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Abstract

In this paper we analyze performance-based remuneration for risk-averse managers in a Black-Scholes-type model. We assume that the firm's performance is influenced by an industry and a firm-specific risk. A relative performance compensation which rewards a manager relative to the exogenous performance of the firms in his peer group, can filter out the industry-specific risk and lower the compensation costs to the firm. However, if all managers of the firms in the peer group receive an endogenous relative performance compensation, we show that the managers may herd in their investment decisions and choose an inferior investment despite the presence of a more profitable alternative. This herding behavior is driven by the managers' risk-aversion and the endogenous relative performance compensation.

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1 Introduction

Delegating the management of a firm to a manager creates a conflict of interests between the owners of the firm and the manager. In practice, a variety of compensation schemes have been designed, depending on the absolute or relative performance of managers. The problem with absolute performance-based bonus payments is that it is sometimes difficult to distinguish between good management and luck. One potential solution is a relative performance-based bonus payment, in which a comparison group of companies in the same industry is considered and managers are rewarded if their (delegated) firm does better than the average firm in this group. Under certain assumptions a relative compensation can serve as a sufficient statistic to disentangle the firm's performance into a firm-specific component (which the manager can influence) and an industry-specific component (which the manager cannot influence).

This paper analyses the effect of relative compensation on risk-averse managers and shows that it can lead to a “bad” equilibrium outcome. While relative compensation has the potential merit of lowering the information asymmetry between the shareholders and managers, it can lead to a socially non-optimal outcome. If all managers are compensated relatively to each other, it is optimal for the managers to choose the same investment strategies. Herding can be interpreted as a hedging device for the risk-averse managers. Under certain conditions herding can lead to the existence of a bad equilibrium outcome, in which all managers choose an inferior investment project. The conclusion of our research is that, relative management compensation is not unambiguously superior to absolute performance compensation and might potentially lead to inferior outcomes.

The key assumption in our model is that all managers are compensated relatively to each other. The strategic behavior of managers is different, if their compensation is based on some exogenous benchmark, that they cannot influ-

ence. For example, if a firm is small, a major stock market index as the Dow Jones can be regarded as exogenous. An exogenous relative performance compensation can filter out some macroeconomic or industry-specific risk, which is beyond the control of the manager, and thus lower the compensation costs to the firm. However, in order to get a more precise estimation of the luck component, the comparison group may be narrowed down to a smaller number of very similar firms in the same industry. If there is a group of firms, whose managers are all compensated relatively to each other, their strategic decisions influence each other. In this paper we consider an endogenous reference index, where the managers of all the firms are compensated such that each manager gets a bonus whose magnitude depends on the the average stock price of the other firms. Under certain conditions risk-averse managers will prefer to make the same decisions as the other managers in the reference group in order to avoid risk.

Example 1. *Consider as an example the management of Ford, GM and Chrysler. As these companies are all car manufacturers, they are exposed to the same macroeconomic shocks. If Ford is doing better than GM and Chrysler, this seems to be very likely due to better management and not to luck. Assume, that the managers of these firms could either invest into new environmentally friendly technology and green cars or continue building SUVs. If the remuneration of GM's manager depends only on the stock price (or the overall performance) of the company, the environmentally friendly technology might be risky, but strictly preferred by the risk-averse manager and the risk neutral shareholders.*

However, assume now that all three managers are compensated based on the relative performance to each other. Given that GM and Chrysler have chosen SUVs over green cars, the manager of Ford can reduce his payoff risk by building SUVs as well. Hence, an overall bad decision might be profitable for the risk-averse manager if it reduces a risk that he cannot hedge. The same argument would apply to the managers of GM and Chrysler leading to a potential "bad

equilibrium".

Comparing a “good” manager to an endogenous index of firms with “bad” managers, can lead to bad outcome for all firms. In our paper the herding motive of the agents is purely motivated by their risk-aversion and the relative performance component in their contracts. Imitating the actions of the other agents in the peer group reduces unhedgeable risk of a manager. This herding behavior implies that there can be two equilibrium outcomes - one in which every manager chooses the good investment project and one in which all the managers undertake the inferior investment project. If some of the managers in the comparison group are of a “low” type, i.e. they cannot choose the better investment project either because of a lack of ability or a behavioral bias, the “bad” outcome can be the only remaining equilibrium.

In the present paper, we consider relative performance evaluation in a two-period Black-Scholes economy with many managers of different types. The managers can choose among investment opportunities with different drift and volatility. The principal agent problem for the shareholders is to choose a compensation scheme that induces the managers (agent) to pick the right investment project under the lowest costs for the shareholders (principal). Our model focuses on illustrating the direct effect of the compensation externality and risk-aversion on the strategic behavior. Among the herding literature, Gumbel (2005) is the most similar one to ours and presents an information model of delegated portfolio management in which herding behavior of risk-averse managers can also occur. Our model differs from Gumbel (2005) in two aspects. First, our setting is not a conventional information model. We focus only on the herding aspect and leave other components out, e.g. acquisition of information, which are not necessary to show our results. Second, we work in a more realistic Black-Scholes economy and analyze how herding can occur in a model with $N \geq 2$ managers, allowing the existence of different types of managers. The herding idea in our paper is also dif-

ferent from the informational cascade described in Bikhchandani et. al. (1998). In their setup every manager receives a private signal about the quality of an investment project, while in our model, investment projects are exogenously given. In our model relative compensation poses a direct externality of one manager's action on the other managers' outcomes. A somewhat different herding story is presented in Scharfstein and Stein (1990), where managers have career concerns and investors in the future periods will judge them partly on their performance compared with other managers. These reputational concerns represent an indirect externality for the managers, which can lead to the "sharing-the-blame" effect which drives managers to herd. In most of the literature on optimal managerial contracting in information models, the security returns are modeled quite simply. Dybvig et. al (2009) represents an exception here and derives optimal managerial contracts using a rich model of security returns. However, they do not consider the effect of endogenous relative compensation. Empirical evidence on relative performance evaluation is mixed. For instance, Murphy (1985), Antle and Smith (1986), Gibbons and Murphy (1990), and Janakiraman et. al (1992) support it, while Barro and Barro (1990), Jensen and Murphy (1990), and Aggarwal and Samwick (1999a, b) reject its use.

The remainder of the paper is organized as follows. Section 2 introduces the underlying assumptions in our model and particularly the two investment possibilities the management faces. Here we allow for different drifts, but the same volatility in the investment projects. Section 3 considers four different cases for the manager's remuneration and solves the corresponding principal-agent problems. Section 4 moves to the case where the investment possibilities are with different volatilities. In the subsequent Section 5, numerical analyses are carried out to answer the question when the managers exhibit herding behavior and all choose the bad project. Section 6 concludes the paper and Section 7 discusses some results in the main text by releasing some relevant assumptions and collects all the proofs.

2 Model

Assume we have $N \geq 2$ firms. Each firm has a manager and shareholders. We assume that all the managers are risk-averse and have the same CRRA utility function

$$U(x) = \frac{1}{1-\gamma} x^{1-\gamma}, \quad \gamma > 1,$$

while the shareholders are risk-neutral.¹ The managers have an outside option, which will yield him the utility \bar{U} . Hence, the certainty equivalent payoff of the outside option to the managers is CE :

$$CE = ((1-\gamma)\bar{U})^{\frac{1}{1-\gamma}}.$$

The management of the firms decides about investment decisions. For simplicity, we assume that the investment decisions have purely impact on the stock prices of the firms. The shareholders decide about the remuneration of the managers. We consider only two periods. At time 0 the compensation scheme of the manager is fixed. Then the manager decides which investments to make. At time T the payoffs are realized and the manager receives his compensation.

We assume that the stock price of firm i follows a geometric Brownian motion under the real world measure \mathbb{P}^2 :

$$\frac{dS_i(t)}{S_i(t)} = \mu_i dt + \sigma^* dW^*(t) + \sigma dW_i(t) \quad i = 1, \dots, N$$

Here μ_i denotes the drift and σ and σ^* the volatilities. The uncertainty of the stock price can be decomposed into randomness concerning all companies in a certain industry (given by $\sigma^* dW^*(t)$) and randomness specific to the firm (given

¹None of the results change, if we allow the shareholders to be risk-averse as long as they are less risk averse than the manager. We do not consider the case $\gamma < 1$ for simplicity.

