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Comparative Statics of Equilibria with Respect to Stochastic Tax Rates

Revised from the Center for Risk Management Research Working Papers 2016-03 and 2017-02

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Abstract

This paper studies equilibrium comparative statics in the finite horizon CCAPM with respect to changes in stochastic tax rates imposed on agents' endowments and dividends. We show that under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate unambiguously reduces current asset prices. The paper also finds that there exists a bound $\overline{B}$ such that for a coefficient of relative risk aversion less than $\overline{B}$, an increase in a future dividend tax rate reduces current price of tradable assets. At the same time, for a coefficient of relative risk aversion greater than $\overline{B}$, an increase in a future dividend tax rate boosts the current price of tradable assets. Finally, for a coefficient of relative risk aversion equal to $\overline{B}$, an increase in a future dividend tax rate leaves current price of tradable assets unchanged. As a special case, under additional assumptions, $\overline{B}$ is equal to 1. Also, under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices, while an increase in a future endowment tax rate boosts current asset prices.

1. Introduction

Taxes are a part of individuals' and corporations' budget constraints. Therefore, taxes clearly affect equilibrium commodity and asset prices and allocations. Also, changes in various tax rates, especially income tax, are driven by the constantly changing political balance of power, and the direction of those changes seems to have been anything but predictable. Thus, it seems entirely appropriate to regard future taxation as stochastic.

But if taxation is stochastic, then it is clearly a risk factor affecting equilibrium asset prices through stochastic discount factors and after-tax dividends. Since this risk cannot be eliminated or substantially reduced by diversification, standard finance theory suggests that it ought to be an asset-pricing risk factor, which ought to affect asset prices and allocations.

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Surprisingly, however, there has been very little research done to date on the effects of stochastic taxes on equilibrium asset prices and allocations. The research done so far, relies on the CCAPM with identical agents and twice-differentiable utility functions and focuses primarily on resolving the so-called "Equity Premium Puzzle." See Magin (2015a), Edelstein and Magin (2016) and (2013), DeLong and Magin (2009), Sialm (2009) and (2006). While resolving the Equity Premium Puzzle is critically important for confirming the validity of the Lucas-Rubenstein CCAPM with identical agents, the role of insecure property rights (stochastic taxation) in economic theory is much broader. For example, do equilibria exist in the finite horizon CCAPM with stochastic taxation? Do sufficiently small changes in stochastic tax rates preserve the existence and completeness of equilibria? Magin (2015b) finds that under reasonable assumptions, equilibria exist for all stochastic tax rates imposed on agents' endowments and dividends except for a closed set of measure zero. Moreover, sufficiently small changes in stochastic taxation preserve the existence and completeness of equilibria. The next natural question to ask would be: Does an increase in current and future taxes reduce current prices of tradable assets?

This paper studies comparative statics of equilibria in the finite horizon CCAPM with respect to changes in stochastic tax rates imposed on agents' endowments and dividends. The Sonnenschein-Mantel-Debreu Theorem states that if we exclude prices close to zero, then no further restrictions other than Continuity, Homogeneity and Walras' Law can be imposed on the aggregate excess demand function of an exchange economy. As a result, comparative statics results are fairly rare in general equilibrium.[1] Here, we develop a technique which we believe to be new, and which is potentially applicable in other situations. We show that, although the sign of a derivative of a complex object of interest may be ambiguous, as is typically the case in general equilibrium, the sign of the derivative of this complex object of interest is always the same as that of the derivative of some other simple and more intuitive object. While the signs of the derivative of this simple and more intuitive object in general equilibrium are often indeterminate, many of them have a natural sign; the presence of the opposite sign is viewed as possible but somewhat rare and pathological. Our methods give intuitive signs to derivatives in cases where the natural sign may not be obvious, by showing that they are the same as the signs in cases where there is an obvious natural sign.

