General Equilibrium Theory and Climate Change[1]

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[1] This project grew out of work, joint with M. Ali Khan and Metin Uyanik, concerning externalities in production economies with bads and a continuum of agents.

GE with Climate Change

The mitigation of climate change requires the reduction of the greenhouse gas emission and other forms of pollution control. Without governmental intervention, market mechanism have proven insufficient to achieve the necessary reduction. Two most commonly used regulatory schemes are:

- Imposing quota on the emission of pollutant;
- Imposing tax on the emission of pollutant.

We introduce the *Quota Equilibrium Model* and the *Tax Equilibrium Model* that incorporate regulatory schemes to control total net pollution emission:

- Existence of Equilibrium in the Quota Equilibrium Model;
- Welfare Properties of Quota Equilibrium.
- Comparison between Quota Equilibrium and Tax Equilibrium.

The Classical Feasibility Constraint

The feasibility constraint at equilibrium in the classical GE theory is given as:

- The demand is no greater than the supply, giving rise to the notion of *free-disposal equilibrium*;
- The demand equals to the supply, giving rise to the notion of *non-free-disposal equilibrium*.

Neither free-disposal equilibrium nor non-free-disposal equilibrium is suitable to model climate change since:

- Free-disposal equilibrium implies that bads can be freely disposed at equilibrium, which is incompatible with the goal of reducing pollution;
- Non-free-disposal equilibrium requires all pollution emission to be eliminated, which is impractical in the near term.

Our Feasibility Constraint

We generalize the Arrow-Debreu model by considering the *quota-compliance region* as our feasibility constraint. Suppose the society wishes to regulate the first k commodities. The quota-compliance region $\mathcal{Z}(m)$ is:

- Let $m = (m_1, m_2, \ldots, m_k) \in \mathbb{R}^k_{\leq 0}$ be the negative of the quota for the first k commodities;
- $\mathcal{Z}(m) = \{m\} \times \prod_{k<n\leq \ell} \mathcal{Z}(m)_n$ is a convex subset of $\mathbb{R}^\ell_{\leq 0}$, where $\mathcal{Z}(m)_n$ is either $\{0\}$ or $\mathbb{R}_{\leq 0}$ for $k < n \leq \ell$.

Production Economy with Quota

$\mathcal{E} \equiv \{(X, P_\omega, e_\omega, \theta)_{\omega\in\Omega}, (Y_j)_{j\in J}, (m^{(j)})_{j\in J}, \mathcal{Z}(m)\}$ is production economy where:

- $\Delta = \{p \in \mathbb{R}^\ell : \sum_{k=1}^{\ell} |p_k| = 1\}$ is the price set; we allow *negative prices*;
- Ω is a finite set of agents. For every agent $\omega \in \Omega$, its consumption set $X(\omega)$ is a non-empty, closed and convex subset of $\mathbb{R}^\ell_{\geq 0}$. We write X_ω for $X(\omega)$;
- J is a finite set of producers. Our economy has two types of firms: private firms and a single government firm. The government firm, denoted as firm 0, has the production set $Y_0 = \{0\}$. For each private firm $j \in J$, its production set $Y_j \subset \mathbb{R}^\ell$ is a non-empty subset. We write $Y = \prod_{j\in J} Y_j$;
- The set of allocations is $\mathcal{A} = \prod_{\omega\in\Omega} X_\omega$.

Production Economy with Quota (Continued)

- Each agent's preference is characterized by a preference map $P_\omega : \mathcal{A} \times Y \times \Delta \to \mathcal{P}$ that is continuous,[2] where $\mathcal{P}$ is the set of all continuous, transitive, irreflexive and convex preferences on $\mathbb{R}^\ell_{\geq 0}$. We allow for very general form of *externality* since agent's preference depends on the allocation of all agents, the price and the production;
- $e(\omega) \in \mathbb{R}^\ell_{\geq 0}$ is the initial endowment of the agent ω;
- $\theta(\omega)(j) \in \mathbb{R}_{\geq 0}$ represents agent ω's share of firm j such that $\sum_\Omega \theta(\omega)(j) = 1$ for all $j \in J$.
- The government chooses to regulate the first k commodities and assigns quotas on regulated commodities to the firms. For each $j \in J$, define $m^{(j)} \in \mathbb{R}^k_{\leq 0}$ to be the negative of the quota for the firm j. Let $m = \sum_{j \in J} m^{(j)}$. The *quota-compliance region* is $\mathcal{Z}(m) = \{m\} \times \prod_{k < n \leq \ell} \mathcal{Z}(m)_n$.

