

On optimal option purchasing strategy with market impact

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Context and Motivations

Life insurance liabilities are characterized by three main features :

- Long term duration
- Large volumes
- Significant market risk exposure

The complexity of the products and low interest rates environment

→ The use of derivatives to hedge financial risks embedded within insurance liability guarantees

Context and Motivations

- Guarantees embedded within insurance liabilities hold a convex risk profile w.r.t the underlying stock → Insurance companies need to buy some convex hedge assets
- Such type of contracts can be hedged via dynamic hedging, static hedging or semi-static hedging :
 - **Dynamic hedging** : Holding at any time t , Δ_t shares of the underlying asset.
 - **Static hedging** : Suggests replicating the embedded guarantees with a static position in put options.
 - **Semi-static hedging** : Similar to static hedging but different in that the insurer constructs a hedging portfolio at each rebalancing date by following an optimal hedging strategy.

Context and Motivations

- In 2014, the NAIC Capital Market Bureau reported a 2 trillion dollars of notional value of derivatives use in the insurance business, mainly for hedging purposes (94% of the total notional value), 25% are aimed for hedging equity risk, and options accounted for 44% of the total notional value.
 - Trading such volumes can not come without a cost ; its impact should be an important driver for prices, yet, there are no known practices for splitting the targeted quantity to minimize such cost.
- The need to take into account market impact as it is usual practice in the stock market.
- Minimize the cost of market impact by splitting the target value.

Introduction

It is common practice in the stock market to split large orders to minimize the market impact generated by such trades. This problem was studied in the literature by many authors :

- Optimal execution of a stock portfolio under market impact : [Almgren and Chriss(2000), ?]
- Option pricing and hedging : [Abergel and Loeper(2013), ?]

Our interest

- Define an option execution price when the size of a trade is a driver of the option price.
- In the spirit of Almgren and Chriss, set a rigorous optimal execution problem for an option buyer who seeks to minimize the cost of his strategy.

Introduction

What we aim to do

- Define a price of an option under market impact under a Black-Scholes framework.
- Set a framework for the optimal purchasing problem that minimizes one of the two criteria :
 - The expected cost.
 - The mean-variance of the cost.
- Solve the problem using appropriate methods.
- Extend the framework to local volatility models.

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A review of the Almgren-Chriss framework I

We consider a trading strategy on an asset S described by the asset position x_t held at time $t \in [0, T]$.

- The path $x = (x_t)_{t \in [0, T]}$ is absolutely continuous and satisfies $x_0 = X$ and $x_{T+} = 0$.
- The unaffected asset price S is a semi-martingale.
- When the strategy x is used, the price is changed from S_t to \tilde{S}_t which is given in the Almgren-Chriss framework by :

$$\tilde{S}_t = S_t + \int_0^t g(\dot{x}_s) ds + h(\dot{x}_t),$$

where $g(x) = \gamma x$ describes permanent impact, $h(x) = \eta x$ instantaneous (or temporary) impact and $S_t = S_0 + \sigma W_t$.

A review of the Almgren-Chriss framework II

Almgren and Chriss consider a mean-variance optimization problem.

- The mean-variance functional is given by

$$\mathbb{E}[\mathcal{C}(x)] + \lambda \text{Var}[\mathcal{C}(x)],$$

where $\mathcal{C}(x) = \int_0^T \tilde{S}_t dx_t$ is the cost function.

- The optimal purchasing strategy is then found by minimizing the mean-variance functional.

The optimal purchasing problem for a portfolio of options

We consider a trading strategy on an option P described by the option position x_t held at time $t \in [0, T]$.