The first major finding of this paper is Theorem 3.2. It analyzes comparative statics of current asset prices with respect to changes in current dividend taxes. Dividends are paid in units of a consumption good. It states that although the sign of a derivative of current equilibrium asset prices with respect to current dividend tax rates may be ambiguous, it is always the same as that of the derivative of the current equilibrium aggregate consumption with respect to current dividend tax rates. Specifically, if the current equilibrium aggregate consumption is a normal good, then under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate reduces current asset prices. Otherwise, if the current aggregate equilibrium consumption is an inferior good, then an increase in the current dividend tax rate boosts current asset prices. While inferior goods have been demonstrated to exist, it is generally believed that they are rare: at any given time and price level, the demand for the vast majority of goods moves in the intuitive direction with respect to changes in after-tax income.[2] So it is reasonable to assume that the aggregate consumption is a normal good. Therefore, it is natural to conclude that under reasonable assumptions, an increase in the current dividend tax rate reduces current asset prices.

Corollary 3.4. of the above theorem states that although the sign of a derivative of before-tax and

[1] See sections 17.E-17.G in Mas-Colell, Whinston and Green (1995) for discussion. Quah (2003) is a rare exception.

[2] In his 1957 book "A Theory of the Consumption Function" Milton Friedman develops the Permanent Income Hypothesis and establishes a strong positive correlation between the permanent consumption and the permanent income.

after-tax rates of return for tradable assets with respect to current dividend tax rates may be ambiguous, it is always the opposite of that of the derivative of the current equilibrium aggregate consumption with respect to current dividend tax rates. Specifically, if the current equilibrium aggregate consumption is a normal good, then under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate boosts before-tax and after-tax rates of return for tradable assets. Otherwise, if the current equilibrium aggregate consumption is an inferior good, then an increase in the current dividend tax rate reduces before-tax and after-tax rates of return for tradable assets. Since it is reasonable to assume that the current equilibrium aggregate consumption is a normal good, it is natural to conclude that under reasonable assumptions, an increase in the current dividend tax rate boosts before-tax and after-tax rates of return for tradable assets. Assuming that the real risk-free rate of return is constant, we can also conclude that an increase in the current dividend tax rate boosts before-tax and after-tax risk premiums for tradable assets.

The second major finding of this paper is Theorem 3.5. It analyzes comparative statics of current asset prices with respect to changes in future dividend taxes. It states that there exists a bound $\overline{B}$ such that for a coefficient of relative risk aversion less than $\overline{B}$, an increase in a future dividend tax rate reduces current prices of tradable assets. At the same time, surprisingly, for a coefficient of relative risk aversion greater than $\overline{B}$, an increase in a future dividend tax rate boosts current prices of tradable assets. Finally, for a coefficient of relative risk aversion equal to $\overline{B}$, an increase in a future dividend tax rate leaves current prices of tradable assets unchanged. As a special case, under additional assumptions, $\overline{B}$ is equal to 1.

Theorem 4.1. analyzes comparative statics of current asset prices with respect to changes in current endowment taxes. It states that although the sign of a derivative of current equilibrium asset prices with respect to current endowment tax rates may be ambiguous, it is always the same as that of the derivative of the current equilibrium aggregate consumption with respect to current endowment tax rates. Specifically, if the current equilibrium aggregate consumption is a normal good, then under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current endowment tax rate reduces current asset prices. Otherwise, if the current equilibrium aggregate consumption is an inferior good, then an increase in the current endowment tax rate boosts current asset prices. Since it is reasonable to assume that the current equilibrium aggregate consumption is a normal good, it is natural to conclude that under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices.

Theorem 4.3. analyzes comparative statics of current asset prices with respect to changes in future endowment taxes. It states that although the sign of a derivative of current equilibrium asset prices with respect to future endowment tax rates may be ambiguous, it is always the opposite to that of the derivative of the future equilibrium aggregate consumption with respect to future endowment tax rates. Specifically, if the future equilibrium aggregate consumption is a normal good, then under reasonable assumptions, without assuming identical agents, an increase in the future endowment tax rate boosts current asset prices. Otherwise, if the future equilibrium aggregate consumption is an inferior good, then an increase in the future endowment tax rate reduces current asset prices. Since it is reasonable to assume that the future equilibrium aggregate consumption is a normal good, it is natural to conclude that under reasonable assumptions, an increase in the future endowment tax rate boosts current asset prices.

