High-Dimensional Factor Models and the Factor Zoo

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Factor Zoo in Asset Pricing

- Factor Zoo: An ever-growing list of characteristic premia
- Suppose we observe C characteristics of M assets over T periods
- Goal: Parsimonious model with $K_C \ll C$ pricing factors
- Standard approach:
  1. **Sort** assets into $10C$ portfolios → C HML (10-1) factors
  2. **Factor zoo**: Reduce dimension from C to K_C factors
- Current literature: Methods in Step 2

  → PCA and extensions, Lasso, Ridge, ML, neural nets, random forests, ...

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- Current literature: Methods in Step 2
  $\rightarrow$ PCA and extensions, Lasso, Ridge, ML, neural nets, random forests, ...
- **This paper**: Reverse the order of sorting and dimension reduction
  1. **Factor zoo**: Summarize C characteristics in K_C **characteristic factors**
  2. **Sort** assets into $10K_C$ portfolios $\rightarrow$ K_C HML factors
- Difference: Resolve factor zoo in **characteristic space** instead of return space

Challenge: 3-dimensional Data

- Challenge: Data is **3-dimensional** (characteristics of assets over time)
- Characteristics are correlated in all 3 dimensions
- PCA and factor models: Eliminate one dimension $\rightarrow$ loss of information
- High-dimensional data form **tensors**
- **Tensor Factor Models (TFM)**:
  - Exploit dependencies along **all 3 dimensions** simultaneously
  - Allow for complex dependencies across characteristics (vs. univariate/bivariate sort)
- **Methodology**:
  1. Dimension reduction: Apply TFM to asset pricing characteristics
     $\rightarrow$ K_C **characteristic factors** that summarize the information in C characteristics
  2. Portfolios: Sort on K_C factors $\rightarrow$ K_C pricing factors
  3. Compare TFM factors to Fama-French and PCA factors

Main Findings

1. **Characteristic data**:
   - Parsimonious TFM models capture over 90% of the variation in the data
   - Compress data by more than 95%
   - Good fit for most characteristics, highest errors for MOM and REV
   - Stable over time, robust results

2. **Asset pricing implications**:
   - TFM characteristic factors have higher means and SR than standard PCA factors
   - TFM factors are related to the cross-section of returns
   - TFM factors yield smaller pricing errors than FF and PCA factors
   - Robust in-sample and out-of-sample results

Outline

1. From 2-dimensional PCA/SVD to Tensor Factor Models (TFM)
2. Empirical application: Characteristics of mutual funds
3. Estimation and fit of TFM
4. Construct pricing factors from TFM
5. Asset pricing tests, comparison to benchmark models (FF, PCA, ...)

Tensor Factor Models

From Matrices to Tensors

- Notation:

scalar:	$x \in \mathbb{R}$
vector:	$\mathbf{x} \in \mathbb{R}^{I}$
matrix:	$\mathbf{X} \in \mathbb{R}^{I_1} \times \mathbb{R}^{I_2}$
3rd-order tensor (cuboid):	$\mathcal{X} \in \mathbb{R}^{I_1} \times \mathbb{R}^{I_2} \times \mathbb{R}^{I_3}$

- Methods apply to $n > 3$ dimensions
- Paper: Summary of tensor operations
- Tensors have **fibers** (vectors) and **slices** (matrices)
- No transpose operator → n-mode tensor multiplication $\times_n$: $\mathcal{X} \times_1 \mathbf{A}_1 \times_2 \mathbf{A}_2$
  → multiply each n-dimension tensor fiber with each row of $\mathbf{A}_n$
- Data: **Characteristic** c of **asset** m in **quarter** → $\mathcal{X} : (T \times M \times C)$

Tensor Factor Models: Intuition

- Objective: Capture characteristics in small number of factors
- Possibility: Eliminate one dimension → PCA
  - PCA for each period $t = 1, \dots, T$
  - PCA for each asset $m = 1, \dots, M$
  - PCA for each characteristic $c = 1, \dots, C$
- $T + M + C$ separate factor models exploit only 2-dim. correlations
- Loss of information, order of factors might change
- **Tensor factor model (TFM)**: Estimate $T + M + C$ 2-dimensional PCA **simultaneously**
- **TFM exploit dependencies in all 3 dimensions**
- The $T + M + C$ models are **connected** and subject to **restrictions**