- The path $x = (x_t)_{t \in [0, T]}$ is absolutely continuous and satisfies $x_0 = X$ and $x_{T+} = 0$.
- The unaffected option price P is a semi-martingale on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$.
- When the strategy x is used, the price is changed from P_t to \tilde{P}_t where :

$$\tilde{P}_t = P_t + \text{impact term},$$

A heuristic approach to derive the option price under market impact constraint

We consider a universe with two agents :

- Option end-users : buy options to hedge an external risk/ complex product.
- Option market-makers : sell options at their replication price :
 - The option replication price is inspired from [Leland(1985)] transaction costs framework.
 - We consider a geometric Brownian motion for the unaffected underlying price, i.e $S_t = S_0 e^{-\frac{1}{2}\sigma^2 t + \sigma W_t}$ and proportional impact cost :

$$\tilde{S}_t = S_t(1 + \eta \dot{x}_t + \gamma(x_t - x_0)),$$

The transaction costs approach revisited

The transactions costs framework :

- Introduced by [?] to hedge derivatives under proportional transaction costs.
- Based on an approximate replication of the European-type options with terminal payoff V_T under the Black-Scholes framework.
- Leads to the Black-Scholes formula with a suitably enlarged volatility.

The setting

- We consider an option issued at time 0 with maturity \hat{T} .
- The underlying asset is given under the martingale measure by the SDE :

$$dS_t = \sigma S_t dW_t, \quad 0 \leq t \leq \hat{T}.$$

- The cash evolves with zero risk-free rate.
- The hedging is performed over a discrete grid with n revision s
 $t_0 < t_1 < \dots < t_n = \hat{T}$.
- It involves proportional impact rate I_0 on the underlying asset.

Review of Leland's framework I

The current value of the option process at time $t \in [0, \hat{T}]$ is defined by

$$V_t^n = V_0^n + \int_0^t \Delta_u^n dS_u - \sum_{t_i \leq t} l_0 S_{t_i} | \Delta_{i+1}^n - \Delta_i^n |$$

where :

- $t_i = t_i^n = i/n$, $0 \leq i \leq n$, $t_0 = 0$, $t_n = \hat{T}$ are the revision dates.
- $\Delta^n = \Delta_i^n$ on the interval $]t_{i-1}, t_i]$, $\Delta_{n+1}^n := \Delta_n^n$.
- Δ_i^n is $\mathcal{F}_{t_{i-1}}$ -measurable.

Δ^n corresponds to the trading strategy. The number of shares of the risky asset that the holder possesses in the period i is then Δ_i^n .

Review of Leland's framework II

The option price solves the following PDE :

$$\begin{cases} \partial_u \tilde{P}(u, S) + \frac{1}{2} \tilde{\sigma}^2 S^2 \partial_{SS} \tilde{P}(u, S) = 0, & (u, S) \in [0, \hat{T}[\times]0, \infty[\\ \tilde{P}(\hat{T}, s) = (K - s)^+, & s \in]0, \infty[. \end{cases}$$

where $\tilde{\sigma}$ is the "enlarged volatility" and defined by

$$\tilde{\sigma}^2 = \sigma^2 + \sigma l_0 n^{1/2} \sqrt{\frac{8}{\pi}}$$

Remark

- *Leland's framework is known to present problems when the number of revision dates n tends to infinity.*
- *This issues is avoided by choosing n of a reasonable order and finding the option price at time 0 that the market-maker will charge the end-user.*

The Leland framework revisited

In our framework :

- Leland's transaction costs coefficient l_0 is replaced by the impact cost term $\eta\dot{x}_t + \gamma(x_t - x_0)$ at an arbitrary time $t \in [0, T]$ where $T < \hat{T}$.
- We fix the hedging discretization step h and replace the number of discretization n by $\frac{\hat{T}-t}{h}$.
- We rewrite the enlarged volatility at t as :

$$\tilde{\sigma}_t^2 = \sigma^2 + (\tilde{\eta}\dot{x}_t + \tilde{\gamma}(x_t - x_0))\sqrt{\hat{T} - t}\sigma,$$

where $\tilde{\eta} = \eta\sqrt{\frac{8}{h\pi}}$ and $\tilde{\gamma} = \gamma\sqrt{\frac{8}{h\pi}}$.

[Lépinette and Quoc(2014)] extends the framework to local volatility in which case, we need only replace the constant volatility σ by $\sigma(t, S)$.