[2] $\mathcal{P}$ is endowed with the closed convergence topology.

Property Rights in the Quota Model

- $-m^{(j)}$ represents firm j's quota right to emit regulated commodities. The profits attributed to this property right flow through to the shareholders of firm j;
- The government determines the shareholdings of the government firm;
- The allocation of quotas between the government firm and the privates firms plays a key role in the effect of the quota scheme on income distribution;
- The emissions of regulated commodities of a firm are determined endogeneously, and may be above or below the firm's assigned quota;
- When the government firm's quota is 0, our model captures the pollution rights in *cap and trade*. When private firms' quotas are 0, we refer to our model as *global quota economy*;
- The government may impose cap and trade on one regulated commodity, while imposing a global quota on another regulated commodity.

An Example on Quota Property Rights

Let $\mathcal{E}$ be a finite production economy with quota where:

- There are three commodities CO_2, coal and electricity, which we denote by c_1, c_2 and c_3;
- There are two agents with identical consumption sets $X_1 = X_2 = \{0\} \times \mathbb{R}^2_{\geq 0}$ and endowments $e(1) = e(2) = (0, 1, 0)$. Given the total net emission v of CO_2, the two agents have the same utility function $u_v(c_1, c_2, c_3) = c_3 - \frac{v^2}{2}$;
- There is a single private firm with the production set $Y = \{(r, -r, r) : r \in \mathbb{R}_{\geq 0}\}$;
- Let m be the negative of the quota on total net CO_2 emission. The quota-compliance region $\mathcal{Z}(m)$ is $\{m\} \times \mathbb{R}_{\leq 0} \times \{0\}$. The quota $m^{(0)}$ for the government firm and the quota $m^{(1)}$ for the private firm will be specified later;
- We have $\theta(1)(0) = \theta(2)(0) = \frac{1}{2}$, and $\theta(1)(1) = 1$ and $\theta(2)(1) = 0$.

An Example on Quota Property Rights (Continued)

We focus on the case where $m \in (-2, 0)$. We shall see that the allocation of quotas play a key role in agents' welfare:

- Suppose $m^{(0)} = m$ and $m^{(1)} = 0$. Then, for any equilibrium $(\bar{x}, \bar{y}, \bar{p})$, the private firm's equilibrium production is $(-m, m, -m)$, the equilibrium price is $\bar{p} = (-\frac{1}{2}, 0, \frac{1}{2})$ and both agents' equilibrium consumption of electricity are $\frac{-m}{2}$;
- Suppose $m^{(0)} = 0$ and $m^{(1)} = m$. Then, for any equilibrium $(\bar{x}, \bar{y}, \bar{p})$, the private firm's production is $(-m, m, -m)$, the equilibrium price is $\bar{p} = (-\frac{1}{2}, 0, \frac{1}{2})$ and the equilibrium consumption of electricity of the first agent is $-m$ while the equilibrium consumption of electricity of the second agent is 0.

The Quota Budget and Demand Sets

Each firm's profit at a given price p is $p \cdot y(j) + \pi_k(p) \cdot m^{(j)}$. Its supply set is $S_j^m(p) = \underset{z \in Y_j}{\operatorname{argmax}} \left(p \cdot z + \pi_k(p) \cdot m^{(j)}\right)$. Note that $S_j^m(p) = \underset{z \in Y_j}{\operatorname{argmax}}\, p \cdot z$.

Given $(y, p) \in Y \times \Delta$, the *quota budget set* $B_\omega^m(y, p)$ is:

$$\left\{ z \in X_\omega : p \cdot z \leq p \cdot e(\omega) + \sum_{j \in J} \theta_{\omega j} \left(p \cdot y(j) + \pi_k(p) \cdot m^{(j)}\right) \right\}.$$

The *quota demand set* $D_\omega^m(x, y, p)$ of the agent ω is:

$$\{z \in B_\omega^m(y, p) : w \succ_{x,y,\omega,p} z \implies w \notin B_\omega^m(y, p)\}.$$

The quota demand set consists of all $P_\omega(x, y, p)$ maximal elements of the budget set $B_\omega^m(y, p)$.