²It is straightforward to let the volatilities be different among the firms. Adding the complexity is not necessary to make our point.

by $\sigma dW_i(t)$). We assume that W^* and W_i are independent.

Assume, the firm i's manager can choose between two different investment projects which lead to the following two possible stock prices (good project results in \bar{S}_i and bad project in \tilde{S}_i)

$$\begin{aligned}\bar{S}_i &= S_0 \exp \left(\left(\bar{\mu} - \frac{1}{2}(\sigma^*)^2 - \frac{1}{2}\bar{\sigma}^2 \right) T + \sigma^* W^*(T) + \bar{\sigma} \bar{W}_i(T) \right) \\ \tilde{S}_i &= S_0 \exp \left(\left(\tilde{\mu} - \frac{1}{2}(\sigma^*)^2 - \frac{1}{2}\tilde{\sigma}^2 \right) T + \sigma^* W^*(T) + \tilde{\sigma} \tilde{W}_i(T) \right).\end{aligned}$$

Distinguishing between an industry-wide risk and a firm-specific risk, we can introduce the following notations:

$$\bar{S}_i = \bar{X}_i Y, \quad \tilde{S}_i = \tilde{X}_i Y,$$

where

$$\begin{aligned}Y &= \exp \left(-\frac{1}{2}(\sigma^*)^2 T + \sigma^* W^*(T) \right) \\ \bar{X}_i &= S_0 \exp \left(\left(\bar{\mu}_i - \frac{1}{2}\bar{\sigma}^2 \right) T + \bar{\sigma} \bar{W}_i(T) \right) \\ \tilde{X}_i &= S_0 \exp \left(\left(\tilde{\mu}_i - \frac{1}{2}\tilde{\sigma}^2 \right) T + \tilde{\sigma} \tilde{W}_i(T) \right).\end{aligned}$$

The only difference between the two projects is that the good project \bar{S}_i has the parameters $(\bar{\sigma}^2, \bar{\mu}, \bar{W})$, while the bad project \tilde{S}_i has instead $(\tilde{\sigma}^2, \tilde{\mu}, \tilde{W})$. We mean “good project” by assuming

$$\begin{aligned}E[\bar{S}_i] &\geq E[\tilde{S}_i] \Leftrightarrow \bar{\mu} \geq \tilde{\mu} \\ E \left[\frac{\bar{S}_i^{1-\gamma}}{1-\gamma} \right] &\geq E \left[\frac{\tilde{S}_i^{1-\gamma}}{1-\gamma} \right] \Leftrightarrow \bar{\mu} - \tilde{\mu} \geq \frac{1}{2} (\bar{\sigma}^2 - \tilde{\sigma}^2) \gamma.\end{aligned} \quad (2.1)$$

For the simple case that $\bar{\sigma}^2 = \tilde{\sigma}^2 = \sigma^2$ and $\bar{\mu} > \tilde{\mu}$, the above two inequalities are trivially satisfied. Intuitively, $\bar{\mu} > \tilde{\mu}$ implies that a good project results in a higher expected return. In the appendix, we treat the more general case with

$\bar{\sigma}^2 \neq \tilde{\sigma}^2$.

Note that firm $j \neq i$ can only invest in two projects as well. Compared to firm i , the only difference lies in the firm-specific risks: $\bar{W}_j(T)$ and $\tilde{W}_j(T)$. For different firms, the correlation between the same investment projects is higher than for different ones:

$$d\bar{W}_i d\tilde{W}_j = d\tilde{W}_i d\bar{W}_j = \rho_L$$

$$d\bar{W}_i d\bar{W}_j = d\tilde{W}_i d\tilde{W}_j = \rho_H$$

with $\max(\rho_L, 0) < \rho_H$.³ It is intuitive to assume that the firm-specific risks of two firms are more correlated, when both the firms decide for the same project, whatever the project is good or bad.

In the subsequent section, we will discuss how diverse remuneration schemes influence the management's investment decisions. We consider four different cases for the manager's remuneration:

1. A bonus scheme based on the observable stock price of the firm S .
2. A bonus scheme based on the unobservable X (recall that S has an industry-wide risk Y and a firm-specific risk X .)
3. A relative compensation only for firm i , i.e. manager i 's compensation depends on the stock price of all the other companies within the industry, while the other firms are compensated independently of firm i and use an absolute compensation as in case 1.
4. All managers obtain a relative compensation.

³The condition $\rho_H > 0$ is not necessary in the case $\bar{\sigma} = \tilde{\sigma}$. As we will see later, only the difference $\rho_H - \rho_L$ matters. However, from an economic point of view it is sensible to require similar investment projects to be positively correlated. For the more general case $\bar{\sigma} \neq \tilde{\sigma}$, where this assumption becomes important.

3 Performance Based Compensation

3.1 Case 1: A bonus scheme based on S

Take firm i as an example, in which the stock price of the firm is S_i and the manager of firm i obtains a fraction α_1 of S_i .⁴ The shareholders receive what remains $(1 - \alpha_1)S_i$. The shareholders optimally determine the compensation parameter α_1 under the constraint that \bar{S}_i is implemented:

$$\begin{aligned} \text{Shareholders} \quad & \max_{\alpha_1} E[(1 - \alpha_1)\bar{S}_i] \\ \text{subject to} \quad & E[U(\alpha_1\bar{S}_i)] \geq \bar{U} \quad (\text{PC}) \\ & E[U(\alpha_1\bar{S}_i)] \geq E[U(\alpha_1\tilde{S}_i)] \quad (\text{IC}). \end{aligned}$$

where PC stands for participation constraint and IC for incentive constraint. Note that the incentive constraint is always satisfied due to (2.1). The participation constraint holds if

$$\alpha_1 \geq \alpha_1^* := \frac{CE}{(E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}} (E[Y^{1-\gamma}])^{\frac{1}{1-\gamma}}} = \frac{CE}{S_0} \exp\left(-\left(\bar{\mu} - \gamma\frac{1}{2}\sigma^2 - \gamma\frac{1}{2}(\sigma^*)^2\right)T\right).$$

When the manager's salary is a fixed fraction of S_i , the first-best solution can be implemented if the shareholder provides a compensation $\alpha_1^*S_i$. The cost of compensation is given by

$$\begin{aligned} E[\alpha_1^*\bar{S}_i] &= \frac{((1 - \gamma)\bar{U})^{\frac{1}{1-\gamma}}}{(E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}} (E[Y^{1-\gamma}])^{\frac{1}{1-\gamma}}} E[\bar{X}_i Y] \\ &= CE \cdot \frac{E[\bar{X}_i]}{(E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}}} \cdot \frac{E[Y]}{(E[Y^{1-\gamma}])^{\frac{1}{1-\gamma}}} \\ &=: CE \cdot k_X \cdot k_Y. \end{aligned}$$

⁴In practice, managers typically receive a fixed and a variable component. In our problem, we focus only on the variable component and assume that a certain amount of utility is obtained by the variable component. We do not explicitly derive the relationship between the fixed and variable component. The same compensation scheme has been used by Basak, Pavlova and Shapiro (2003). With such type of compensation, the managers have an explicit incentive to increase the stock price at time T .

Lemma 1. *It holds $k_X > 1$ for $\sigma > 0$ and $k_Y > 1$ for $\sigma^* > 0$. Hence, $k_X > 1$ and $k_Y > 1$ can be interpreted as a risk premium that the manager demands, because he cannot hedge these risks that are part of his remuneration.*

The costs to the shareholders are then

$$E[\alpha_1^* \bar{S}_i] = CE \cdot \exp\left(\left(\frac{\gamma}{2}\sigma^2 + \frac{\gamma}{2}(\sigma^*)^2\right)T\right).$$

The positive risk premium increases in degree of risk aversion γ , the level of riskiness σ and σ^* .