Simplifying the execution price and the impact function

Proposition

In the Black & Scholes framework, the put option effective price is written as the following :

$$\begin{aligned}\tilde{P}(t, S_t, \dot{x}_t, x_t) &= P(t, S_t) \\ &+ \frac{1}{2} \{ \tilde{\eta} \dot{x}_t + \tilde{\gamma} (x_t - x_0) \} \sigma S_t^2 (\hat{T} - t)^{3/2} \Gamma(t, S_t),\end{aligned}$$

where :

- $\tilde{\eta}$ controls the temporary impact strength in $\$ \times \text{hour} / \text{N}$ of options.
- $\tilde{\gamma}$ controls the permanent impact strength and is in $\$ \text{N}$ shares.
- x_t is the quantity held at time t and \dot{x}_t is the speed of trading in number of options per time unit.

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The setting

Buy X put options over a finite time horizon $[0, T]$. We define :

- $x = (x_t)_{t \in [0, T]}$ execution strategy
- $x_0 = X < 0$, $x_T = 0$ (liquidating a short position)
- Assume x_t is continuous and adapted
- The option effective price

$$\tilde{P}_t = P_t + \frac{1}{2} \tilde{\eta} \dot{x}_t \sigma S_t^2 (\hat{T} - t)^{3/2} \Gamma(t, S_t).$$

The cost function

The cost arising from the strategy x is

$$\mathcal{C}(x) = \int_0^T \tilde{P}_t \dot{x}_t dt.$$

which we can be rewritten as :

$$\begin{aligned} \mathcal{C}(x) = & -XP_0 - \int_0^T \sigma x_t S_t \Delta(t, S_t) dW_t \\ & + \frac{1}{2} \tilde{\eta} \sigma \int_0^T \dot{x}_t^2 S_t^2 (\hat{T} - t)^{3/2} \Gamma(t, S_t) dt, \end{aligned}$$

and seek to minimize the two criteria :

- The expected cost $\mathbb{E}[\mathcal{C}(x)]$
- The mean-variance case $\mathbb{E}[\mathcal{C}(x)] + \lambda \text{Var}[\mathcal{C}(x)]$

The optimal strategy : Minimizing the expected value

We consider the temporary market impact only, i.e. $\tilde{\gamma} = 0$

Theorem

The optimal strategy x^* resulting in minimizing the expected cost under the Black & Scholes framework is characterized by :

$$\dot{x}^*(t) = \frac{K_1}{(\hat{T} - t)^{3/2}}$$
$$x^*(t) = \frac{K_1}{(\hat{T} - t)^{1/2}} + K_2$$

where $K_1 = \frac{X}{2\left(\hat{T}^{-\frac{1}{2}} - (\hat{T} - T)^{-\frac{1}{2}}\right)}$ and $K_2 = -2K_1(\hat{T} - T)^{-1/2}$.

The mean-variance case

The variance of the cost can be expressed as :

$$\begin{aligned} \mathbb{V}[\mathcal{C}(x)] &= \mathbb{E} \left[\left(\int_0^T \tilde{P}_t \dot{x}_t dt - \mathbb{E} \left[\int_0^T \tilde{P}_t \dot{x}_t dt \right] \right)^2 \right] \\ &= \mathbb{E} \left[\int_0^T x_t^2 \sigma^2 S_t^2 \partial_S \tilde{P}^2(t, S_t) dt \right] \\ &\quad + \{ \text{terms arising from uncertainty in the drift part} \}. \end{aligned}$$

The mean-variance objective function can be reasonably approximated as the following :

$$\begin{aligned} \mathbb{E}[\mathcal{C}(x)] + \lambda \mathbb{V}[\mathcal{C}(x)] &\approx \\ \mathbb{E} \left[\int_0^T \frac{1}{2} \tilde{\eta} \sigma \dot{x}_t^2 S_t^2 (\hat{T} - t)^{3/2} \Gamma(t, S_t) dt + \tilde{\lambda} \int_0^T x_t^2 \sigma^2 S_t^2 \Delta^2(t, S_t) dt \right]. \end{aligned}$$

A dynamic programming framework

We define $\mathcal{X}(T, X)$ the set of all adapted and absolutely continuous strategies that satisfy :

- The boundary conditions $x_0 = X < 0$, $x_T = 0$
- The integrability conditions $\mathbb{E} \left[\int_0^T \dot{x}_t^2 S_t^2 (\hat{T} - t)^{3/2} \Gamma(t, S_t) dt \right] < \infty$
 and $\mathbb{E} \left[\int_0^T x_t^2 S_t^2 \Delta^2(t, S_t) dt \right] < \infty$