$\mathcal{Z}(m)$-compliant Quota Equilibrium

A $\mathcal{Z}(m)$-compliant quota equilibrium is $(\bar{x}, \bar{y}, \bar{p}) \in \mathcal{A} \times Y \times \Delta$ such that the following conditions hold:

- $\bar{x}(\omega) \in D^m_\omega(\bar{x}, \bar{y}, \bar{p})$ for all $\omega \in \Omega$;
- $\bar{y}(j) \in S^m_j(p)$ for all $j \in J$. So every firm is profit maximizing given the price $\bar{p}$;
- $\sum_{\omega\in\Omega} \bar{x}(\omega) - \sum_{\omega\in\Omega} e(\omega) - \sum_{j\in J} \bar{y}(j) \in \mathcal{Z}(m)$. This feasibility constraint implies that the total net amount of the first k commodities equals the pre-specified quota.

The classical free-disposal equilibrium and non-free-disposal equilibrium are special cases of our equilibrium concept.

Theorem

Under moderate regularity conditions, every finite production economy with a feasible quota $-m$ has a $\mathcal{Z}(m)$-compliant quota equilibrium.

Florenzano(2009) proved a special case where the quota-compliance region $\mathcal{Z}(m)$ is a cone, and all quotas are 0.

Constrained Pareto Ranking

It is too much to hope that quota equilibria are full Pareto optimal. After all, the equlibrium with 0 quota yields an immediate return to a pre-industrial society.

Suppose the only externality arises from the total net emission of the first k commodities. For two feasible consumption-production pairs $(g, h), (g', h')$ with the same total net pollution emission, we say (g, h) *constrained Pareto dominates* (g', h') if:

- for all $\omega \in \Omega$, the agent ω *does not* strictly prefer $g'(\omega)$ to $g(\omega)$;
- there exists some $\omega_0 \in \Omega$ such that the agent ω_0 strictly prefers $g(\omega_0)$ over $g'(\omega_0)$.

A feasible consumption-production pair is *Constrained Pareto optimal* if no feasible consumption-production pair with the same total net emission of the first k commodities Pareto dominates it.

The Welfare Property of Quota Equilibrium

Theorem

Suppose the only externality arises from the total net emission of the first k commodities, and $\mathcal{Z}(m)_n = \{0\}$ for all $k < n \leq \ell$, i.e., we require non-free-disposal for non-regulated commodities at equilibrium. Under moderate conditions, if $(\bar{f}, \bar{y}, \bar{p})$ is a $\mathcal{Z}(m)$-compliant quota equilibrium, then $(\bar{f}, \bar{y})$ is constrained Pareto optimal.

- It is possible that the equilibrium consumption-production pair of one quota Pareto dominates the equilibrium consumption-production pair of a possibly different quota;
- Once the government has established the quota, constrained Pareto optimality can be achieved via market forces, and no further government intervention is required;

Production Economy with Tax

A production economy with tax $\mathcal{F}$ is given by:

- Consumption sets, preferences, endowments, production sets, agents' shares of private firms are the same as in the production economy with quota;
- $\theta_0 : \Omega \to \mathbb{R}_{\geq 0}$ is the government's rebate share to agents such that $\sum_{\omega \in \Omega} \theta_0(\omega) = 1$. The government rebates its revenue from tax to agents according to θ_0;
- The global quota economy and the tax economy allocate property rights to agents in identical ways, namely, through the government rebate scheme;
- The interpretation of the compliance region $\mathcal{V}$ is that:[3] we allow for arbitrary net emission for the first k commodities, but charge a tax on these commodities;
- $t \in \mathbb{R}^k$ is the tax rate on the regulated commodities.

[3] $\mathcal{V}$ takes the form of $\prod_{n \leq \ell} \mathcal{V}_n$ where $\mathcal{V}_n = \mathbb{R}_{\leq 0}$ for all $n \leq k$ and $\mathcal{V}_n$ is either $\{0\}$ or $\mathbb{R}_{\leq 0}$ for all $n > k$.

The Tax Budget and Demand Sets

For $(x, y) \in \mathcal{A} \times Y$, the total net emission of the first k commodities is

$$C(x, y) = \pi_k\big(\sum_{\omega \in \Omega} e(\omega) + \sum_{j \in J} y(j) - \sum_{\omega \in \Omega} x(\omega)\big),$$

Given $(x, y, p) \in \mathcal{A} \times Y \times \Delta$, the *tax budget set* $B^t_\omega(x, y, p)$ is

$$\Big\{ z \in X_\omega : p \cdot z \leq p \cdot e(\omega) + \sum_{j \in J} \theta_{\omega j} p \cdot y(j) + \theta_0(\omega) t \cdot C(x, y) \Big\}.$$

The *tax demand set* $D^t_\omega(x, y, p)$ of the agent ω consists of all $P_\omega(x, y, p)$ maximal elements of the budget set $B^t_\omega(x, y, p)$.