3.2 Case 2: A bonus scheme based on X

In the unrealistic case, where the shareholders can observe the “luck” component of the firm’s output, they can write a more “efficient” contract. If S_i and Y can be observed, then the realization of X_i is known as well. Hence, the shareholders can directly contract on X_i . We assume that the manager gets a constant fraction α_2 of \bar{X}_i respectively \tilde{X}_i . The optimal compensation parameter α_2 can be determined similarly as in Case 1. The optimization problem of the shareholders for \bar{X}_i being implemented is given by

$$\begin{aligned} \text{Shareholders} \quad & \max_{\alpha_2} E[S_i - \alpha_2 \bar{X}_i] \\ \text{subject to} \quad & E[U(\alpha_2 \bar{X}_i)] \geq \bar{U} \quad (\text{PC}) \\ & E[U(\alpha_2 \bar{X}_i)] \geq E[U(\alpha_2 \tilde{X}_i)] \quad (\text{IC}). \end{aligned}$$

Note that the incentive constraint is always satisfied due to (2.1). The participation constraint holds if

$$\alpha_2 \geq \alpha_2^* := \frac{CE}{(E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}}} = \frac{CE}{S_0} \exp\left(-\left(\bar{\mu} - \gamma \frac{1}{2}\sigma^2\right)T\right).$$

When the manager’s salary is a fixed fraction of X_i , the first-best solution can be implemented if the shareholder provides a compensation $\alpha_2^* \bar{X}_i$. The cost of

compensation is given by

$$\begin{aligned}
 E[\alpha_2^* \bar{X}_i] &= \frac{((1 - \gamma)\bar{U})^{\frac{1}{1-\gamma}}}{(E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}}} E[\bar{X}_i] \\
 &= CE \cdot \frac{E[\bar{X}_i]}{(E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}}} \\
 &= CE \exp\left(\left(\frac{\gamma}{2}\sigma^2\right) T\right) \\
 &= CE \cdot k_X \leq CE \cdot k_X \cdot k_Y = E[\alpha_1^* \bar{S}_i]
 \end{aligned}$$

because $k_Y > 1$ for $\sigma^* > 0$. The compensation scheme directly based on X_i is cheaper for the shareholders. Thus, as expected including more information is beneficial.

3.3 Case 3: A relative compensation only for firm i

Unfortunately, in practice, the shareholders do not observe Y . However, by comparing the *realized* performance between different firms they can filter out the influence of luck on the overall output. Assume there are N firms in the economy and the performance of manager i is compared to the geometric average of the performance of the other $N - 1$ firms. In more detail, we assume that the manager of firm i receives a relative compensation:

$$\beta \frac{S_i}{\left(\prod_{j \neq i} S_j\right)^{1/(N-1)}}.$$

Here, we consider the case in which firm i is the exclusive firm using relative compensation scheme. All the other firms offer a compensation scheme as in case 1, i.e. $\alpha_1 S_j$, $j \neq i$. From the analysis for case 1, we already know that manager $j \neq i$ will choose good projects when $\alpha_1 \geq \alpha_1^*$. The shareholders in firm i solve the following optimization problem, if they want the good project \bar{S}_i to

be implemented.

$$\begin{aligned}
 \text{Shareholders} \quad & \max_{\beta} \left(E \left[\bar{S}_i - \beta \frac{\bar{S}_i}{\left(\prod_{j \neq i} \bar{S}_j \right)^{1/(N-1)}} \right] \right) \\
 \text{subject to} \quad & EU \left(\beta \frac{\bar{S}_i}{\left(\prod_{j \neq i} \bar{S}_j \right)^{1/(N-1)}} \right) \geq \bar{U} \quad (\text{PC}) \\
 & EU \left(\beta \frac{\bar{S}_i}{\left(\prod_{j \neq i} \bar{S}_j \right)^{1/(N-1)}} \right) \geq EU \left(\beta \frac{\tilde{S}_i}{\left(\prod_{j \neq i} \tilde{S}_j \right)^{1/(N-1)}} \right) \quad (\text{IC})
 \end{aligned}$$

Lemma 2. *The expected utility of choosing \bar{S}_i given that $N - 1$ firms choose the good project is*

$$EU \left(\beta \frac{\bar{S}_i}{\left(\prod_{j \neq i} \bar{S}_j \right)^{1/(N-1)}} \right) = \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left(\frac{1}{2} (1-\gamma)^2 \sigma^2 \frac{N}{N-1} (1-\rho_H) T \right) \quad (3.1)$$

The expected utility of choosing \tilde{S}_i given that $N - 1$ firms choose the good project is

$$\begin{aligned}
 EU \left(\beta \frac{\tilde{S}_i}{\left(\prod_{j \neq i} \tilde{S}_j \right)^{1/(N-1)}} \right) &= \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left((1-\gamma)(\tilde{\mu} - \bar{\mu}) T \right. \\
 &\quad \left. + \frac{1}{2} (1-\gamma)^2 \sigma^2 \left(2(\rho_H - \rho_L) + \frac{N}{N-1} (1-\rho_H) \right) T \right). \quad (3.2)
 \end{aligned}$$

Lemma 3. *The incentive constraint is always satisfied.*

The minimum number of shares β^* such that the participation constraint is satisfied is equal to

$$\beta^* = CE \exp \left(\frac{1}{2} (\gamma - 1) \sigma^2 \frac{N}{N-1} (1 - \rho_H) T \right).$$

The costs to the shareholders are then

$$E \left[\beta^* \frac{\bar{S}_i(T)}{\left(\prod_{1 \leq j \leq N-1} \bar{S}_j(T) \right)^{1/(N-1)}} \right] = CE \exp \left(\frac{1}{2} \sigma^2 \frac{N}{N-1} (1 - \rho_H) \gamma T \right).$$

Lemma 4. *If $\rho_H \geq 1/N := \rho_H^*$, then the compensation scheme of case 3 is cheaper than the compensation scheme of case 2 (and accordingly also of case 1).*

Since in equilibrium, all the managers choose the same (good) project, ρ_L does not play a role in the cost. The higher the correlation coefficient ρ_H , the less volatile the relative compensation $\bar{S}_i(T) / \left(\prod_{1 \leq j \leq N-1} \bar{S}_j(T) \right)^{1/(N-1)}$ becomes. If there is a perfect positive correlation $\rho_H = 1$, the relative compensation becomes risk-less.⁵

3.4 Case 4: All managers obtain a relative compensation.

The interesting case is when all managers receive a relative compensation. Then manager i 's decision will affect the choice of manager j , which in turn has an effect on manager i . Here, all managers make their decisions simultaneously.⁶ Therefore, the optimal investment decision is determined endogenously among managers. In this section, we examine pure Nash equilibria. The important question is when a bad Nash equilibrium exists, in which all the managers choose bad projects. Assuming all the managers are identical (particularly with the same risk aversion), a Nash equilibrium can be characterized up to relabeling by the number N_G of managers that choose the good project in the economy.

Similarly as in Case 3, there are N firms in the economy and the performance of manager i is compared to the geometric average of the performance of the other $N - 1$ firms. Assume that from the other $N - 1$ firms, $N_G - 1$ choose the

⁵In our setup we can perfectly filter out the industry-specific risk. In practice, a benchmark index will not completely filter out the luck component. It is straightforward to extend our model to capture this effect. However, this is not going to change our results qualitatively.

⁶In practice, there is always some time delay between the decisions of different managers. However, what is important is not when the decision is made, but when it becomes public knowledge. We think it is not unrealistic to assume that at the time when the managers decide about their investment projects, they do not know what their peers have done, and hence we can treat the decision making process as a simultaneous event.

good project \bar{S} , while $N_B = N - N_G$ choose the bad project \tilde{S} . Without loss of generality we can label firm i as N_G and assume that the first $N_G - 1$ firms choose the good project. To simplify notation, we denote the expected utility of firm i given that $N_G - 1$ of the other firms choose \bar{S} as

$$EU(S_i | N_G - 1) = \frac{\beta^{1-\gamma}}{1-\gamma} \left(\frac{S_i}{\left(\prod_{1 \leq j \leq N_G-1} \bar{S}_j \right)^{1/(N-1)} \left(\prod_{N_G+1 \leq j \leq N} \tilde{S}_j \right)^{1/(N-1)}} \right)^{(1-\gamma)}$$

for $S_i = \bar{S}_i$ or \tilde{S}_i . We define $p = \frac{N_G-1}{N-1}$ as the fraction of the good firms among the other firms.