And define the objective function :

$$U(0, S_0, X) := \inf_{x \in \mathcal{X}(T, X)} \mathbb{E} \left[\int_0^T \left\{ \dot{x}_t^2 S_t^2 (\hat{T} - t)^{3/2} \Gamma(t, S_t) + \lambda x_t^2 \sigma^2(S_t) S_t^2 \Delta^2(t, S_t) \right\} dt \right],$$

A finite fuel problem

To solve these kind of optimal execution problems, usual practice is to :

- Set α s.t : $\alpha_t = -\dot{x}_t$ and

$$\mathcal{A}(T, X) = \{x_t^\alpha \in \mathcal{X}(T, X) \mid x_t^\alpha := X - \int_0^t \alpha_s ds, \quad 0 \leq t \leq T\},$$

- Restrict the solution to Markovian processes of the form $\alpha(t, S_t, x_t)$ and solve :

$$U(t, S, x) = \inf_{\alpha \in \mathcal{A}(T, X)} \mathbb{E}_t \left[\int_t^T \left\{ \alpha_u^2 S_u^2 (\hat{T} - u)^{3/2} \Gamma(u, S_u) + \lambda \sigma^2 (x_u^\alpha)^2 S_u^2 \Delta^2(u, S_u) \right\} du \right],$$

- Derive the Hamilton-Jacobi-Bellman equation to find a nonlinear PDE.

The value function

For a given strategy $\alpha(\cdot, \cdot, \cdot)$, the value function $U(t, S, x)$ is defined as

$$U(t, S, x) = \inf_{\alpha \in \mathcal{A}(T, X)} \mathbb{E}_t \left[\int_t^T \left\{ \alpha_u^2 S_u^2 (\hat{T} - u)^{3/2} \Gamma(u, S_u) + \lambda \sigma^2 (x_u^\alpha)^2 S_u^2 \Delta^2(u, S_u) \right\} du \right],$$

where \mathbb{E}_t is the expectation conditional to $S_t = s$ and $x_t^\alpha = x$.

The finite fuel constraint

- U should satisfy a singular terminal condition of the form

$$\lim_{t \rightarrow T} U(t, S, x) = \begin{cases} 0 & \text{if } x = 0 \\ +\infty & \text{if } x \neq 0, \end{cases}$$

- To solve the problem, we substitute the infinite penalty problem with a finite terminal condition and consider the parametrized value function

$$U_\varepsilon(t, s, x) = \inf_{\alpha \in \mathcal{A}(T, X)} \mathbb{E}_t \left[\int_t^T \left\{ \alpha_u^2 S_u^2 (\hat{T} - u)^{3/2} \Gamma(u, S_u) + \lambda \sigma^2 (x_u^\alpha)^2 S_u^2 \Delta^2(u, S_u) \right\} du + \frac{1}{\varepsilon} \psi(x_T^\alpha) \right].$$

With terminal condition

$$U_\varepsilon(T, s, x) = \frac{1}{\varepsilon} \psi(x) \begin{cases} 0 & \text{if } x = 0 \\ \gg 1 & \text{if } x \neq 0. \end{cases}$$

The PDE of the optimal execution framework

Theorem

Let U_ε^* be a regular function which solves the PDE :

$$\begin{cases} \partial_t U_\varepsilon^* + \frac{1}{2}\sigma^2 S^2 \partial_{SS} U_\varepsilon^* + \lambda x^2 \sigma^2 S^2 \Delta^2(t, S) - \frac{(\partial_x U_\varepsilon^*)^2}{4(\hat{T}-t)^{3/2}\Gamma(t, S)} = 0 \\ U_\varepsilon^*(T, S_T, x_T) = \frac{1}{\varepsilon}\psi(x_T^\alpha). \end{cases}$$

Then U_ε^* is the unique solution to the optimal execution problem.