2-dimensional PCA/SVD/Factor Model

- K-factor **Singular Value Decomposition (SVD)** of a matrix $\mathbf{X}$:

$$\mathbf{X} = \hat{\mathbf{X}}_K + \mathbf{E}_K$$

$$\hat{\mathbf{X}} = \mathbf{U}_{1,K}\,\mathbf{H}_K\,\mathbf{U}_{2,K}^{\top}$$

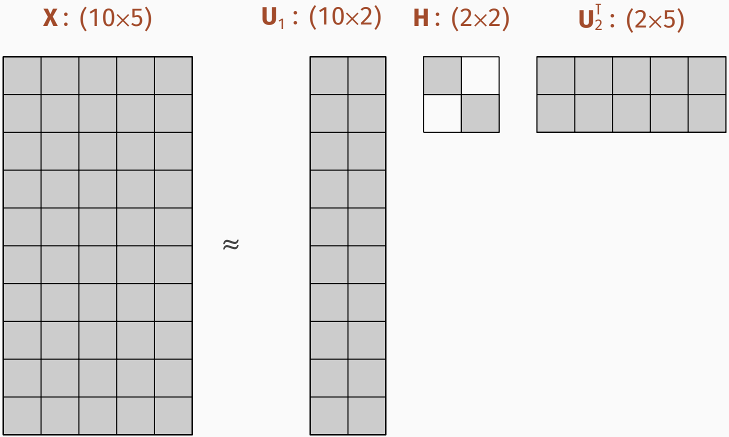
- $\mathbf{H}_K, \mathbf{U}_{1,K}, \mathbf{U}_{2,K}$ are matrices of eigenvalues/vectors of $\mathbf{X}\mathbf{X}^{\top}$ and $\mathbf{X}^{\top}\mathbf{X}$
- **Factor** representation:

$$\mathbf{X} = \mathbf{F}_K\,\mathbf{B}_K^{\top} + \mathbf{E}_K,$$

$$\mathbf{F}_K = \mathbf{U}_{1,K}\,\mathbf{H}_K, \quad \mathbf{B}_K^{\top} = \mathbf{U}_{2,K}^{\top}$$

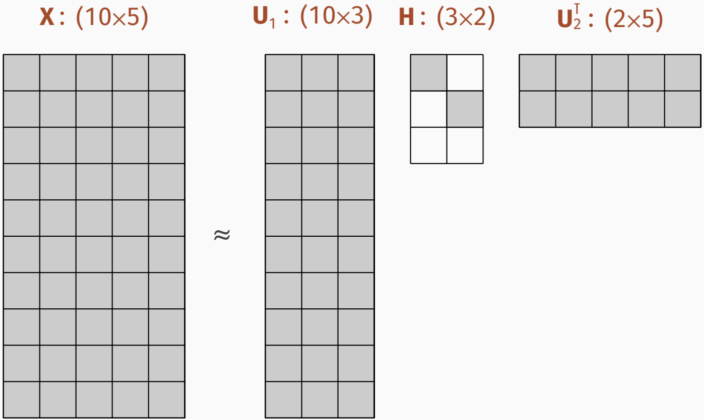
- **Factor** matrix $\mathbf{F}_K$, **loading** matrix $\mathbf{B}_K^{\top}$
- **Tucker decomposition** extends matrix SVD to tensors
- Note: $\mathbf{F}_K$ are **PCA** factors, not **asset pricing** factors ($\mathbf{f}_t$)

Truncated SVD: $K = 2$



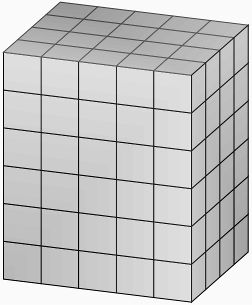
Asymmetric SVD: $K_1 = 3, K_2 = 2$

- In principle, we could write an SVD with different $K_1 \neq K_2$
- But $\mathbf{H}$ is **diagonal** $\rightarrow$ equivalent to $K = \min(K_1, K_2)$
- Tensor SVD: $\mathcal{H}$ is **not** diagonal $\rightarrow$ different K_i possible



Tucker Decomposition: Intuition

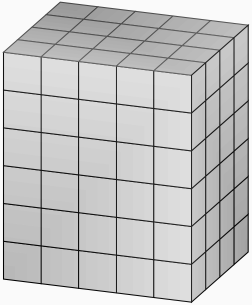
$\mathcal{X}$: (6×5×4)



$\mathcal{X}$ is a (large) **data tensor** ($T \times M \times C$)

Tucker Decomposition: Intuition

$\mathcal{X}$: (6×5×4)