The paper is organized as follows. Section 2 introduces necessary definitions to incorporate stochastic taxation imposed on agents' endowments and assets' dividends and used to finance public good into the CCAPM. Section 3 studies comparative statics of equilibria with respect to changes in stochastic taxation of dividends. Section 4 studies comparative statics of equilibria with respect to changes in stochastic taxation of endowments. Section 5 concludes.

2. Definitions

First, we need to introduce several definitions to incorporate stochastic taxation imposed on agents' endowments of a single consumption good and assets' dividends and used to finance public good G into the CCAPM.[3] Let ET be the event-tree, I be the set of finitely living investors-consumers, K be the set of assets traded on financial markets, such that

$$max\left[|ET|,\ |I|,\ |K|\right] < \infty.$$

Next, we consider the finite horizon CCAPM with stochastic taxation $\tau = (\tau_e,\ \tau_d) \in [0,\ 1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|}$ and with assets in strictly positive supply $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}^{|K|}_{++}$, where agents maximize their utility functions of consumption and a public good G

$$U_i(c_i,\, G) = \sum_{\xi\in ET} \Pr(\xi)\cdot b_i^{T(\xi)}\cdot [u_i(c_i(\xi)) + v_i(G(\xi))],$$

subject to their budget constraints

$$c_i - (1-\tau_{e_i})\cdot e_i(\tau_{e_i}) = W(q,\, (1-\tau_d)\cdot d(\tau_d))\cdot z_i\ \forall i\in I,$$

where the spending on the public good $G = \{G(\xi)\}_{\xi,\in ET} \in \mathbb{R}^{|ET|}_{+}$ is given by

$$G(\xi) = \sum_{i\in I}\tau_{e_i}(\xi)\cdot e_i(\xi,\, \tau_{e_i}) + \sum_{k\in K}\tau_d(\xi,\, k)\cdot d(\xi,\, k,\, \tau_d)\cdot\delta(k)\ \forall\xi\in ET,$$

$\tau_{e_i} = \{\tau_{e_i}(\xi)\}_{\xi\in ET} \in [0,\ 1]^{|ET|}$, s.t. $\tau_{e_i}(\xi) \in [0,\ 1]$ be the stochastic tax imposed on the individual endowment of agent $i \in I$ at node $\xi \in ET$,

$e_i(\tau_{e_i}) = \{e_i(\xi,\ \tau_{e_i})\}_{\xi\in ET} \in \mathbb{R}^{|ET|}_{+}$, s.t. $e_i(\xi,\ \tau_{e_i}) \in \mathbb{R}_+$ be the individual endowment of agent $i \in I$ at node $\xi \in ET$,[4]

$c_i = \{c_i(\xi)\}_{\xi\in ET} \in \mathbb{R}^{|ET|}_{+}$, s.t. $c_i(\xi) \in \mathbb{R}_+$ be the vector of consumption of agent $i \in I$ at node $\xi \in ET$,

$\tau_d = \{\tau_d(\xi,\ k)\}_{(\xi,\ k)\in ET\times K} \in [0,\ 1]^{|ET\times K|}$, s.t. $\tau_d(\xi,\ k)$ be the tax imposed on the dividend paid by asset $k \in K$ at node $\xi \in ET$,

$d(\tau_d) = \{d\,(\xi,\ k,\ \tau_d)\}_{(\xi,\ k)\in ET\times K} \in \mathbb{R}^{|ET\times K|}$, s.t. $d\,(\xi,\ k,\ \tau_d)$ be the dividend paid by asset $k \in K$ at node $\xi \in ET$,[4]

$z_i = \{z_i(\xi,\ k)\}_{(\xi,\ k)\in ET\times K} \in \mathbb{R}^{|ET\times K|}$, s.t. $z_i(\xi,\ k)$ be the number of shares of asset $k \in K$ held by agent $i \in I$ at node $\xi \in ET$,

$q = \{q(\xi,\ k)\}_{(\xi,\ k)\in ET\times K} \in \mathbb{R}^{|ET\times K|}$, s.t. $q(\xi,\ k)$ be the price of asset $k \in K$ at node $\xi \in ET$,

$W(q,\ (1-\tau_d)\cdot d(\tau_d))$ be the Payoff matrix with stochastic taxation.[5]

Next, we introduce the notion of a budget set.