Moreover, the optimal execution rate $\alpha_t^* = -\dot{x}_t^*$ is such that :

$$\alpha_t^* = \frac{\partial_x U_\varepsilon^*(t, S_t, x_t^*)}{4(\hat{T}-t)^{3/2}S_t^2\Gamma(t, S_t)}.$$

Deriving HJB : A multiplicative state variable

Let S_t and x_t^κ be the state variables and define the control variable κ such that $\alpha_t = x_t \kappa_t$

$$\begin{aligned}dS_t &= \sigma(S_t)S_t dW_t \\dx_t^\kappa &= -\kappa_t x_t^\kappa dt\end{aligned}$$

where $\kappa_t > 0$, $x_0 = X$ and x_t increasing and bounded by 0.

This form implies that :

- $\forall t \in [0, T]$, $x_t^\kappa = X e^{-\int_0^t \kappa_s ds}$ and $x_u^\kappa = x_t^\kappa e^{-\int_t^u \kappa_s ds}$ for $u > t$
- We define $\mathcal{K}(T, X)$ the set of admissible control processes κ such that x belongs to $\mathcal{X}(T, X)$.
- Using such parametrization offers a straightforward way to reduce the problem through writing :

$$U_\varepsilon(t, s, x) =: x^2 u_\varepsilon(t, s).$$

Deriving the HJB equation

- The reduced value function u_ε is defined by :

$$u_\varepsilon(t, S) = \inf_{\kappa \in \mathcal{K}} \mathbb{E}_t \left[\int_t^T e^{-\int_t^u 2\kappa_s ds} \left\{ \kappa_u^2 (\hat{T} - u)^{3/2} S_u^2 \Gamma(u, S_u) + \lambda \sigma^2 S_u^2 \Delta^2(u, S_u) \right\} du + \frac{1}{\varepsilon} e^{-\int_t^T \kappa_s ds} \right].$$

- By means of Itô's formula, u_ε verifies the HJB equation :

$$\partial_t u_\varepsilon + \frac{1}{2} \sigma^2 S^2 \partial_{SS} u_\varepsilon + \inf_{\kappa} \left\{ \kappa^2 (\hat{T} - t)^{3/2} S^2 \Gamma(t, S) - 2\kappa u_\varepsilon \right\} + \lambda \sigma^2 S^2 \Delta^2(t, S) = 0$$

$$\kappa^* \text{ is given by } \kappa^*(t, S) = \frac{u_\varepsilon(t, S)}{(\hat{T} - t)^{3/2} S^2 \Gamma(t, S)}.$$

The PDE of the reduced value function

Theorem

Let u_ε^* be a regular function verifying the following PDE

$$\begin{cases} \partial_t u_\varepsilon^* + \frac{1}{2} \sigma^2 S^2 \partial_{SS} u_\varepsilon^* + \lambda \sigma^2 S^2 \Delta^2(t, S) - \frac{1}{(\hat{T}-t)^{3/2} S^2 \Gamma(t, S)} u_\varepsilon^2 = 0 \\ u_\varepsilon^*(T, s) = \frac{1}{\varepsilon}. \end{cases}$$

Then u_ε^* is the unique solution to the reduced optimization problem. The optimal trading rate κ_t^* is given by :

$$\kappa^*(t, S) = \frac{u_\varepsilon(t, S)}{(\hat{T} - t)^{3/2} S^2 \Gamma(t, S)}.$$

Localization and boundary conditions

We localize the initial domain of the PDE to $[0, T] \times [0, S_{\max}]$ and add the complementary conditions :

- When $S = 0$ $P(t, S) \approx K$, $S^2\Gamma(t, S) \approx 0$ and $S^2\Delta^2(t, S) \approx 0$, thus we find the variational equation :

$$\min \{ \partial_t u_\varepsilon, u_\varepsilon \} = 0 \quad \text{and} \quad u_\varepsilon(T) = \frac{1}{\varepsilon}.$$

- When $S = S_{\max} \gg K$ we again have $S^2\Gamma(t, S) \approx 0$ and $S^2\Delta^2(t, S) \approx 0$. The variational inequality arising from this condition is :

$$\min \left\{ \partial_t u_\varepsilon + \frac{1}{2} \sigma^2 S^2 \partial_{SS} u_\varepsilon, u_\varepsilon \right\} = 0$$