$\mathcal{V}$-compliant Tax Equilibrium

A $\mathcal{V}$-compliant tax equilibrium is $(\bar{x}, \bar{y}, \bar{p}) \in \mathcal{A} \times Y \times \Delta$ such that the following conditions hold:

- The equilibrium price vector of the first k commodities is $-t$;
- $\bar{x}(\omega) \in D^t_\omega(\bar{x}, \bar{y}, \bar{p})$ for all $\omega \in \Omega$;
- $\bar{y}(j) \in \operatorname*{argmax}_{z \in Y_j} \bar{p} \cdot z$ for all $j \in J$. So every firm is profit maximizing given the price $\bar{p}$;
- $\sum_{\omega \in \Omega} \bar{x}(\omega) - \sum_{\omega \in \Omega} e(\omega) - \sum_{j \in J} \bar{y}(j) \in \mathcal{V}$.

Three key questions on tax equilibrium:

- What is the connection between tax and quota equilibrium?
- Does tax equilibrium exist in general?
- How does tax equilibrium compare with quota equilibrium?

Connection between Tax and Quota Equilibria

Every quota equilibrium can be realized as a tax equilibrium where the tax rate is set to be the negative of the equilibrium price vector of the first k commodities, and the government's rebate share defined accordingly.

Theorem

If $(\bar{x}, \bar{y}, \bar{p})$ is a quota equilibrium, then $(\bar{x}, \bar{y}, \bar{p})$ is a tax equilibrium of a production economy with tax rate $t = -\pi_k(\bar{p})$ and rebate share $\tilde{\theta}_0(\omega) = \frac{\sum_{j \in J} \theta_{\omega j} \pi_k(\bar{p}) \cdot m^{(j)}}{\pi_k(\bar{p}) \cdot m}$.

Every tax equilibrium can be realized as a global quota equilibrium where the quota is the total net pollution emission.

Theorem

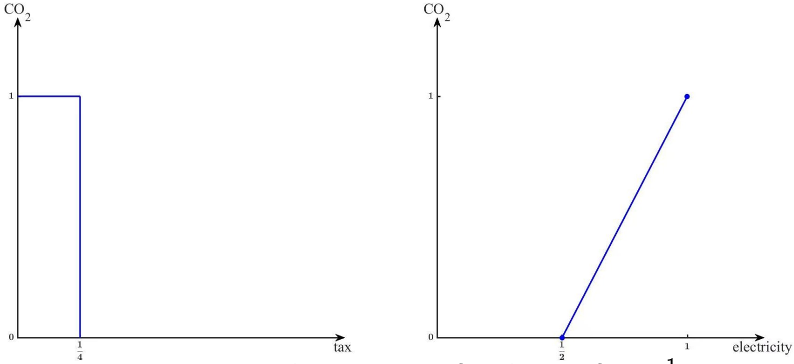
If $(\bar{x}, \bar{y}, \bar{p})$ is a tax equilibrium, then $(\bar{x}, \bar{y}, \bar{p})$ is a global quota equilibrium where the global quota is $C(\bar{x}, \bar{y})$, and each private firm's quota is 0.

An Economy with Electricity Generation and CO_2 Sequestration: Linear Case

Let $\mathcal{F}$ be a finite production economy with tax where:

- There are three commodities: CO_2, coal and electricity, which we denote by c_1, c_2 and c_3, respectively;
- There is a single agent with consumption set $X = \{0\} \times \mathbb{R}^2_{\geq 0}$, endowment $e = (0, 1, 0)$. Given the total net emission v of CO_2, the utility function $u_v(c_1, c_2, c_3) = c_3 - v^2$;
- There are two producers with production sets $Y_1 = \{(r, -r, r) : r \in \mathbb{R}_{\geq 0}\}$ and $Y_2 = \{(-2r, 0, -r) : r \in \mathbb{R}_{\geq 0}\}$;
- The compliance region $\mathcal{V} = \mathbb{R}^3_{\leq 0}$.