DEFINITION: *Define the budget set of agent $i \in I$ as follows*

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[3] See Appendix A for more definitions related to the finite horizon CCAPM with stochastic taxation.

[4] Consistent with the Dividend Clientele Hypothesis (DCH), it is reasonable to assume that individual endowments e_i are decreasing functions $e_i(\tau_{e_i})$ of individual endowment tax rates τ_{e_i} and assets' dividends d are decreasing functions $d(\tau_d)$ of dividend tax rates τ_d. See Kawano (2013), for example, for a review of the DCH. She estimated that a one percentage point decrease in the dividend tax rate relative to the long-term capital gains tax rate leads to a 0.04 percentage point increase in dividend yields. Several papers, including Chetty and Saez (2005), Brown, Liang and Weisbenner (2007) have documented an increase in dividend payments in response to the 2003 tax changes.

[5] See Appendix A for the definition of the Payoff matrix $W(q,\ (1-\tau_d)\cdot d(\tau_d))$ with stochastic taxation.

$$B(q,\ (1-\tau_{e_i})\cdot e_i(\tau_{e_i}),\ (1-\tau_d)\cdot d(\tau_d)) =$$
$$\left\{c_i \in \mathbb{R}_+^{|ET\times I|}\, |\exists z_i \in \mathcal{Z}\ s.t.\ c_i - (1-\tau_{e_i})\cdot e_i(\tau_{e_i}) = W(q,\ \ (1-\tau_d)\cdot d(\tau_d))\cdot z_i\right\}.$$

Next, we introduce the notion of an equilibrium for the CCAPM with stochastic taxation and with assets in strictly positive supply.

DEFINITION: *An equilibrium for the CCAPM with stochastic taxation* $\tau = (\tau_e,\ \tau_d) \in [0,1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|}$ *and with assets in strictly positive supply* $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}_{++}^{|K|}$ *is a pair*

$$(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I}\,,\ \overline{q}(\tau)) \in \mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|} \times Q$$

such that all agents utilities are maximized subject to the budget constraints

$$(\overline{c}_i(\tau),\, \overline{z}_i(\tau)) \in$$
$$\arg\max\left\{U_i(c_i,\ G) \mid (c_i,\ z_i) \in B(\overline{q}(\tau),\ \ (1-\tau_{e_i})\cdot e_i(\tau_{e_i}),\ (1-\tau_d)\cdot d(\tau_d))\right\}$$

$\forall i \in I$,

the equilibrium in commodities markets is given by

$$\sum_{i\in I} \overline{c}_i(\tau) = \sum_{i\in I} (1-\tau_{e_i})\cdot e_i(\tau_{e_i}) + \sum_{k\in K} (1-\tau_d(k))\cdot d(k,\, \tau_d)\cdot \delta(k),$$

the equilibrium in assets markets is given by

$$\sum_{i\in I} \overline{z}_i(\tau) = \delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}_{++}^{|K|}$$

and the spending on the public good $G = \{G(\xi)\}_{\xi\in ET} \in \mathbb{R}_+^{|ET|}$ *is given by*

$$G(\xi) = \sum_{i\in I} \tau_{e_i}(\xi)\cdot e_i(\xi,\, \tau_{e_i}) + \sum_{k\in K} \tau_d(\xi,\, k)\cdot d(\xi,\, k,\, \tau_d)\cdot \delta(k)\ \forall \xi \in ET.$$

DEFINITION: *We define the set of no-arbitrage security prices as*

$$Q = \{q \in \mathbb{R}^{|ET\times K|} \mid \exists \pi \in \mathbb{R}_{++}^{|ET|},\, \pi\cdot W(q,\, (1-\tau_d)\cdot d(\tau_d)) = 0\}.$$