The shareholders in firm i solve the following optimization problem, if they want the good project \bar{S}_i to be implemented.

$$\begin{aligned} \text{Shareholders} \quad & \max_{\beta} \left(E \left[\bar{S}_i - \beta \frac{\bar{S}_i}{\left(\prod_{1 \leq j \leq N_G-1} \bar{S}_j \right)^{1/(N-1)} \left(\prod_{N_G+1 \leq j \leq N} \tilde{S}_j \right)^{1/(N-1)}} \right] \right) \\ \text{subject to} \quad & EU(\bar{S}_i | N_G - 1) \geq \bar{U} \quad (\text{PC}) \\ & EU(\bar{S}_i | N_G - 1) \geq EU(\tilde{S}_i | N_G - 1). \quad (\text{IC}) \end{aligned}$$

An important assumption is that if two firms make the same investment decisions, there is more co-movement in the output than if they invest into different projects. This is captured by the assumption $\rho_H > \max(\rho_L, 0)$.

Lemma 5. *The expected utility of choosing \bar{S}_i given that $N_G - 1$ firms choose the good project is*

$$\begin{aligned} EU(\bar{S}_i | N_G - 1) = & \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left((1-p)(1-\gamma)(\bar{\mu} - \tilde{\mu})T \right. \\ & \left. + \frac{1}{2}(1-\gamma)^2 \sigma^2 \left(2(1-p)^2(\rho_H - \rho_L) + \frac{N}{N-1}(1-\rho_H) \right) T \right) \end{aligned}$$

The expected utility of choosing \tilde{S}_i given that $N_G - 1$ firms choose the good project

is

$$EU(\tilde{S}_i|N_G - 1) = \frac{\beta^{1-\gamma}}{1-\gamma} \exp\left(p(1-\gamma)(\tilde{\mu} - \bar{\mu})T\right) + \frac{1}{2}(1-\gamma)^2\sigma^2\left(2p^2(\rho_H - \rho_L) + \frac{N}{N-1}(1-\rho_H)\right)T$$

where $p = \frac{N_G-1}{N-1}$.

Lemma 6. *The incentive constraint $EU(\bar{S}_i|N_G - 1) \geq EU(\tilde{S}_i|N_G - 1)$ is satisfied if and only if*

$$\bar{\mu} - \tilde{\mu} \geq (\gamma - 1)(\rho_H - \rho_L)\sigma^2\left(1 - 2\left(\frac{N_G - 1}{N - 1}\right)\right) \quad (3.3)$$

For $N_G = 0, 1$, (3.3) is satisfied automatically. For $N_G > 1$, the higher the number N_G , the lower the RHS of (3.3) and the more likely the above inequality holds. For this case, there is a critical number $N_G^* - 1$ of other managers to choose the good project such that the incentive constraint is always satisfied for $N_G \geq N_G^*$. In other words, if the fraction p of the managers who choose the good project satisfies

$$p \geq p^* := \frac{N_G^* - 1}{N - 1} = \max\left\{0, \frac{1}{2}\left(1 - \frac{\bar{\mu} - \tilde{\mu}}{(\gamma - 1)\sigma^2(\rho_H - \rho_L)}\right)\right\}, \quad (3.4)$$

then the incentive constraint holds. As p^* owns a value smaller than 0.5, the inequality (3.3) is always satisfied if at least half of the other managers choose good projects, i.e. $p \geq 0.5$.

Definition 1. *A Nash equilibrium with N managers is described by their choices (S_1, \dots, S_N) . Up to relabeling a Nash equilibrium is completely characterized by the number N_G of good projects and the number $N_B = N - N_G$ of bad projects. For $N_G = 0$ and $N_G = N$ we call the Nash equilibrium symmetric, as all agents choose the same action.*

As the utility function of each agent is the same, we will suppress the index i and j in the following.

Proposition 1. *For $N \geq 3$, asymmetric pure Nash equilibria, i.e. $1 \leq N_G \leq N - 1$, do not exist. $N_G = N$ is always a Nash equilibrium. $N_G = 0$ is a Nash equilibrium if and only if $\bar{\mu} - \tilde{\mu} \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$.*

The main takeaway for $N \geq 3$ is that both extreme outcomes, the one where every manager chooses the good project and the one where all managers pick the bad project, can be equilibria. In other words, the bad equilibrium, i.e. herding behavior, may arise when the mean return spread $\bar{\mu} - \tilde{\mu}$ is smaller than $(\gamma - 1)(\rho_H - \rho_L)\sigma^2$. The more risk averse the managers, the higher this magnitude and the more likely this bad equilibrium might occur. Moreover, herding behavior is justified by the fact that the managers' expected utility is identical in the good and bad equilibrium, i.e.

$$EU(\bar{S}_i|N - 1) = EU(\tilde{S}_i|0) = \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left\{ \frac{1}{2}(1-\gamma)^2\sigma^2 \frac{N}{N-1}(1-\rho_H)T \right\}.$$

It implies that the managers have no incentive to ensure that the good equilibrium results.

Proposition 2. *For $N = 2$, there exists an asymmetric Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) = (\gamma - 1)(\rho_H - \rho_L)\sigma^2$. $N_G = 2$ is a Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) \geq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$. $N_G = 0$ is a Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$.*

3.5 Different types of managers

Here, we restrict ourselves to $N \geq 3$. We add another ingredient to our model: different types of managers. The idea is that some managers are better than others. Some managers might have more information or better business relationships than other managers. Another interpretation is that the “bad” managers suffer from some form of behavioral bias. We model the difference in abilities by assuming that there is a number of “bad” managers who can only implement the bad project. Equivalently, we can say that there is an upper bound \bar{N}_G on the

number of managers who can choose between the good and the bad project. This is a restriction to the possible Nash equilibria to the set $\{N_G | 0 \leq N_G \leq \bar{N}_G\}$. From our previous analysis we already know that $N_G \in \{1, \dots, \bar{N}_G - 1\}$ cannot be a Nash equilibrium.

Proposition 3. *If $\bar{N}_G < N_G^*$ (N_G^* defined in (3.4)), then \bar{N}_G is not a Nash equilibrium and $N_G = 0$ is the exclusive Nash equilibrium. If $\bar{N}_G \geq N_G^*$, then $N_G = \bar{N}_G$ is the exclusive Nash equilibrium.*

Note that in the proof of the above proposition, we do not need to examine whether the “bad” managers have the incentive to deviate because they do not have the choice to implement the good project.

4 Different Volatilities for Different Investment Projects

So far we have assumed that the good and the bad investment project differ only in their drift $\bar{\mu}$ respectively, $\tilde{\mu}$. In Appendix 7.2 we treat the more general case with $(\bar{\mu}, \bar{\sigma}) \neq (\tilde{\mu}, \tilde{\sigma})$ in both components. Our results still hold in the more general case.

Proposition 4. *For $N \geq 3$, asymmetric pure Nash equilibria, i.e. $1 \leq N_G \leq N - 1$, do not exist. $N_G = N$ is always a Nash equilibrium. $N_G = 0$ is a Nash equilibrium if and only if*

$$\bar{\mu} - \tilde{\mu} \leq \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 - (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}(\gamma - 1)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\tilde{\sigma} \bar{\sigma} \rho_L).$$

The incentive constraint $EU[\bar{S}_i | N_G - 1] \geq EU[\tilde{S}_i | N_G - 1]$ is satisfied if and only if

$$\bar{\mu} - \tilde{\mu} > \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 + (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}(\gamma - 1)(1 - 2p)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma} \tilde{\sigma} \rho_L)$$

The critical number $N_G^ - 1$ of other managers to choose the good project, such that the incentive constraint is always satisfied, (respectively, the critical fraction*

p^*) is given by:

$$p^* := \frac{N_G^* - 1}{N - 1} = \max \left\{ 0, \frac{1}{2} \left(1 - \frac{(\bar{\mu} - \tilde{\mu}) - \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 + (\gamma - 1)(1 - \rho_H))}{\frac{1}{2}(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L)} \right) \right\}.$$

5 Numerical Analysis

In this section we want to analyze how relevant our findings are for a range of realistic parameter values. We are interested in four questions; each will be analyzed in a separate subplot:⁷

- a) How different are the costs to the shareholders for each of the compensation schemes considered in case 1, case 2 and case 3, if we consider different levels of risk aversion for the manager?
- b) What is the minimal spread between $\bar{\mu}$ and $\tilde{\mu}$ to ensure that the bad equilibrium cannot occur for different levels of risk aversion for the managers in case 4? Recall that the condition for a good Nash equilibrium holds trivially, while the bad equilibrium requires $\bar{\mu} - \tilde{\mu}$ to be bounded by the magnitude $(\gamma - 1)(\rho_H - \rho_L)\sigma^2$. In other words, this magnitude is the minimum spread between $\bar{\mu}$ and $\tilde{\mu}$ above which the bad equilibrium cannot occur.
- c) What is the minimal fraction of good managers p^* below which the bad equilibrium is the only equilibrium outcome as a function of the risk aversion in case 4?
- d) For a fixed risk aversion, how many good managers do we need in case 4 to avoid that the bad equilibrium is the only equilibrium outcome?