Properties of Tax Equilibrium



- $\mathcal{V}$-compliant tax equilibrium exists if and only if $t \leq \frac{1}{4}$;
- Total net emission of $\mathrm{CO_2}$ is 1 if $t < \frac{1}{4}$;
- If $t = \frac{1}{4}$, for every $v \leq 1$, there is a unique $\mathcal{V}$-compliant tax equilibrium such that the total net $\mathrm{CO_2}$ emission is v;
- Hence, it is impossible to achieve any pre-specified total net $\mathrm{CO_2}$ emission $v < 1$ by setting an emission tax.

Associated Quota Economy

Let $\mathcal{F}'$ be a finite production economy with quota which is defined exactly the same as $\mathcal{F}$ except that the disposal region of $\mathcal{F}'$ is given by $\mathcal{Z}(m) = \{m\} \times \mathbb{R}^2_{\leq 0}$, and there is a government firm with production set $Y_0 = \{0\}$.

- Since there is only one agent, the quota allocation between the government and private firms does not matter;
- For $-1 \leq m \leq 0$, there is a $\mathcal{Z}(m)$-compliant quota equilibrium for $\mathcal{F}'$, under which the total net CO_2 emission is $-m$.

To conclude:

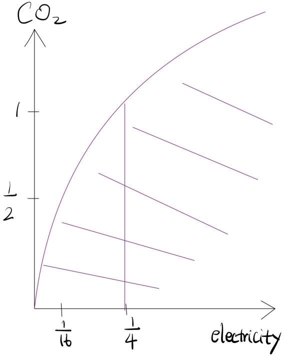
- Tax equilibrium may not exist and, in some cases, it is impossible to get under the pre-specified total net pollution emission by setting tax rate;
- Quota equilibrium always exists, and quota can be chosen to ensure that the total net pollution emission of every quota equilibrium equals a pre-specified level.

Pareto Ranking Among Equilibria

The consumption-production pair of the $\mathcal{V}$-compliant tax equilibrium $(\hat{x}, \hat{y}, \hat{p})$, where $\hat{x} = (0, 0, \frac{5}{8})$, $\hat{y} = \big((1, -1, 1), (-\frac{3}{4}, 0, -\frac{3}{8})\big)$ and $\hat{p} = (-\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$, is full Pareto optimal.

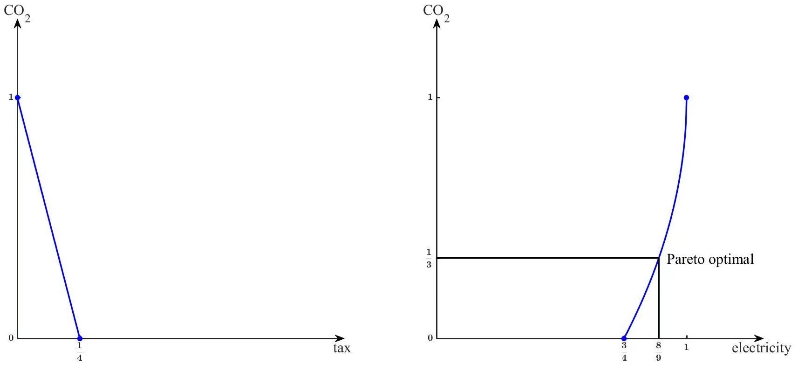
- For every $0 \leq v \leq 1$, there exists a $\mathcal{V}$-compliant tax equilibrium with tax rate $\frac{1}{4}$ and the total net $\mathrm{CO_2}$ emission v. We can not ensure full Pareto optimality by setting the tax rate alone;
- $(\hat{x}, \hat{y}, \hat{p})$ is the unique $\mathcal{Z}(-\frac{1}{4})$-compliant quota equilibrium. We can ensure full Pareto optimality of the quota equilibrium by setting the right quota.

An Economy with Electricity Generation and CO_2 Sequestration: Non-Linear Case



Let $\mathcal{G}$ be a finite production economy with tax which is the same as $\mathcal{F}$ (linear case) except that the production set of the second firm is given by $Y_2 = \{(-a, 0, -r^2) : (r \in \mathbb{R}_{\geq 0}) \wedge (0 \leq a \leq 2r)\}$.

Properties of Tax Equilibrium (Non-Linear Example)



- $\mathcal{G}$ has a unique $\mathcal{V}$-compliant tax equilibrium if and only if the tax rate $t \leq \frac{1}{4}$;
- There is a one-to-one correspondence between tax rate and total net CO_2 emission. In particular, given a tax rate $t \leq \frac{1}{4}$, the total net CO_2 emission is $1 - 4t$.
- The agent's utility is uniquely maximized at $t = \frac{1}{5}$;

Associated Quota Economy (Non-Linear Example)

Let $\mathcal{G}'$ be a finite production economy with quota which is defined exactly the same as $\mathcal{G}$ except that the compliance region of $\mathcal{G}'$ is given by $\mathcal{Z}(m) = \{m\} \times \mathbb{R}^2_{\leq 0}$, and there is a government firm with production set $Y_0 = \{0\}$.