DEFINITION: *Given* $(q,\ (1-\tau_d)\cdot d(\tau_d)) \in Q\ \times \mathbb{R}^{|ET\times K|}$, *markets are complete if there exists a unique normalized price vector* $\overline{\pi} = \{\overline{\pi}(\xi')\}_{\xi'\in ET} \in \mathbb{R}_{++}^{|ET|}$ *with*

$$\overline{\pi}\cdot W(q,\, (1-\tau_d)\cdot d(\tau_d)) = 0.$$

3. Comparative Statics of Equilibria with Respect to the Dividend Tax τ_d

For the rest of this section we will assume

D1 (Assets): *Assets are in strictly positive supply, i.e.,*

$$\sum_{i\in I} z_i = \delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}_{++}^{|K|}.$$

D2 (Preferences): *Agents' preferences* $\succeq_i$ *on* $\mathbb{R}_+^{|ET|} \times \mathbb{R}_+^{|ET|}$ *are given by the utility function*

$$U_i(c_i,\, G) = \sum_{\xi \in ET} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi)) + v_i(G(\xi))],$$

where $u_i \in C^2$ *such that* $u_i'(\cdot) > 0$ *and* $u_i"(\cdot) < 0\ \forall i \in I$.

D3 (Consumption): $\forall\,(\xi,\ \xi') \in ET \times ET^+(\xi)\ \exists \bar{i} \in I$ *such that the equilibrium consumption* $\overline{c}_{\bar{i}}(\tau)$ *is such that*

$$\frac{u_{\bar{i}}'\big(\overline{c}_{\bar{i}}(\xi',\,\tau)\big)}{u_{\bar{i}}'\big(\overline{c}_{\bar{i}}(\xi,\,\tau)\big)} = \frac{u_{\bar{i}}'\left(\displaystyle\sum_{i \in I} \overline{c}_i(\xi',\,\tau)\right)}{u_{\bar{i}}'\left(\displaystyle\sum_{i \in I} \overline{c}_i(\xi,\,\tau)\right)}.$$

In case of the CRRA utility function, it means that agent's $\bar{i}$ consumption is growing at the same rate as the aggregate consumption, i.e.,

$$\frac{\overline{c}_{\bar{i}}(\xi',\,\tau)}{\overline{c}_{\bar{i}}(\xi,\,\tau)} = \frac{\displaystyle\sum_{i \in I} \overline{c}_i(\xi',\,\tau_{e_i})}{\displaystyle\sum_{i \in I} \overline{c}_i(\xi,\,\tau_{e_i})}.$$

This assumption makes sense. Indeed, there has been very little research done to date on the comparative statics in the context of the CCAPM. The research done so far, relies on the CCAPM with identical agents. The assumption D3 is significantly weaker than the assumption of identical agents. The assumption D3 merely states that there exists an "average" agent whose consumption is growing at the same rate as the aggregate consumption. The assumption D3 is motivated by the fact that most distributions in economics have a connected support with the exception of tails. If the there exists no "average" agent, then the distribution of agents does not have a connected support.[6]

D4 (Dividends): *Assets' dividends* d *paid in units of consumption are differentiable with respect to the dividend tax* τ_d, *i.e.,*

$$\exists \frac{\partial d}{\partial \tau_d} = \left\{ \left\{ \frac{\partial d(\xi',\, k_1,\, \tau_d)}{\partial \tau_d(\xi,\ k_2)} \right\}_{(k_1,\, k_2) \in K \times K} \right\}_{(\xi',\, \xi) \in ET \times ET} = \left\{ \frac{\partial d(\xi',\, \tau_d)}{\partial \tau_d(\xi)} \right\}_{(\xi',\, \xi) \in ET \times ET}.$$

Moreover, dividends $d(\xi',\ k,\ \tau_d)$ *are unaffected by any other tax rates, except* $\tau_d\,(\xi', k)\ \forall\,(\xi',\ k) \in ET \times K$, *i.e.,* $\frac{\partial d(\xi',\ k_1,\, \tau_d)}{\partial \tau_d(\xi,\ k_2)} = 0$ *for* $(\xi',\ k_1) \neq (\xi,\ k_2)$.