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Results I

Parameter	Value
σ	30%
T (the strategy horizon)	1/12 (years)
\hat{T} (the option maturity)	1 (years)
S_0	1
K	S_0
Action	Buy
x_0	-1
$\tilde{\eta}$	0.05
Trading frequency	4 trades per day
λ	0, 100

TABLE – Parameters for buying options under market impact over 1m horizon

Results II

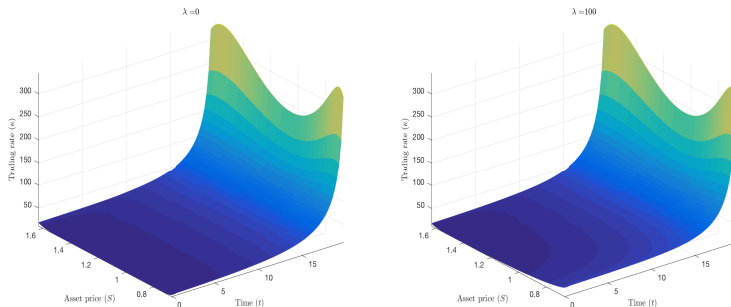


FIGURE – The rate of trading κ as a function of the underlying price S and time t for different values of λ ($\lambda = 0$ top left, $\lambda = 1$ top right, $\lambda = 10$ bottom left, $\lambda = 100$ bottom right). The strike $K = S_0$ is fixed at time 0.

Results III

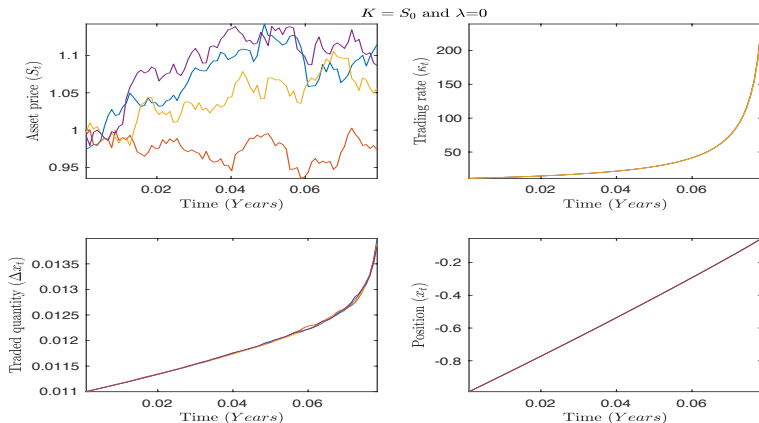


FIGURE – Sample paths of the evolution of the fundamental price, trading rate, inventory and traded quantity throughout the execution for $\lambda = 0$.

Results IV

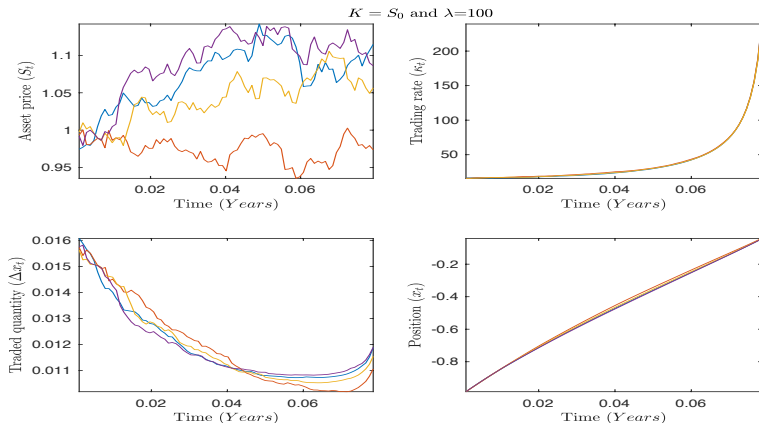


FIGURE – Sample paths of the evolution of the fundamental price, trading rate, inventory and traded quantity throughout the execution for $\lambda = 0$.