- For $0 \leq m \leq 1$, there is a $\mathcal{Z}(m)$-compliant quota equilibrium for $\mathcal{G}'$, and the total net CO_2 emission equals $-m$. Hence, quota can be chosen to ensure that the total net CO_2 emission of every quota equilibrium equals a pre-specified level;
- Emission tax equilibrium may not exist for certain tax rates. However, there exists a one-to-one correspondence between tax rate and the total net CO_2 emission of the corresponding equilibrium. The government can limit the total net CO_2 emission under any pre-specifed level $v \leq 1$ by setting the emission tax rate to be no less than $\frac{1-v}{4}$.

Pareto Ranking Among Equilibria (Non-Linear Example)

The consumption-production pair of the $\mathcal{V}$-compliant tax equilibrium $(\bar{x}, \bar{y}, \bar{p})$, where $\bar{x} = (0, 0, \frac{21}{25})$, $\bar{y} = \big((1, -1, 1), (-\frac{4}{5}, 0, -\frac{4}{25})\big)$ and $\bar{p} = (-\frac{1}{5}, \frac{3}{10}, \frac{1}{2})$, is full Pareto optimal.

- As there is a one-to-one correspondence between tax rate and total net CO_2 emission, full Pareto optimality can be achieved by setting the tax rate to be $\frac{1}{5}$;
- $(\bar{x}, \bar{y}, \bar{p})$ is the only $\mathcal{Z}(-\frac{1}{5})$-compliant quota equilibrium. Thus, once the government sets the quota to be $\frac{1}{5}$, full Pareto optimality of the equilibrium consumption-production pair is achieved without further intervention from the government.

Concluding Remarks

To model climate change, we define the quota equilibrium model and the tax equilibrium model, incorporating regulatory schemes to control the total net pollution emission:

- Under moderate conditions, quota equilibrium always exists, and is constrained Pareto optimal;
- Full Pareto optimality of quota equilibrium can often be achieved by setting the right quota;
- The property rights specified in quota equilibrium have a major impact on the distribution of welfare among the agents;
- Every quota equilibrium can be realized as a tax equilibrium with a corresponding tax rate and government rebate scheme;
- Every tax equilibrium can be realized as a global quota equilibrium;

Concluding Remarks (Continued) and Future Work

- Unlike quota equilibrium, tax equilibrium may not exist for certain tax rates and, in some cases, it is impossible to get under the pre-specified total net pollution emission by setting tax rate;
- In some cases, one can choose among tax equilibrium and achieve full Pareto optimality, through tax alone.

Future Works

The paper suggests the following promising directions for future works:

- General results on the existence of a quota so that the quota equilibrium is full Pareto optimal;
- The second welfare theorem in our setting;
- A Multi-periods model to allow for trading long-term emission rights, choosing an optimal emission-reduction path. Such model could also capture volatility of emission price;
- A Multi-governments model to allow for international trade. Climate change policies could have a significant impact on domestic firms;
- Computable General equilibrium model based on our setting;
- A continuum agents model and its existence of equilibrium.

Technical Conditions

Theorem

Let $\mathcal{E}$ be a finite production economy with quota such that:

1. *for $\omega \in \Omega$, we have $0 \in X_\omega$, $\hat{X}_\omega$ being compact and non-empty, and $e_\omega \in \mathrm{int}\big(X_\omega - \sum_{j\in J}\theta_{\omega j}(Y_j + \{E(m^{(j)})\})\big)$;*
2. *for all feasible consumption-production pair $(x, y) \in \mathcal{A} \times Y$ and all $\omega \in \Omega$, the preference $P_\omega(x, y, p)$ has the non-satiation property for all $p \in \Delta$;*
3. *the aggregated production set $\bar{Y} = \left\{\sum_{j\in J} y(j) : y \in Y\right\}$ is closed and convex, and for each $j \in J$, the set $\hat{Y}_j$ is relatively compact and non-empty, and the set $\bar{Y} + \mathcal{Z}(m)$ is closed.*

Then, there exists a $\mathcal{Z}(m)$-compliant quota equilibrium.