D5 (Dividend Tax): *Dividend tax rates* τ_d *are unaffected by each other, i.e.,* $\frac{\partial \tau_d(\xi',\, k_1)}{\partial \tau_d(\xi,\ k_2)} = 0$ *for* $(\xi',\ k_1) \neq (\xi,\ k_2)$ *and*

[6] Alternatively, we can assume that $\exists \bar{i} \in I$ s.t. $\frac{\partial \overline{c}_{\bar{i}}(\xi',\, \tau)}{\partial \tau_d(\bar{\xi})} = 0\ \forall \bar{\xi} \in \left[ET \setminus ET\left(\xi'\right)\right]$, where $ET(\xi') = \{\xi \in ET \mid \xi \geqslant \xi'\}$. This assumption also makes sense. This is a generalization of the traditional case of identical agents, where the equilibrium consumption of the representative agent at node $\xi' \in ET$

$$\overline{c}(\xi',\,\tau) = \big(1 - \tau_e(\xi')\big) \cdot e(\xi',\,\tau_e) + \sum_{k \in K} \big(1 - \tau_d(\xi',\ k)\big) \cdot d(\xi',\,k,\,\tau_d) \cdot \tfrac{\delta(k)}{|I|}$$

is unaffected by any dividend tax rate, except $\tau_d\left(\xi'\right)$. In contrast, in this paper we assume that $\overline{c}_{\bar{i}}(\xi',\,\tau)$ is unaffected by any tax rates, except $\tau_d\left(\bar{\xi}\right)\ \forall \bar{\xi} \in ET\left(\xi'\right)$.

$$\frac{\partial\tau_{e_i}}{\partial\tau_d} = \frac{\partial e_i}{\partial\tau_d} = 0\ \forall i \in I.$$

This assumption also makes sense. It means that endowment tax rates τ_{e_i} and endowments e_i are unaffected by dividend tax rates τ_d.

We start by showing that equilibrium comparative statics with respect to various stochastic tax rates is possible in an open neighborhood of every stochastic tax rate for which an equilibrium exists.

DEFINITION: *An equilibrium*

$$\left(\left\{(\overline{c}_i,\ \overline{z}_i)\right\}_{i\in I},\ \overline{q}\right) \in \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

for the CCAPM with stochastic taxation $\tau = (\tau_e,\ \tau_d)$ *is called regular if*

$$det\left[D_q ED(\overline{q},\ \ (1-\tau_e)\cdot e(\tau_e),\ \ (1-\tau_d)\cdot d(\tau_d))\right] \neq 0,$$

where $D_q ED$ *is the Jacobian of the excess demand function* ED.[7]

LEMMA 3.1: *Suppose Assumptions D1-D5 hold and agents' endowments* e_i *of consumption are differentiable with respect to the endowment tax* τ_{e_i}*, i.e.,* $e_i \in C^1\left([0,1]^{|ET|}\right)\ \forall i \in I$*. Let*

$$\left(\left\{(\overline{c}_i,\ \overline{z}_i)\right\}_{i\in I},\ \overline{q}\right) \in \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

be a regular equilibrium in which markets are complete for the CCAPM with stochastic taxation $\overline{\tau} = (\overline{\tau}_e,\ \overline{\tau}_d)$*. Then* $\exists$ *an open neighborhood* $O_{\overline{\tau}} \subset [0,\ 1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|}$ *of* $\overline{\tau}$ *and a function*

$$h : O_{\overline{\tau}} \longrightarrow \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q \times \mathbb{R}_{++}^{|ET|}$$

defined as

$$h(\tau) = \left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ \overline{q}(\tau), \overline{\pi}(\tau)\right),$$

where

$$\left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ \overline{q}(\tau)\right) \in \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

be an equilibrium in which markets are complete and

$$\overline{\pi}(\tau) = \left\{\overline{\pi}(\xi',\ \tau)\right\}_{\xi'\in ET} \in \mathbb{R}_{++}^{|ET|}$$

be a unique normalized price vector with

$$\overline{\pi}(\tau)\cdot W(\overline{q}(\tau), (1-\tau_d)\cdot d(\tau_d)) = 0\ \forall \tau \in O_{\overline{\tau}}$$

and

$$\overline{c}_i, \overline{z}_i \in C^1\left(O_{\overline{\tau}}\right)\ \forall i \in I.$$

[7] The differentiability of the excess demand function for goods and assets with respect to prices is established on p. 24 of the paper.