⁷We only consider the simple case with $\bar{\sigma} = \tilde{\sigma} = \sigma$. The results for $\bar{\sigma} \neq \tilde{\sigma}$ are quantitatively similar.

To answer these questions, we choose the parameters as follows:

$$\begin{aligned} \gamma = 3, CE = 100, \bar{\mu} = 0.10, \tilde{\mu} = 0.08, \sigma = 0.20, \\ \sigma^* = 0.10, T = 1, N = 30, \rho_H = 0.5, \rho_L = -0.5. \end{aligned} \quad (5.1)$$

We have fixed the (reservation) certainty equivalent (CE) for managers with different levels of risk aversion, which implicitly assumes that they own different reservation utilities. This is a plausible assumption.

In Figure 1 and Table 1, we analyze the four questions for a particular parameter set. The good investment project has a drift of $\bar{\mu} = 0.1$ while the bad investment project merely has a drift of $\tilde{\mu} = 0.08$. In the first plot we show the costs to the shareholders of the three different compensation schemes labeled as case 1 to 3. Obviously, a relative compensation scheme seems to be substantially cheaper, because we have chosen $\rho_H > 1/N$. The higher the risk aversion of the manager, the more expensive a performance-based compensation becomes and the larger the cost advantage of a relative compensation scheme. In the second plot we look at the minimal spread that is necessary to enforce that there is only a unique “good” equilibrium outcome. This means, if the spread $\bar{\mu} - \tilde{\mu}$ is below the plotted line for a given risk aversion, the “bad” equilibrium is possible. For a risk aversion of $\gamma = 3$ the minimal spread, which we need to avoid a bad equilibrium, is already relatively high, namely 0.08. This implies that the existence of bad equilibria is a relevant question. In the third plot, we show the relationship between p^* (the fraction of good managers among the other managers) and the risk aversion. If the fraction of bad managers is larger than $1 - p^*$, then there is only a bad equilibrium outcome. Note that p^* converges to 0.5 for an increasing risk aversion γ . For $\gamma = 3$, the critical fraction is 0.375, i.e. if more than 72.5% of the managers choose the bad investment, then the unique outcome is the bad equilibrium. In the fourth plot, we are interested in how the number of good managers influences the equilibria. We fix $\gamma = 3$. For instance, in a group of

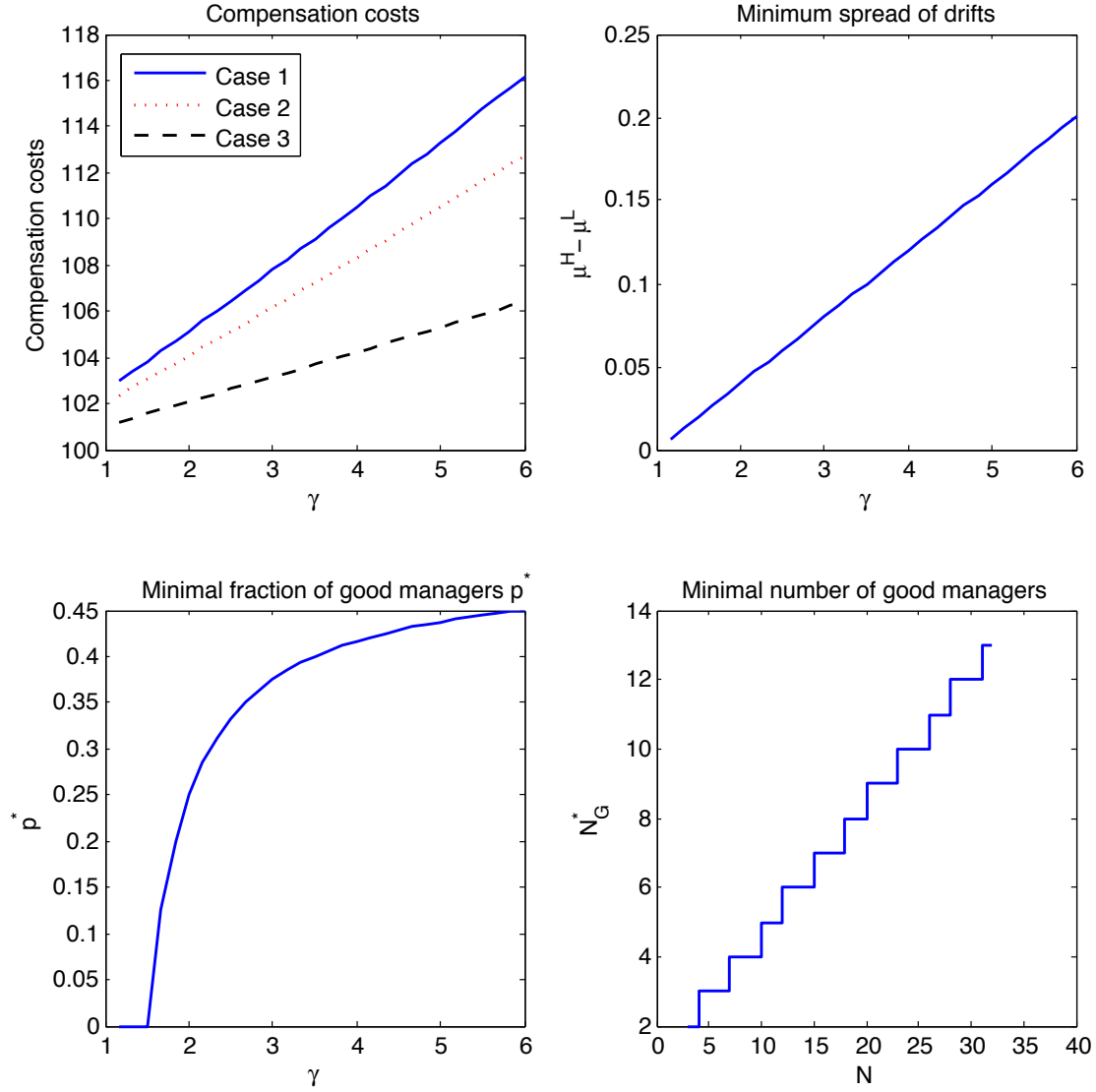


Figure 1: Cost of compensation and occurrence of bad and good equilibria with parameters: $\gamma = 3$, $CE = 100$, $\bar{\mu} = 0.10$, $\tilde{\mu} = 0.08$, $\sigma = 0.20$, $\sigma^* = 0.10$, $T = 1$, $N = 30$, $\rho_H = 0.5$, $\rho_L = -0.5$.

γ	cost of compensation (case 1)	cost of compensation (case 2)	cost of compensation (case 3)	minimum spread $\bar{\mu} - \tilde{\mu}$	$p^* = \frac{N_G^* - 1}{N - 1}$	N_G^*
1.5	103.821	103.045	101.564	0.02	0.	1
2.	105.127	104.081	102.091	0.04	0.25	9
2.5	106.449	105.127	102.62	0.06	0.333333	11
3.	107.788	106.184	103.152	0.08	0.375	12
3.5	109.144	107.251	103.687	0.1	0.4	13
4.	110.517	108.329	104.225	0.12	0.416667	14
4.5	111.907	109.417	104.765	0.14	0.428571	14
5.	113.315	110.517	105.309	0.16	0.4375	14

Table 1: Cost of compensation, minimum spread $\bar{\mu} - \tilde{\mu}$, p^* and N_G^* as a function of γ for $N = 30$.

$N = 30$ managers, it is sufficient to have 18 bad managers to end up in the bad equilibrium, which can be also read in Table 1.