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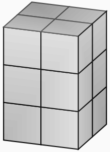
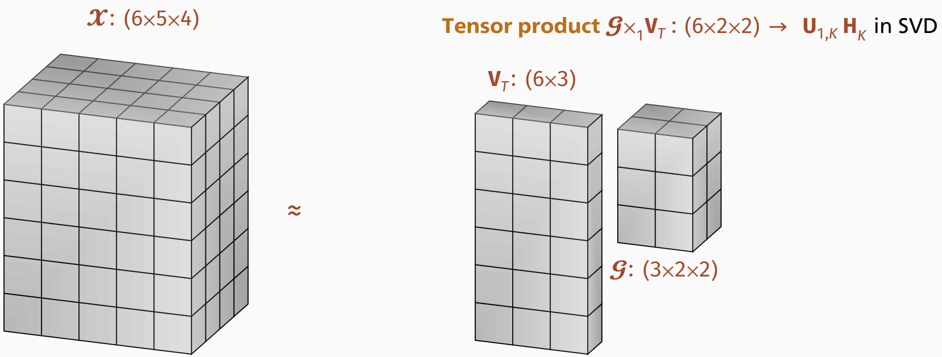
$\mathcal{G}$ is a (small) **core tensor** $(K_T \times K_M \times K_C) \rightarrow$ $\mathbf{H}_K$ in SVD

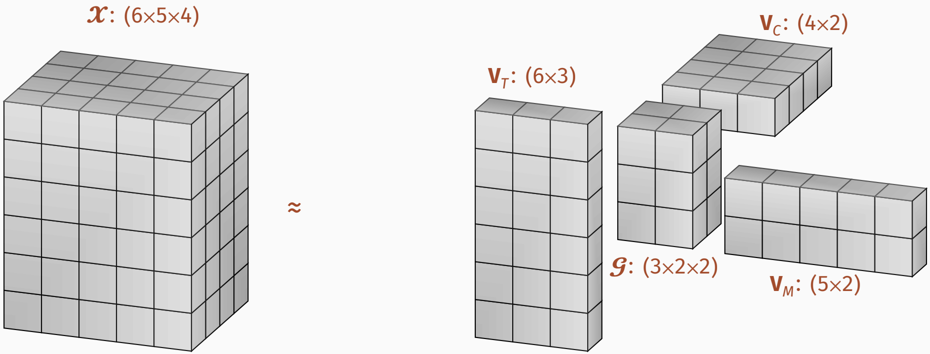


$\mathcal{G}$: (3×2×2)

Tucker Decomposition: Intuition



Tucker Decomposition: Intuition



- TFM: $\mathcal{X} \approx \mathcal{G} \times_1 \mathbf{V}_T \times_2 \mathbf{V}_M \times_3 \mathbf{V}_C$
- SVD: $\mathbf{X} \approx \mathbf{U}_{1,K}\, \mathbf{H}_K\, \mathbf{U}_{2,K}^T = \mathbf{H}_K \times_1 \mathbf{U}_{1,K} \times_2 \mathbf{U}_{2,K}$

Tucker Tensor Decomposition

- K-factor SVD of $(T \times N)$ matrix: $\mathbf{X} = \hat{\mathbf{X}} + \mathbf{E}$

$$\hat{\mathbf{X}}(K) = \mathbf{U}_{1,K}\, \mathbf{H}_K\, \mathbf{U}_{2,K}^{\top} = \mathbf{H}_K \times_1 \mathbf{U}_{1,K} \times_2 \mathbf{U}_{2,K}$$

- **Tucker decomposition** of $(T \times M \times C)$ tensor: $\mathcal{X} = \hat{\mathcal{X}} + \mathcal{E}$

$$\hat{\mathcal{X}}(K_T \times K_M \times K_C) = \mathcal{G} \times_1 \mathbf{V}_T \times_2 \mathbf{V}_M \times_3 \mathbf{V}_C$$

$$\mathcal{G} : \ (K_T \times K_M \times K_C)$$

$$\mathbf{V}_T : \ (T \times K_T), \quad \mathbf{V}_M : (M \times K_M), \quad \mathbf{V}_C : (C \times K_C)$$

- $\mathcal{G}$ is the **core tensor** and $\mathbf{V}_T, \mathbf{V}_M, \mathbf{V}_C$ are **loading matrices**
- $\mathcal{G}$ **not diagonal** $\rightarrow$ number of factors K_T, K_M, K_C can differ across dimensions
- $\mathcal{G}, \mathbf{V}_T, \mathbf{V}_M, \mathbf{V}_C$ to minimize MSE($\mathcal{E}$)
- **Data compression**: DoF$/N$, where N is the sample size (see paper)

Tucker Decomposition vs. SVD

Similarities:

- Representations are not unique and can be rotated
- Normalization: $\mathbf{U}_i, \mathbf{V}_i$ are orthonormal

Differences:

- SVD: $\mathbf{H}$ is diag. eigenvalue matrix, $\mathbf{U}_i$ are eigenvector matrices
- Tucker: Notions of eigenvalues/eigenvectors do not exist
- Core tensor $\mathcal{G}$ is not (necessarily) diagonal
- Numbers of factors for each mode can be different: (K_T, K_M, K_C)

- SVD: Closed-form solution (aside from numerical computation of eigenvalues/vectors)
- TFM: Higher Order Orth. Iteration (HOOI) $\rightarrow$ linear, robust, fast even for large data sets
- Monte Carlo simulation: Small estimation errors for sample sizes similar to data set

Properties of Tucker Decomposition

- 3-dim. TFM implies 2-dim. factor models in each dimension
- Paper: TFM can be written as the **sum of 2-dim. factor models**
  - (M, C) dimensions: Sum of K_T factor models of order $\min(K_M, K_C)$
  - (T, C) dimensions: Sum of K_M factor models of order $\min(K_T, K_C)$
  - (T, M) dimensions: Sum of K_C factor models of order $\min(K_T, K_M)$
- All 2-dim. factor models stem from the same 3-dim. TFM and are **interconnected**
- The TFM imposes **restrictions** on the 2-dim. factor models
- **Data compression** of n-dim. TFM: $\mathcal{O}(Q^n)$, where $Q = \max(K_i)/\min(N_i)$
- Higher dimension $n \rightarrow$ higher dimension reduction

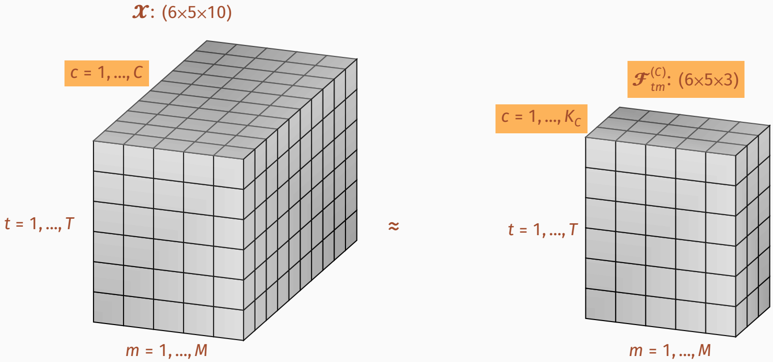
Factor Representation of TFM

- The Tucker model can be written in **factor form** in each dimension
- Example: **Characteristic** factors

$$\mathcal{X} = \mathcal{F}^{(C)}_{tm} \times_3 \mathbf{V}^{(C)} + \mathcal{E}$$
$$\mathcal{F}^{(C)}_{tm} = \mathcal{G} \times_1 \mathbf{V}^{(T)} \times_2 \mathbf{V}^{(M)}$$

- Characteristic factors: $\mathcal{F}^{(C)}_{tm}$ is a $(T \times M \times K_C)$-dim. **characteristic factor tensor**
- Similar to factor representation of SVD/PCA: Factor **tensor** instead of **matrix**
- Each m-slice is a $(T \times K_C)$-dim. matrix $\mathbf{F}^{(C)}_{tm}$: TS of K_C characteristic factors of asset m
- The characteristic factors $\mathbf{F}^{(C)}_{tm}$, $m = 1, \ldots, M$
  - summarize the information in all C characteristics of asset m
  - exploit information across time, other assets, and other characteristics
  - are based only on characteristics but use no return information
  - can be used to construct asset pricing factors
- Dimension reduction in characteristic space rather than in return space

Factor Zoo: C Characteristics to K_C Characteristic Factors



- $\mathcal{F}^{(C)}_{tm}$ is a tensor of **characteristic factors**
- Fix asset m: mth vertical slice of $\mathcal{F}^{(C)}_{tm}$ represents time series of K_C factors of asset m
- $\mathcal{F}^{(M)}_{tc}$: **Mutual fund factors** summarize info. in M assets
- $\mathcal{F}^{(T)}_{mc}$: **Time factors** summarize info. in T periods

What are "Time" Factors?