Results V

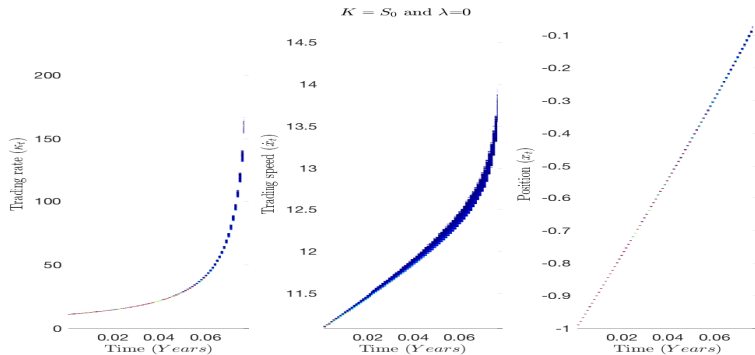


FIGURE – Heatmaps showing the density of inventory and trading speed throughout the execution for $\lambda = 0$.

Results VI

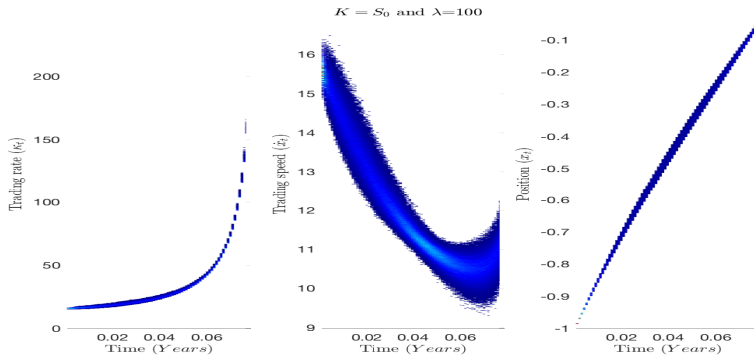


FIGURE – Heatmaps showing the density of inventory and trading speed throughout the execution for $\lambda = 100$.

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The option price PDE

The option price with market impact can be extended to local volatility models (see [?]). We focus on the CEV model, i.e. $\sigma(S) = \sigma_0 S^{\beta/2-1}$, and rewrite the valuation PDE of \tilde{P} as :

$$\begin{cases} \partial_u \tilde{P}(u, S) + \frac{1}{2} \tilde{\sigma}^2(t, S) S^2 \partial_{SS} \tilde{P}(u, S) = 0, & (u, S) \in [t, \hat{T}[\times]0, \infty[\\ \tilde{P}(\hat{T}, s) = (K - s)^+, & s \in]0, \infty[\end{cases}$$

where

$$\tilde{\sigma}^2(t, S) = \sigma^2(S) + (\tilde{\eta} \dot{x}_t + \tilde{\gamma}(x_t - x_0)) \sqrt{\hat{T} - t} \sigma(S),$$

The option price implied volatility

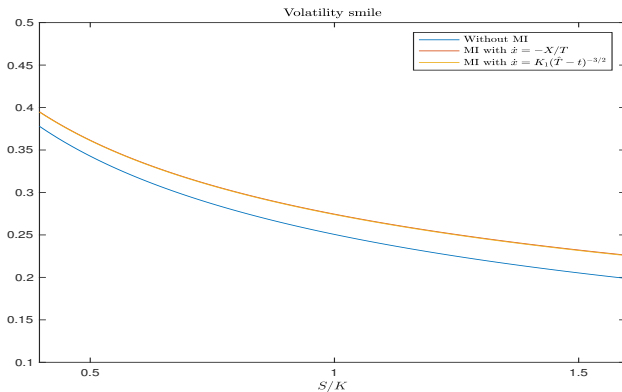


FIGURE – Volatility smile as a function of the moneyness for the CEV model with and without market impact.