Table 2 illustrates the effect of the firm-specific risk σ on the cost of compensations and the other relevant magnitudes. We have the following observations. First, the cost of compensation increases in the volatility. The advantage of using the relative compensation is particularly demonstrated for high-volatile projects. Second, the required mean return spread $\bar{\mu} - \tilde{\mu}$ between the good and bad project increases in σ . For instance, the spread needs to be 32% such that the bad equilibrium outcome does not result. In other words, the more volatile the projects, the more likely the bad equilibrium will occur. Accordingly, the fraction of good managers p^* and the number of N_G^* should be higher to avoid a bad equilibrium. But p^* and N_G^* do not increase unlimitedly and are capped by $1/2$ and $N/2$.

σ	cost of compensation (case 1)	cost of compensation (case 2)	cost of compensation (case 3)	minimum spread $\bar{\mu} - \tilde{\mu}$	$p^* = \frac{N_G^* - 1}{N - 1}$	N_G^*
0.05	101.893	100.376	100.194	0.005	0	1
0.1	103.045	101.511	100.779	0.02	0	1
0.15	104.996	103.433	101.761	0.045	0.277778	10
0.2	107.788	106.184	103.152	0.08	0.375	12
0.25	111.488	109.829	104.969	0.125	0.42	14
0.3	116.183	114.454	107.232	0.18	0.444444	14
0.35	121.988	120.172	109.971	0.245	0.459184	15
0.4	129.046	127.125	113.217	0.32	0.46875	15

Table 2: Cost of compensation, minimum spread $\bar{\mu} - \tilde{\mu}$, p^* and N_G^* as a function of different firm-specific risk σ . σ^* is fixed at 0.10.

How ρ_H (or $\rho_H - \rho_L$) influences the cost of compensation, the minimum spread $\bar{\mu} - \tilde{\mu}$, p^* and N_G^* is exhibited in Table 3. The cost of compensation in cases 1 and 2 does not depend on the correlation coefficient ρ_H , while the cost of compensation in case 3 decreases in ρ_H . If there is a perfect positive correlation ($\rho_H = 1$), all the risks of the relative compensation can be eliminated, therefore the lowest cost results. $\rho_H - \rho_L$ has the same effect on the other three magnitudes as σ^2 , as only the product $(\rho_H - \rho_L)\sigma^2$ enters the relevant formulas.

ρ_H	cost of compensation (case 1)	cost of compensation (case 2)	cost of compensation (case 3)	minimum spread $\bar{\mu} - \tilde{\mu}$	$p^* = \frac{N_G^* - 1}{N - 1}$	N_G^*
0.1	107.788	106.184	105.745	0.048	0.291667	10
0.2	107.788	106.184	105.091	0.056	0.321429	11
0.3	107.788	106.184	104.441	0.064	0.34375	11
0.4	107.788	106.184	103.794	0.072	0.361111	12
0.5	107.788	106.184	103.152	0.08	0.375	12
0.6	107.788	106.184	102.514	0.088	0.386364	13
0.7	107.788	106.184	101.88	0.096	0.395833	13
0.8	107.788	106.184	101.249	0.104	0.403846	13
0.9	107.788	106.184	100.623	0.112	0.410714	13
1.	107.788	106.184	100.	0.12	0.416667	14

Table 3: Cost of compensation, $\bar{\mu} - \tilde{\mu}$, p^* and N_G^* as a function of ρ_H . ρ_L is fixed at -0.5 .

6 Conclusion

The present paper investigates the question of how different relative compensation schemes influence the investment decisions of the management. In a two period Black-Scholes type model, risk-averse managers are compensated relatively to other managers in their industry group and can choose between a good and a bad investment project. Relative compensation can reduce the costs of compensation to the shareholders as it can help to distinguish between the firm-specific risk (which the manager can influence) and an industry-specific risk (luck). However, when all managers are rewarded relatively, “all the managers choosing the good project” is only one possible pure Nash equilibrium. For sufficiently high risk-aversion or low mean spread between the good and the bad project, “all the

managers choosing the bad project” can also be a Nash equilibrium. We show that the only possible Nash equilibria in our model are consistent with herding behavior, i.e. all managers will either choose the good or the bad project, but no asymmetric pure equilibria exist.

In an extension we consider the case where a fraction of the managers in an industry are of a low type, i.e. they cannot select the good investment project. Rewarding all managers in such an industry relatively can lead to a situation, where the only Nash equilibrium is that every manager chooses the bad project. The magnitudes of the quantities reported in this paper are subject to the many limitations and simplifications of our model. However, we see these results as providing a useful perspective on the directional effects of relative compensation. The conclusion of our research is that, relative management compensation is not unambiguously superior to absolute performance compensation and might potentially lead to inferior outcomes. Rewarding the managers according to their rank rather than the level of relative performance (c.f. a tournament scheme as in Lazear and Rosen (1981)) might mitigate the incentive to herd on a bad investment. However it is not clear if it will in general lead to a more efficient outcome.

7 Appendix

7.1 Proofs for the Main Text

Lemma 1. *It holds $k_X > 1$ for $\sigma > 0$ and $k_Y > 1$ for $\sigma^* > 0$. Hence, $k_X > 1$ and $k_Y > 1$ can be interpreted as a risk premium that the manager demands, because he cannot hedge these risks that are part of his remuneration.*

Proof. It holds

$$E[\bar{X}_i] = S_0 \exp\{\bar{\mu}T\}, \quad (E[\bar{X}_i^{1-\gamma}])^{\frac{1}{1-\gamma}} = S_0 \exp\left\{\bar{\mu}T - \frac{1}{2}\sigma^2\gamma T\right\}$$

$$E[Y] = 1, \quad (E[Y^{1-\gamma}])^{\frac{1}{1-\gamma}} = \exp\left\{-\frac{1}{2}(\sigma^*)^2\gamma T\right\}.$$

Hence, $k_X > 1$ for $\sigma > 0$ and $k_Y > 1$ for $\sigma^* > 0$. \square

Lemma 2. *The expected utility of choosing \bar{S}_i given that $N - 1$ firms choose the good project is*

$$EU\left(\beta \frac{\bar{S}_i}{\left(\prod_{j \neq i} \bar{S}_j\right)^{1/(N-1)}}\right) = \frac{\beta^{1-\gamma}}{1-\gamma} \exp\left(\frac{1}{2}(1-\gamma)^2\sigma^2 \frac{N}{N-1}(1-\rho_H)T\right) \quad (7.1)$$

The expected utility of choosing \tilde{S}_i given that $N - 1$ firms choose the good project is

$$EU\left(\beta \frac{\tilde{S}_i}{\left(\prod_{j \neq i} \tilde{S}_j\right)^{1/(N-1)}}\right) = \frac{\beta^{1-\gamma}}{1-\gamma} \exp\left((1-\gamma)(\tilde{\mu} - \bar{\mu})T\right. \\ \left. + \frac{1}{2}(1-\gamma)^2\sigma^2 \left(2(\rho_H - \rho_L) + \frac{N}{N-1}(1-\rho_H)\right)T\right). \quad (7.2)$$

Proof. The detailed proof is left out in this place. In the following Section 3.4, we introduce and derive $EU(\bar{S}_i|N_G - 1)$ and $EU(\tilde{S}_i|N_G - 1)$ to denote the expected utility of firm i given that $N_G - 1$ of the other firms choose \bar{S} . The expected utility in (7.1) equals $EU(\bar{S}_i|N_G - 1)$ for $N_G = N$ and the expected utility in (7.2) equals $EU(\tilde{S}_i|N_G - 1)$ for $N_G = N$. \square

Lemma 3. *The incentive constraint is always satisfied.*

Proof. Based on Lemma 2, the incentive constraint is satisfied if and only if

$$\bar{\mu} - \tilde{\mu} \geq -(\gamma - 1)(\rho_H - \rho_L)\sigma^2.$$

This condition is trivially satisfied, because we have assume $\gamma > 1$ and $\rho_H > \rho_L$. \square

Lemma 4. *If $\rho_H \geq 1/N := \rho_H^*$, then the compensation scheme of case 3 is cheaper than the compensation scheme of case 2 (and accordingly also of case 1).*

Proof. The cost in case 2 is $CE \exp \left\{ \frac{1}{2} \gamma \sigma^2 T \right\}$. The compensation scheme of case 3 is cheaper if and only if

$$\frac{1}{2} \sigma^2 \frac{N}{N-1} (1 - \rho_H) \gamma T \leq \frac{1}{2} \gamma \sigma^2 T \Leftrightarrow \rho_H \geq 1/N.$$

□

Lemma 5. *The expected utility of choosing \bar{S}_i given that $N_G - 1$ firms choose the good project is*

$$\begin{aligned} EU(\bar{S}_i | N_G - 1) &= \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left((1-p)(1-\gamma)(\bar{\mu} - \tilde{\mu})T \right) \\ &\quad + \frac{1}{2} (1-\gamma)^2 \sigma^2 \left(2(1-p)^2 (\rho_H - \rho_L) + \frac{N}{N-1} (1-\rho_H) \right) T \end{aligned}$$

The expected utility of choosing \tilde{S}_i given that $N_G - 1$ firms choose the good project is

$$\begin{aligned} EU(\tilde{S}_i | N_G - 1) &= \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left(p(1-\gamma)(\tilde{\mu} - \bar{\mu})T \right) \\ &\quad + \frac{1}{2} (1-\gamma)^2 \sigma^2 \left(2p^2 (\rho_H - \rho_L) + \frac{N}{N-1} (1-\rho_H) \right) T \end{aligned}$$

where $p = \frac{N_G - 1}{N - 1}$.