- SVD/PCA/factor models: "Time" does not play special role
- $\mathbf{X}$: $(T \times N)$ matrix of time series of N variables
- PCA: covariance matrix $\mathbf{X}^\intercal\mathbf{X}$:
  - $(T \times K)$ factor matrix: linear combinations across N
  - $(K \times N)$ loadings matrix
- **Isomorphic** alternative: $\mathbf{X}^\intercal \rightarrow$ PCA of $\mathbf{X}\mathbf{X}^\intercal$:
  - $(N \times K)$ factor matrix: linear combinations across T
  - $(K \times T)$ loadings matrix
- SVD of $\mathbf{X}$ vs. $\mathbf{X}^\intercal$: Roles of $\mathbf{U}_{1,K}, \mathbf{U}_{2,K}$ reversed
- PCA for $\mathbf{X}$: Time series of **cross-sectional factors**
- PCA for $\mathbf{X}^\intercal$: Cross-sectional observations of **time-series factors**
- "Time series factors" have the same interpretation as XS factors
- Example: First factors close to XS mean or TS mean

Mutual Fund Characteristics

Application: Mutual Fund Characteristics

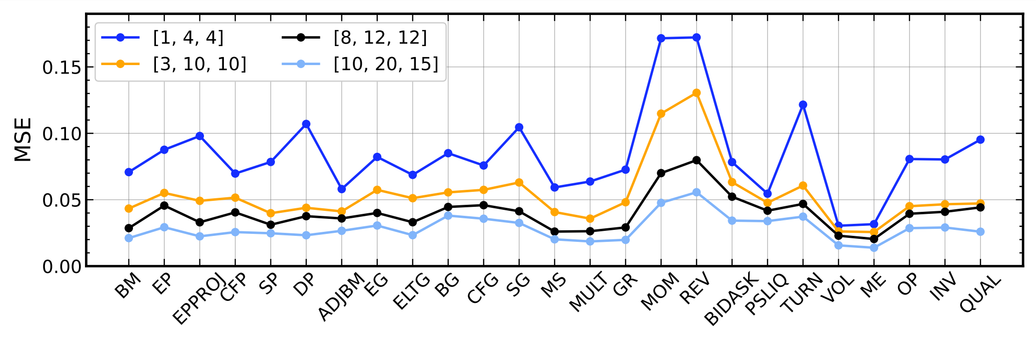
- Characteristics of mutual funds and ETFs (Lettau, Ludvigson, Manoel, 2021):
  - 2010Q3 to 2018Q4 = 34 quarters
  - 934 active equity mutual funds and ETFs
  - 25 characteristics: value/growth, growth of fundamentals, momentum, reversals, size, profitability, investment, liquidity
  - Balanced panel w/o missing values (balanced panel of stocks?)
- Data tensor $\mathcal{X}$: $(34 \times 934 \times 25) = 793{,}900$ obs.
- Methodology:
  1. Estimate Tucker models for (K_T, K_M, K_C) : $\hat{\mathcal{X}}(K_T \times K_M \times K_C)$
  2. Construct characteristic factors $\mathbf{F}_{tm}$ for $m = 1, \ldots, M$
  3. Asset pricing implications

Fit of Tucker Factor Model

- No formal selection method for number of factors (K_T, K_M, K_C)
  → Paper compares fit of many (K_T, K_M, K_C) combinations
- Stable in short samples and across time (subsamples, rolling windows)
- Results do not depend on specific choice of (K_T, K_M, K_C)
- Recall: $M = 934$ mutual funds in the sample

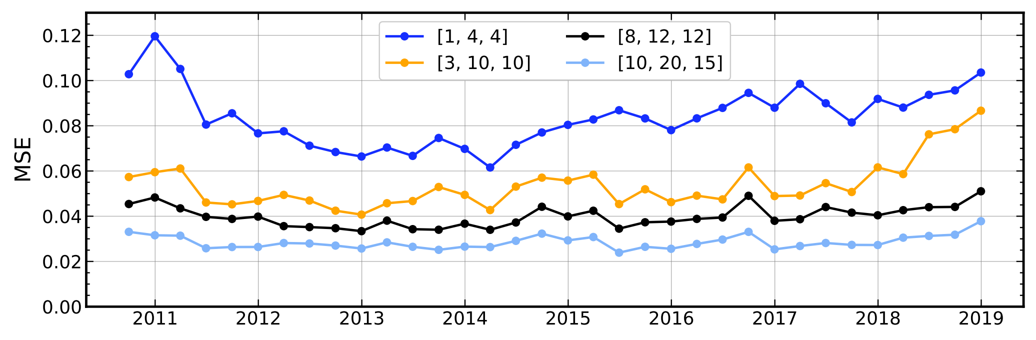
	(1,4,4)	(3,10,10)	(8,12,12)	(10,20,15)
MSE	0.08	0.05	0.04	0.03
R^2	85.3%	90.7%	93.7%	95.2%
Data compression	99.5%	98.8%	98.4%	97.3%

