\right],\end{aligned}$$

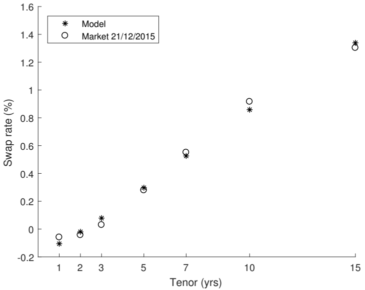
  where $C_t^M(L_t, \tilde{K}(X_T))$ is the Merton (1976) option price with spot L_t and strike $\tilde{K}(X_T) = \zeta_T K - \zeta_T S_T^*$.
- Dependence between L_t and X_t is possible as long as (X_t, L_t) remains jointly a PJD (at the cost of an increased dimension).

Data

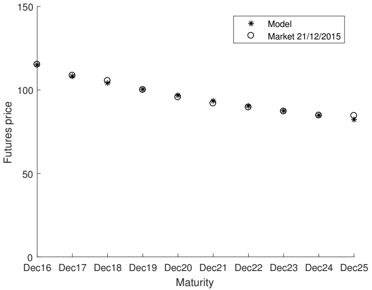
Calibration date: December 21 2015.

- **Euribor swaps (7)**
  Tenors: 1, 2, 3, 5, 7, 10, 15y.
- **Euribor swaptions (12)**
  Expiries: 3m, 6m, 1y
  Tenors: 3, 5, 10, 15y
  Moneyness: ATM
- **Eurostoxx 50 index dividend futures (10)**
  Expiry: 1-10 y
- **Eurostoxx 50 index dividend options (21)**
  Expiry: 2, 3, 4y
  Moneyness: 0.9, 0.95, 0.975, 1, 1.025, 1.05, 1.1
- **Eurostoxx 50 index options (24)**
  Expiry: 3m, 6m, 1y
  Moneyness: 0.8, 0.9, 0.95, 0.975, 1, 1.025, 1.05, 1.1

Swaps and Dividend Futures (21/12/2015)

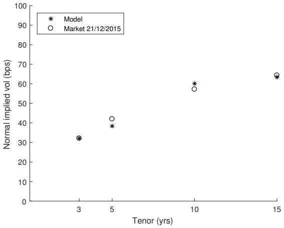


(a) Euribor swap rates

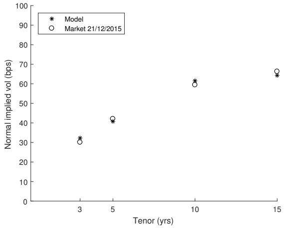


(b) Eurostoxx 50 dividend futures

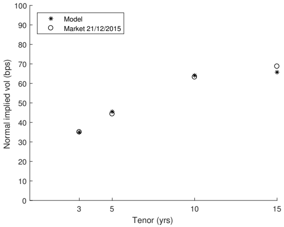
Euribor Swaptions (21/12/2015)



(a) 3 month maturity

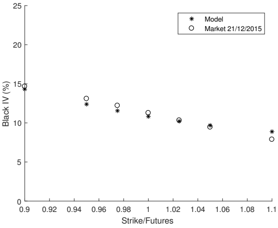


(b) 6 month maturity

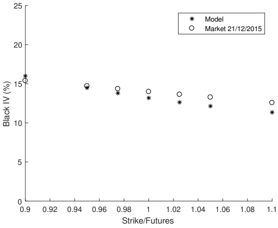


(c) 1 year maturity

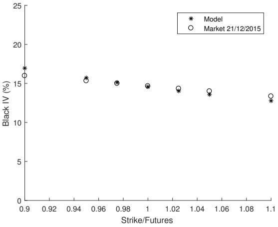
Eurostoxx 50 Dividend Futures Options (21/12/2015)



(a) Dec 2016-2017

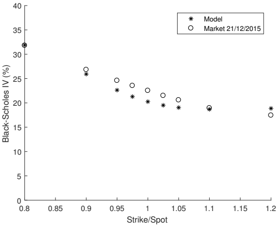


(b) Dec 2017-2018

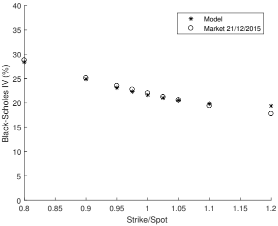


(c) Dec 2018-2019

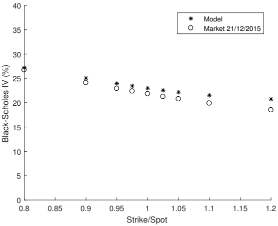
Index Options (21/12/2015)