Proof. Set

$$\begin{aligned} \bar{W}_j &= \sqrt{\rho_H} W + \sqrt{1 - \rho_H} \bar{\bar{W}}_j \\ \tilde{W}_j &= \frac{\rho_L}{\sqrt{\rho_H}} W + \sqrt{\rho_H - \frac{\rho_L^2}{\rho_H}} \bar{W} + \sqrt{1 - \rho_H} \tilde{\tilde{W}}_j \end{aligned}$$

for $j = 1, \dots, N$ and W , \hat{W} , \bar{W}_j and \tilde{W}_j are Brownian motions, which are independent among each other and among all j . This construction yields

$$\begin{aligned} d\bar{W}_i d\bar{W}_i &= d\tilde{W}_i d\tilde{W}_i = dt \\ d\bar{W}_i d\bar{W}_j &= d\tilde{W}_i d\tilde{W}_j = \rho_H dt \\ d\bar{W}_i d\tilde{W}_j &= d\tilde{W}_i d\bar{W}_j = \rho_L dt \end{aligned}$$

Then

$$\begin{aligned} EU(\bar{S}_i | N_G - 1) &= \frac{\beta^{1-\gamma}}{1-\gamma} E \left[\frac{\exp\left((\bar{\mu} - \frac{1}{2}\sigma^2)(1-\gamma)T + (1-\gamma)\sigma\bar{W}_i\right)}{\exp\left(p(1-\gamma)\left(\bar{\mu} - \frac{1}{2}\sigma^2\right)T + \frac{(1-\gamma)\sigma}{N-1} \sum_{j=1}^{N_G-1} \bar{W}_j\right)} \right. \\ &\quad \left. \frac{1}{\exp\left((1-p)(1-\gamma)\left(\tilde{\mu} - \frac{1}{2}\sigma^2\right)T + \frac{(1-\gamma)\sigma}{N-1} \sum_{j=N_G+1}^N \tilde{W}_j\right)} \right] \\ &= \frac{\beta^{1-\gamma}}{1-\gamma} \exp\left((1-p)(1-\gamma)(\bar{\mu} - \tilde{\mu})T + \frac{1}{2}(1-\gamma)^2\sigma^2 TVar(Z)\right) \end{aligned}$$

with

$$\begin{aligned} Z &= \sqrt{\rho_H}W + \sqrt{1-\rho_H}\bar{W}_i - \frac{1}{N-1} \sum_{j=1}^{N_G-1} \left(\sqrt{\rho_H}W + \sqrt{1-\rho_H}\bar{W}_j \right) \\ &\quad - \frac{1}{N-1} \sum_{j=N_G+1}^N \left(\frac{\rho_L}{\sqrt{\rho_H}}W + \sqrt{\rho_H - \frac{\rho_L^2}{\rho_H}}\bar{W} + \sqrt{1-\rho_H}\tilde{W}_j \right). \end{aligned}$$

The other case is analogous. □

Proposition 1. *For $N \geq 3$, asymmetric pure Nash equilibria, i.e. $1 \leq N_G \leq N-1$, do not exist. $N_G = N$ is always a Nash equilibrium. $N_G = 0$ is a Nash equilibrium if and only if $\bar{\mu} - \tilde{\mu} \leq (\gamma-1)(\rho_H - \rho_L)\sigma^2$.*

Proof. $1 \leq N_G \leq N-1$ is a Nash equilibrium if and only if the following two inequalities are satisfied:

1. $EU(\bar{S} | N_G - 1) \geq EU(\tilde{S} | N_G - 1)$,

$$2. EU(\tilde{S}|N_G) \geq EU(\bar{S}|N_G).$$

If N_G is a Nash equilibrium, it cannot be profitable for any player to deviate. First, consider the N_G players, who play \bar{S} . If one of them deviates, he will get $EU(\tilde{S}|N_G - 1)$, while if he continues to play \bar{S} he gets $EU(\bar{S}|N_G - 1)$. Equation 1 ensures that the players with the good projects do not deviate. Next, the players who play \tilde{S} in the Nash equilibrium, get $EU(\tilde{S}|N_G)$ from keeping their strategy and $EU(\bar{S}|N_G)$ from deviating. Therefore, equation 2 ensures that the $N - N_G$ players implement the bad project in equilibrium. Based on the results in Lemma 5, the above two conditions are equivalent to

$$1. \bar{\mu} - \tilde{\mu} \geq (\gamma - 1)(\rho_H - \rho_L)\sigma^2 \left(1 - 2\frac{N_G - 1}{N - 1}\right)$$

$$2. \bar{\mu} - \tilde{\mu} \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2 \left(\frac{N - 3}{N - 1} - 2\frac{N_G - 1}{N - 1}\right).$$

Apparently, these two inequalities do not hold simultaneously for a $1 \leq N_G \leq N - 1$.

$N_G = N$ is a Nash equilibrium if and only if $EU(\bar{S}|N - 1) \geq EU(\tilde{S}|N - 1)$. This condition is the same as

$$\bar{\mu} - \tilde{\mu} \geq -(\gamma - 1)(\rho_H - \rho_L)\sigma^2$$

which holds always.

$N_G = 0$ is a Nash equilibrium if and only if $EU(\tilde{S}|0) \geq EU(\bar{S}|0)$. This condition is equivalent to

$$(\bar{\mu} - \tilde{\mu}) \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2.$$

□

Proposition 2. *For $N = 2$, there exists an asymmetric Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) = (\gamma - 1)(\rho_H - \rho_L)\sigma^2$. $N_G = 2$ is a Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) \geq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$. $N_G = 0$ is a Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$.*

Proof. Denote these two firms by firm 1 and 2. Asymmetric Nash equilibrium exists if and only if the following two inequalities hold:

- a) $EU(\bar{S}_1/\tilde{S}_2) \geq EU(\tilde{S}_1/\tilde{S}_2)$,
- b) and $EU(\tilde{S}_1/\bar{S}_2) \geq EU(\bar{S}_1/\bar{S}_2)$.

Conditions a) and b) can be written as

- a) $(\bar{\mu} - \tilde{\mu}) \geq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$
- b) and $(\bar{\mu} - \tilde{\mu}) \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$.

As a result, if $(\bar{\mu} - \tilde{\mu}) = (\gamma - 1)(\rho_H - \rho_L)\sigma^2$, there is an asymmetric Nash equilibrium.

$N_G = 2$ is a Nash equilibrium if and only if $EU(\bar{S}_1/\bar{S}_2) \geq EU(\tilde{S}_1/\bar{S}_2)$. $N_G = 0$ is a Nash equilibrium if and only if $EU(\tilde{S}_1/\tilde{S}_2) \geq EU(\bar{S}_1/\tilde{S}_2)$. \square

Proposition 3. *If $\bar{N}_G < N_G^*$ (N_G^* defined in (3.4)), then \bar{N}_G is not a Nash equilibrium and $N_G = 0$ is the exclusive Nash equilibrium. If $\bar{N}_G \geq N_G^*$, then $N_G = \bar{N}_G$ is the exclusive Nash equilibrium.*

Proof. If $\bar{N}_G < N_G^*$, the incentive constraint for a good project being implemented cannot be satisfied (see Lemma 6).

Note that $N_G = 0$ is a Nash equilibrium if and only if $(\bar{\mu} - \tilde{\mu}) \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2$.