The objective and value function

- The mean-variance objective function in this case is :

$$\mathbb{E} \left[\int_0^T \dot{x}_t \tilde{P}(t, S_t, x_t, \dot{x}_t) dt + \lambda \int_0^T x_t^2 \sigma^2(S_t) S_t^2 \partial_S \tilde{P}^2(t, S_t, x_t, \dot{x}_t) dt \right].$$

- The value function taken backward, i.e. $V(\tau = T - t, S, x)$ solves the following HJB equation :

$$\begin{aligned} \partial_\tau V = \frac{1}{2} \sigma^2(S) S^2 \partial_{SS} V + \inf_{\alpha} \left\{ -\alpha \partial_x V \right. \\ \left. - \alpha_t \tilde{P}(t, S, x, -\alpha) + \lambda \sigma^2(S) x^2 S_t^2 \partial_S \tilde{P}^2(t, S, x, -\alpha) \right\}. \end{aligned}$$

Which we solve using appropriate terminal and boundary conditions.

The numerical scheme

Let us define a set of nodes $[s_0, s_1, \dots, s_{i_{\max}}]$, $[x_0, x_1, \dots, x_{j_{\max}}]$, discrete times $\tau^n = n\Delta\tau$, and localize the control candidates to values in finite interval $[\alpha_{\min}, \alpha_{\max}]$. The numerical solution is performed in two steps :

- Solve the PDE of the option valuation to determine $\tilde{P}(\tau^n, s_i, x_j, \alpha)$ for each node as well as $\tilde{\Delta}(\tau^n, s_i, x_j, \alpha)$ its partial derivative w.r.t the asset price.
- Let $\alpha_{i,j}^n$ the approximate value of the control variable α at mesh node (τ^n, s_i, x_j) , and given the values of \tilde{P} and $\tilde{\Delta}$ at each mesh node, solve :

$$V_{i,j}^{n+1} = \Delta\tau(\mathcal{L}_h V)_{i,j}^{n+1} + \inf_{\alpha_{i,j}^{n+1} \in [\alpha_{\min}, \alpha_{\max}]} \left\{ V_{i,\hat{j}}^n + \Delta\tau \left(-\alpha_{i,j}^{n+1} \tilde{P}_{i,j}^{n+1}(\alpha_{i,j}^{n+1}) + \lambda\sigma_0(x_j)^2 S_j^\beta (\tilde{\Delta}_{i,j}^{n+1}(\alpha_{i,j}^{n+1}))^2 \right) \right\}.$$

where $V_{i,\hat{j}}$ is obtained by linear interpolation of the discrete value $V_{i,j}^n$.

Results

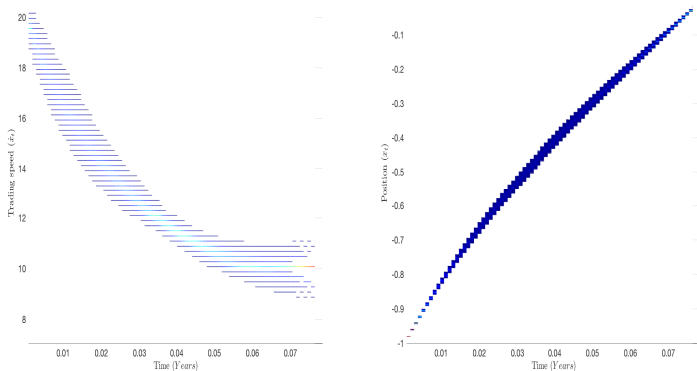


FIGURE – Heatmap showing the density of the trading speed and inventory for $\lambda = 100$.

Summary

In the market impact topic we were able to :

- Define a price of an option under market impact under a Black-Scholes framework.
- Set a framework for the optimal purchasing problem that minimizes :
 - The expected cost.
 - The mean-variance of the cost.
- Solve the problem using appropriate methods.
- Extend the framework to local volatility models.

Limitations and perspectives

- We assumed that the effect of market impact on stock price is not visible to the option buyer.
- The numerical solution is sensitive to the terminal condition through the choice of the penalty ϵ .
- The Black-Scholes framework uses a Taylor approximation and the general case consider is limited to a CEV model. simplicity.

What we would want to do :

- Understand market impact on options from an empirical perspective.
- Consider exotic payoffs for the framework (e.g. look back options) and different risk criteria.
- Assume that market impact on the stock is visible and that the option seller opts for an optimal hedging strategy.
- Improve the numerical scheme for the general case and account for broader volatility models and stochastic interest rates.

QUESTIONS ?
Thank you for you attention



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