(a) 3 month maturity

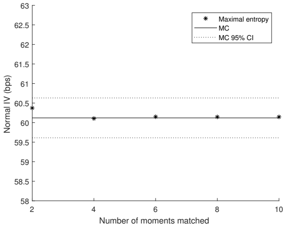


(b) 6 month maturity

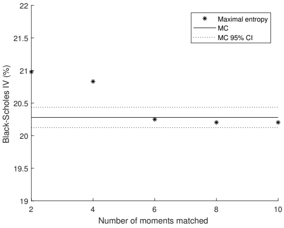


(c) 1 year maturity

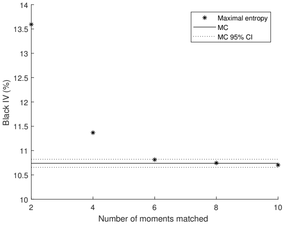
Moments and Option Prices



(a) Swaption, 3m expiry



(b) Index option, 3m expiry



(c) Dividend option, 2y expiry

Overview

Dividend Seasonality

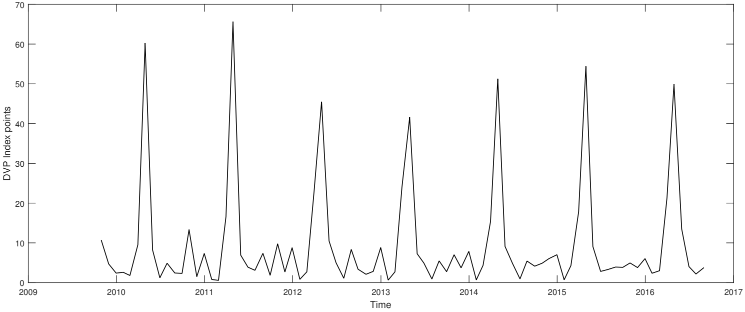


Figure: Monthly dividend payments by Eurostoxx 50 constituents (in index points) from October 2009 until October 2016. Source: Eurostoxx 50 DVP index, Bloomberg.

Dividend Seasonality

- Standard choice to model annual cycles:

$$\delta(t) = \rho_0 + \rho^\top \Gamma(t), \quad \Gamma(t) = \begin{bmatrix} \sin(2\pi t) \\ \cos(2\pi t) \\ \vdots \\ \sin(2\pi K t) \\ \cos(2\pi K t) \end{bmatrix}.$$

- Remark, $\Gamma(t)$ is the solution of a linear ODE:

$$\mathrm{d}\Gamma(t) = \text{blkdiag}\left(\begin{bmatrix} 0 & 2\pi \\ -2\pi & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & 2\pi K \\ -2\pi K & 0 \end{bmatrix} \right) \Gamma(t)\mathrm{d}t.$$

$\rightarrow$ We can add Γ to the state vector!

- For example:

$$\mathrm{d}X_t = \kappa(\delta(t) - X_t)\mathrm{d}t + \mathrm{d}M_t$$

Dividend Swaps

- Dividend swap/forward price:

$$D_{swap}(t, T_1, T_2) = \frac{1}{P(t, T_2)} \frac{1}{\zeta_t} \mathbb{E}_t \left[\zeta_{T_2} \int_{T_1}^{T_2} D_s \, \mathrm{d}s \right]$$
$$= D_{fut}(t, T_1, T_2) + \frac{\mathrm{Cov}_t \left[\zeta_{T_2}, \int_{T_1}^{T_2} D_s \, \mathrm{d}s \right]}{P(t, T_2)\zeta_t}.$$

- In polynomial framework:

$$D_{swap}(t, T_1, T_2) = \frac{w(t, T_1, T_2)^\top H_2(X_t)}{q^\top \mathrm{e}^{G_1(T_2-t)} H_1(X_t)},$$

with $w(t, T_1, T_2) = \int_{T_1}^{T_2} q^\top \mathrm{e}^{G_1(T_2-s)} \, Q \, \mathrm{e}^{G_2(s-t)} \, \mathrm{d}s$ and $QH_2(x) = H_1(x) H_1(x)^\top p$.

Conclusion

- Joint term-structure model for dividends and interest rates.
- Explicit prices for dividend futures/swaps, bonds, and dividend paying stock.
- Moment-based approximations for (path dependent) option prices using principle of maximum entropy.
- Future work:
  - Time-series estimation of S_t^*.
  - SVJ model for bubble component.
  - Single-stock framework with credit risk.

https://ssrn.com/abstract=3016310

Thank you for your attention!

References I

Ackerer, D. and D. Filipović (2016). Linear credit risk models. *Swiss Finance Institute Research Paper* (16-34).

Binsbergen, J. H. v., M. W. Brandt, and R. S. Koijen (2012). On the timing and pricing of dividends. *American Economic Review 102*(4), 1596–1618.

Binsbergen, J. H. v., W. Hueskes, R. S. Koijen, and E. B. Vrugt (2013). Equity yields. *Journal of Financial Economics 110*(3), 503–519.

Binsbergen, J. H. v. and R. S. Koijen (2017). The term structure of returns: Facts and theory. *Journal of Financial Economics*, Forthcoming.

Buehler, H. (2015). Volatility and dividends II-Consistent cash dividends. *Working Paper*.

Buehler, H., A. S. Dhouibi, and D. Sluys (2010). Stochastic proportional dividends. *Working Paper*.

Filipović, D. and M. Larsson (2017). Polynomial jump-diffusion models.

Filipović, D., M. Larsson, and A. B. Trolle (2017). Linear-rational term structure models. *Journal of Finance 72*, 655–704.

Jarrow, R. A., P. Protter, and K. Shimbo (2007). Asset price bubbles in complete markets. *Advances in Mathematical Finance*, 97–121.

References II

Jarrow, R. A., P. Protter, and K. Shimbo (2010). Asset price bubbles in incomplete markets. *Mathematical Finance 20*(2), 145–185.

Kragt, J., F. De Jong, and J. Driessen (2016). The dividend term structure. *Working Paper*.

Lettau, M. and J. A. Wachter (2007). Why is long-horizon equity less risky? A duration-based explanation of the value premium. *Journal of Finance 62*(1), 55–92.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics 3*(1-2), 125–144.

Mixon, S. and E. Onur (2016). Dividend swaps and dividend futures: State of play. *Journal of Alternative Investments*, Forthcoming.

Tunaru, R. (2017). Dividend derivatives. *Quantitative Finance*, 1–19.