Recall the condition $\bar{N}_G < N_G^*$ implies

$$\bar{\mu} - \tilde{\mu} \leq \frac{1}{2} \left(1 - \frac{\bar{N}_G - 1}{N - 1} \right) (\gamma - 1)\sigma^2(\rho_H - \rho_L) \leq (\gamma - 1)(\rho_H - \rho_L)\sigma^2.$$

Hence, $N_G = 0$ is a Nash equilibrium.

If $\bar{N}_G \geq N_G^*$, the condition for $N_G = \bar{N}_G$ being a Nash equilibrium is

$$EU(\bar{S}|\bar{N}_G - 1) \geq EU(\tilde{S}|\bar{N}_G - 1) \Leftrightarrow \bar{\mu} - \tilde{\mu} \geq \frac{1}{2} \left(1 - \frac{\bar{N}_G - 1}{N - 1} \right) (\gamma - 1)\sigma^2(\rho_H - \rho_L).$$

This condition is implied by $\bar{N}_G \geq N_G^*$. \square

7.2 Different Volatilities for Different Investment Projects

Here we want to extend the relative stock price model to the case where $\bar{\sigma}^2 \neq \tilde{\sigma}^2$. The good project \bar{S} has the parameters $(\bar{\sigma}^2, \bar{\mu})$, while the bad project \tilde{S} is linked to the parameters $(\tilde{\sigma}^2, \tilde{\mu})$. Recall that we assume

$$\begin{aligned} E[\bar{S}_t] > E[\tilde{S}_t] &\Leftrightarrow \bar{\mu} > \tilde{\mu} \\ EU[\bar{S}_t] > EU[\tilde{S}_t] &\Leftrightarrow \bar{\mu} - \tilde{\mu} > \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)\gamma \end{aligned}$$

i.e. we consider only drifts that satisfy $\bar{\mu} - \tilde{\mu} > \max(0, \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)\gamma)$. It is crucial to assume that $\rho_H \geq \max(0, \rho_L)$.

Lemma 6. *The expected utility of choosing \bar{S} given that $N_G - 1$ firms choose the good project is*

$$\begin{aligned} EU[\bar{S}|N_G - 1] &= \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left((1-\gamma)(1-p) \left((\bar{\mu} - \tilde{\mu}) - \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2) \right) T \right. \\ &\quad \left. + \frac{1}{2}(1-\gamma)^2 \left((1-p)^2(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L) \right. \right. \\ &\quad \left. \left. + (1-\rho_H) \left(\bar{\sigma}^2 \left(1 + \frac{p}{N-1} \right) + \tilde{\sigma}^2 \frac{1-p}{N-1} \right) \right) T \right) \end{aligned}$$

The expected utility of choosing \tilde{S} given that $N_G - 1$ firms choose the good project is

$$\begin{aligned} EU[\tilde{S}|N_G - 1] &= \frac{\beta^{1-\gamma}}{1-\gamma} \exp \left((1-\gamma)p \left((\tilde{\mu} - \bar{\mu}) - \frac{1}{2}(\tilde{\sigma}^2 - \bar{\sigma}^2) \right) T \right. \\ &\quad \left. + \frac{1}{2}(1-\gamma)^2 \left(p^2(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L) \right. \right. \\ &\quad \left. \left. + (1-\rho_H) \left(\tilde{\sigma}^2 \left(1 + \frac{1-p}{N-1} \right) + \bar{\sigma}^2 \frac{p}{N-1} \right) \right) T \right) \end{aligned}$$

Lemma 7. *The following two inequalities*

1. $EU[\bar{S}|N_G - 1] \geq EU[\tilde{S}|N_G - 1]$
2. $EU[\tilde{S}|N_G] \geq EU[\bar{S}|N_G]$

are equivalent to

1. $\bar{\mu} - \tilde{\mu} > \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 + (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}(\gamma - 1)(1 - 2p)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L)$
2. $\bar{\mu} - \tilde{\mu} < \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 - (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}\left(1 - 2p - \frac{2}{N-1}\right)(\gamma - 1)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L)$

with $p = \frac{N_G - 1}{N - 1}$.

This allows us to generalize Proposition 1:

Proposition 4. *For $N \geq 3$, asymmetric pure Nash equilibria, i.e. $1 \leq N_G \leq N - 1$, do not exist. $N_G = N$ is always a Nash equilibrium. $N_G = 0$ is a Nash equilibrium if and only if*

$$\bar{\mu} - \tilde{\mu} \leq \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 - (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}(\gamma - 1)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L).$$

The incentive constraint $EU[\bar{S}_i | N_G - 1] \geq EU[\tilde{S}_i | N_G - 1]$ is satisfied if and only if

$$\bar{\mu} - \tilde{\mu} > \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 + (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}(\gamma - 1)(1 - 2p)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L)$$

The critical number $N_G^ - 1$ of other managers to choose the good project, such that the incentive constraint is always satisfied, (respectively, the critical fraction p^*) is given by:*

$$p^* := \frac{N_G^* - 1}{N - 1} = \max \left\{ 0, \frac{1}{2} \left(1 - \frac{(\bar{\mu} - \tilde{\mu}) - \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 + (\gamma - 1)(1 - \rho_H))}{\frac{1}{2}(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\bar{\sigma}\tilde{\sigma}\rho_L)} \right) \right\}.$$

Proof. $1 \leq N_G \leq N - 1$ is a Nash equilibrium if and only if the following two inequalities are satisfied:

1. $EU[\bar{S} | N_G - 1] \geq EU[\tilde{S} | N_G - 1]$,
2. $EU[\tilde{S} | N_G] \geq EU[\bar{S} | N_G]$.

Based on the results in Lemma 7, the above two conditions can only hold simultaneously if

$$\begin{aligned}
 (\bar{\sigma}^2 - \tilde{\sigma}^2)(1 - \rho_H)(N - 1) &\leq -(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\tilde{\sigma}\bar{\sigma}\rho_L) && \Leftrightarrow \\
 0 &\leq (\bar{\sigma}^2 + \tilde{\sigma}^2)(\rho_H(N - 2) - (N - 1)) + 2\tilde{\sigma}\bar{\sigma}\rho_L && \Leftrightarrow \\
 0 &\geq \bar{\sigma}^2 + \tilde{\sigma}^2 - \frac{2\tilde{\sigma}\bar{\sigma}\rho_L}{(N - 1) - (N - 2)\rho_H}
 \end{aligned}$$

As one can show easily, that under the assumption $\rho_H \geq \max(0, \rho_L)$, the largest value that $\frac{2\tilde{\sigma}\bar{\sigma}\rho_L}{(N-1)-(N-2)\rho_H}$ can take is $2\tilde{\sigma}\bar{\sigma}$ for $\rho_H = \rho_L = 1$. Hence, if the above inequality does not hold for this extreme case, it can never hold:

$$\begin{aligned}
 0 &\geq \bar{\sigma}^2 + \tilde{\sigma}^2 - 2\tilde{\sigma}\bar{\sigma} && \Leftrightarrow \\
 0 &\geq (\bar{\sigma} - \tilde{\sigma})^2
 \end{aligned}$$

This inequality can at most hold for $\bar{\sigma} = \tilde{\sigma}$, but in Proposition 1 we have already ruled out this case.

$N_G = N$ is a Nash equilibrium if and only if $EU[\bar{S}|N - 1] \geq EU[\tilde{S}|N - 1]$.

This condition is the same as

$$\bar{\mu} - \tilde{\mu} > \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 + (\gamma - 1)(1 - \rho_H)) - \frac{1}{2}(\gamma - 1)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\tilde{\sigma}\bar{\sigma}\rho_L)$$

which holds always as $(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\tilde{\sigma}\bar{\sigma}\rho_L) \geq 0$.

$N_G = 0$ is a Nash equilibrium if and only if $EU[\tilde{S}|0] \geq EU[\bar{S}|0]$. This condition is equivalent to

$$\bar{\mu} - \tilde{\mu} \leq \frac{1}{2}(\bar{\sigma}^2 - \tilde{\sigma}^2)(1 - (\gamma - 1)(1 - \rho_H)) + \frac{1}{2}(\gamma - 1)(\rho_H \bar{\sigma}^2 + \rho_H \tilde{\sigma}^2 - 2\tilde{\sigma}\bar{\sigma}\rho_L).$$

The other statements follow from Lemma 7. □

8 References

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