Benchmark Models: MSE by Characteristic

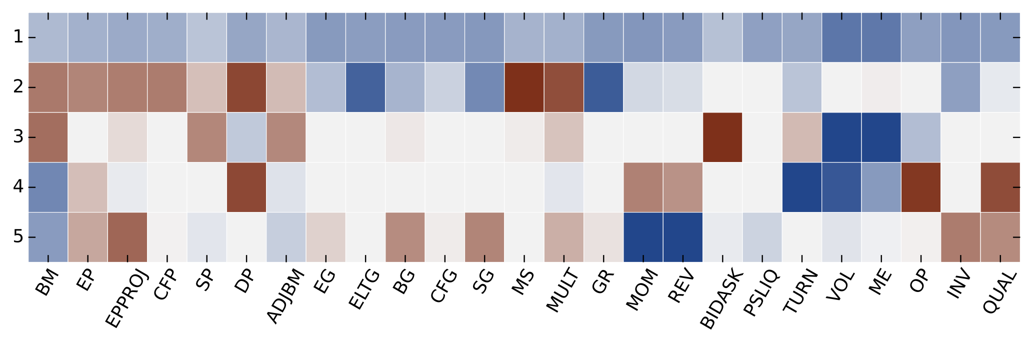


- Worst fit: MOM, REV
- Most characteristics are persistent
- Exceptions: MOM, REV
- Require higher K_T

Benchmark Models: MSE by Quarter



Characteristic Factors: Loading Matrix $V^{(C)}$



Factor 1: Long-only (related to means)

Factor 2: Long growth rates, short price-multiples

Factor 3: Long ME and VOL, short BID-ASK

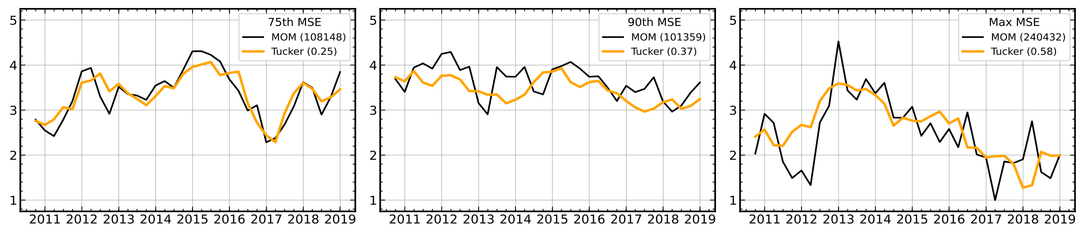
Factor 4: Long TURN, ME, VOL, short DP, OP, QUAL

Factor 5: Long MOM, REV, short EPPROJ

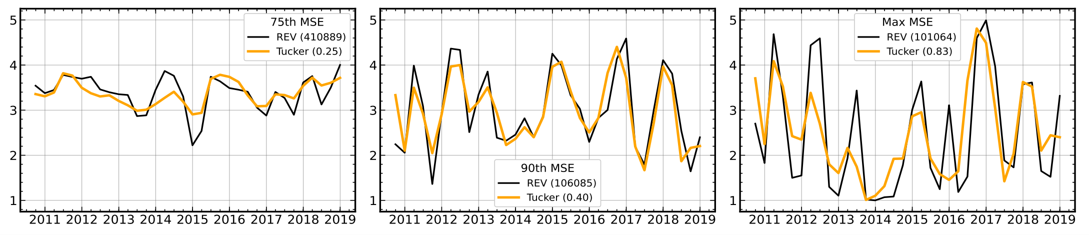
Benchmark Model: Fit of individual Funds

- Columns 1 and 2: 75th and 90th MSE percentiles, column 3: **worst fit**

Momentum



Reversal



Asset Pricing Implications

TFM Characteristic Factors

- Recall: Characteristic factors implied by TFM → $\mathcal{F}_{tm}^{(C)}$ with dim. $(T \times M \times K_C)$
- m-slices of $\mathcal{F}_{tm}^{(C)}$ are $(T \times C)$-matrices $\mathbf{F}_{t(m)}$: time series of K_C characteristic factors of asset m
- $\mathbf{F}_{t(m)}$ summarizes the information in C characteristics of fund m in K_C factors
- TFM characteristic factors have similar properties as PCA factors:
  - The **first** TFM factor is related to **means** across funds
  - **Higher-order** factors are “long/short” factors
  - Example: **Second** TFM factor is related to “value”/“growth” characteristics
- See paper for more details
- Benchmarks: $(K_T, K_M, K_C) = (1, 4, 4), (3, 10, 10)$
- Next: Are TFM characteristic factors related to mutual fund returns?

Tucker Characteristic Factors and the XS of MF Returns

- Standard regression: Excess returns on lagged characteristics
- Instead: Regress excess returns on lagged **TFM characteristic factors**

$$R^e_{m,t+1} = \alpha + \boldsymbol{\beta}^\top \mathbf{F}_{t(m)} + \gamma_t + e_{m,t+1}$$

- Fama-MacBeth (FMB): XS regression in each period, sample means of coefficients
- FMB requires a long time series
- Alternative when T is small (e.g. $T < C$): Panel regression with time fixed effects (γ_t)
- t-stats: entity or time-clustered standard errors
- Between-R^2: XS fit after all time effects are eliminated
- Comparison: Regression on all 25 characteristics → Table: 10 largest coefficients

Tucker Characteristic Factors and the XS of MF Returns

	TFM Characteristic Factors $\mathbf{F}_{t(m)}$										
	1	2	3	4	5	6	7	8	9	10	R^2
β	0.36	0.10	0.34	0.26	0.30	0.29	0.35	−0.05	0.09	0.22	0.50
t-entity	(6.49)	(3.51)	(9.08)	(8.77)	(10.47)	(8.26)	(14.37)	(−1.92)	(3.28)	(6.87)	
t-time	(2.27)	(0.42)	(1.71)	(2.27)	(1.73)	(2.19)	(1.56)	(−0.45)	(0.35)	(0.92)	
	Characteristics										
	MS	MULT	REV	EP	ELTG	BIDASK	ADJBM	CFG	OP	VOL	R^2
β	0.61	0.47	0.44	0.33	0.28	0.18	0.14	0.13	0.08	0.04	0.45
t-entity	(2.48)	(1.57)	(8.66)	(4.94)	(1.97)	(2.17)	(2.39)	(2.14)	(1.48)	(0.18)	
t-time	(1.04)	(0.64)	(2.24)	(1.37)	(0.51)	(0.71)	(0.63)	(0.81)	(0.64)	(0.08)	

- TFM characteristic factors:
  - 8/10 coefficients are statistically significant using entity-clustered s.e.
  - Between-R^2 = 50% → TFM factors capture half of the XS variation in fund returns
- Characteristics: 7/25 significant coefficients, Between-R^2 = 45%

From TFM Characteristic Factors to Pricing Factors

- Standard approach:
  1. Form portfolios based on C characteristics
  2. Decile sorts: $10C$ portfolios
  3. C high-minus-low (HML) factors
  4. Apply dimension reduction (PCA, ML) to large panel of portfolio returns
- Instead:
  1. **Dimension reduction in characteristic space: TFM**
     → K_C TFM characteristic factors
  2. Form decile portfolios based on K_C characteristic factors → $10K_C$ portfolios
  3. **HML pricing factors**: K_C decile-10 minus decile-1 long/short portfolios
- Compare K_C TFM pricing factors to Fama-French and PCA HML factors
- Paper: Out-of-sample TFM, PCA factors → expanding window TFM, PCA estimations

HML Factor Returns: Tucker vs. PCA

- Normalize TFM and PCA factors to have positive means
- Table: 5 Factors with the highest means

			HML Tucker Factors		
	5	9	4	3	10
Mean	5.52	3.66	3.30	3.19	2.56
SR	0.89	0.46	0.72	0.42	0.42
CAPM α	4.88**	5.38*	5.20***	5.58**	5.09**
FF3 α	5.23**	3.60	4.04**	1.44	4.11**
			HML PCA Factors		
	ELTG	REV	GR	QUAL	INV
Mean	2.92	2.39	2.04	1.98	1.72
SR	0.34	0.38	0.24	0.46	0.22
CAPM α	−0.78	0.53	−1.64	3.67**	0.34
FF3 α	−0.18	0.05	−1.11	1.87*	−0.28

Linear Factor Models

- Tucker pricing factors have higher returns and Sharpe ratios than standard PCA factors
- Next: Estimate linear factor models (TFM factors, Fama-French, PCA factors)
- All factors are excess returns $\rightarrow$ time series regressions:

$$R^e_{mt} = \alpha_m + \boldsymbol{\beta}^\top_m \mathbf{f}_t + e_{mt}, \quad m = 1, \ldots, M$$

- Model evaluation: RMSPE of pricing errors α_m
- All specifications: First factor is the market excess return MKT
- Paper: Tucker (1,4,4) model performs (almost) as well as Tucker (3,10,10) model
- RMSPE*: RMSPE when other factors are added
- Note: GRS test rejects H_0 for all models

Factors	L	RMSPE		RMSPE*	
			Tucker	FF	PCA
MKT	1	3.27%	1.90%		2.22%
MKT, SMB	2	2.85%	2.02%		2.71%
MKT, SMB, HML	3	2.59%	2.24%		2.48%
Tucker(1,4,4)					
MKT, 4	2	2.80%		2.47%	2.73%
MKT, 2, 4	3	1.91%		2.46%	1.88%
PCA					
MKT, 3	2	2.58%	1.90%	2.64%	
MKT, 3, 6	3	2.22%	1.91%	2.48%	

Factors	L	RMSPE	RMSPE*		
			Tucker	FF	PCA
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MKT, SMB, HML	3	2.59%	2.24%		2.48%
Tucker(1,4,4)					
MKT, 4	2	2.80%		2.47%	2.73%
MKT, 2, 4	3	1.91%		2.46%	1.88%
PCA					
MKT, 3	2	2.58%	1.90%	2.64%	
MKT, 3, 6	3	2.22%	1.91%	2.48%	
Tucker(1,4,4) OOS					
MKT, 4	2	2.62%		2.55%	2.47%
MKT, 2, 4	3	1.87%		1.83%	2.28%
PCA-OOS					
MKT, 3	2	2.92%	2.42%	2.80%	
MKT, 3, 4	3	2.76%	2.53%	2.73%	

RMSPE by Fund Type

- Sample: Cap-based (C), growth (G), value (V), balanced (B), and "sector"(S), other (O)

	C	B	G	V	S	O
Tucker-IS	1.85%	1.53%	1.65%	1.82%	2.80%	0.99%
Tucker-OOS	1.72%	1.63%	1.68%	1.49%	2.66%	0.86%
CAPM	3.37%	2.25%	2.05%	3.06%	5.84%	1.19%
FF3	2.54%	2.04%	1.85%	1.92%	4.86%	1.20%
PCA-IS	2.12%	1.77%	1.81%	1.92%	3.73%	1.42%
PCA-OOS	2.55%	2.09%	1.89%	2.22%	5.31%	1.31%

RMSPE by Fund Type

- Sample: Cap-based (C), growth (G), value (V), balanced (B), and “sector”(S), other (O)

	C	B	G	V	S	O
Tucker-IS	1.85%	1.53%	1.65%	1.82%	2.80%	0.99%
Tucker-OOS	1.72%	1.63%	1.68%	1.49%	2.66%	0.86%
CAPM	3.37%	2.25%	2.05%	3.06%	5.84%	1.19%
FF3	2.54%	2.04%	1.85%	1.92%	4.86%	1.20%
PCA-IS	2.12%	1.77%	1.81%	1.92%	3.73%	1.42%
PCA-OOS	2.55%	2.09%	1.89%	2.22%	5.31%	1.31%

Pricing Errors and Characteristics

- Suppose returns depend on asset characteristics
- Question: Does factor model capture link between characteristics and returns?
- If factor model is correctly specified: PE should not depend on characteristics:

$$\mathrm{PE}_m = a + \mathbf{b}^\intercal \mathbf{C}_m + e_m$$

- Number of significant coefficients:

Model	# sig. b_i	Largest b_i	Smallest b_i
$\overline{R}^e_m$	18	ME, MS	VOL, BM
Tucker-IS	9	INV, MOM	REV, SG
Tucker-OOS	10	INV, MOM	REV, VOL
CAPM	19	ME, MULT	GR, EPPROJ
FF3	16	MS, MULT	VOL, EPPROJ
PCA-IS	15	ME, MS	VOL, CFP
PCA-OOS	19	MULT, ME	GR, EPPROJ

Conclusion

- Factor models for higher dimensional data
- The Tucker decomposition extends the SVD of matrices to tensors
- Factor zoo: Dimension reduction in characteristic space rather than in return space
- Sample of mutual fund characteristics:
  - Data dimensions: $(T \times M \times C) = (34 \times 934 \times 25)$
  - Parsimonious Tucker models capture $> 90\%$ of variation in data
  - Robust fit across time, funds, and characteristics, stable over time
- TFM characteristic factors are related to fund returns
- Pricing factors derived from the Tucker model
  - have higher mean returns and SR than PCA factors
  - smaller pricing errors than Fama-French and PCA factors
- Full-sample and recursive estimation