PROOF: See Appendix B.

Let us first analyze how a change in the current dividend tax rate $\tau_d(\xi) \in [0,1]^{|K|}$ will affect current equilibrium asset prices $\overline{q}(\xi,\ \tau) \in \mathbb{R}^{|K|}$. Since a change in $\tau_d(\xi)$ might affect various node prices $\overline{\pi}(\xi',\ \tau)$ and after-tax dividends $(1-\tau_d(\xi')) \cdot d(\xi',\ \tau_d)$, $\xi' \in ET^+(\xi)$ differently, the net effect of $\tau_d(\xi)$ on $\overline{q}(\xi,\ \tau)$ is ambiguous. We need to impose Assumptions D1-D5 from above to remove this ambiguity.

We will be able to derive economically intuitive comparative statics of $\overline{q}(\xi,\ \tau)$ with respect to $\tau_d(\xi)$ results without assuming either CRRA utility functions or identical agents. We will start our analysis with the following theorem:

THEOREM 3.2: *Suppose assumptions D1-D5 hold. Let*

$$(\{(\overline{c}_i,\ \overline{z}_i)\}_{i\in I}, \overline{q}) \in \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

be a regular equilibrium in which markets are complete for the CCAPM with stochastic taxation $\overline{\tau} = (\overline{\tau}_e,\ \overline{\tau}_d)$. Let ξ be the initial node of the event tree ET. Then an open neighborhood $O_{\overline{\tau}}$ of $\overline{\tau}$ and a function

$$h(\tau) = (\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I}, \overline{q}(\tau), \overline{\pi}(\tau))\ \forall\tau \in O_{\overline{\tau}},$$

obtained in the previous Lemma 3.1. are s.t.

$$\boxed{\overline{\pi}(\xi',\ \tau) = b_i^{T(\xi')} \cdot \frac{u_i'(\overline{c}_i(\xi',\ \tau))}{u_i'(\overline{c}_i(\xi,\ \tau))} \cdot \Pr(\xi'),} \tag{1}$$

$$\boxed{\overline{q}(\xi,\ \tau) = \sum_{\xi'\in ET^+(\xi)} b_i^{T(\xi')} \cdot \frac{u_i'(\overline{c}_i(\xi',\ \tau))}{u_i'(\overline{c}_i(\xi,\ \tau))} \cdot \Pr(\xi') \cdot (1-\tau_d(\xi')) \cdot d(\xi',\ \tau_d),} \tag{2}$$

$\forall(\xi',\ i) \in ET^+(\xi) \times I$ and $\exists\overline{i} \in I$ such that

$$\boxed{\frac{\partial\overline{\pi}(\xi',\ \tau)}{\partial\tau_d(\xi)} = \overline{\pi}(\xi',\tau) \cdot \left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))\right] \cdot g_{\sum_{i\in I}\overline{c}_i(\xi,\tau)}} \tag{3}$$

$$\boxed{\frac{\partial\overline{q}(\xi,\ \tau)}{\partial\tau_d(\xi)} = \overline{q}(\xi,\ \tau) \cdot \left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))\right] \cdot g_{\sum_{i\in I}\overline{c}_i(\xi,\tau)}} \tag{4}$$

and

$$\boxed{sign\left[\frac{\partial\overline{q}(\xi,\ \tau)}{\partial\tau_d(\xi)}\right] = sign\left[\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\xi,\tau)\right]}{\partial\tau_d(\xi)}\right],} \tag{5}$$

where

$$rr_i(c) = -\left[\frac{u_i''(c)\cdot c}{u_i'(c)}\right]$$

is the coefficient of relative risk aversion of an agent $i \in I$ and

$$g_{\sum_{i \in I} \overline{c}_i(\xi,\, \tau)} = \frac{1}{\left[\sum_{i \in I} \overline{c}_i(\xi,\, \tau)\right]} \cdot \frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\, \tau)\right]}{\partial \tau_d(\xi)}.$$

Moreover, if the Dividend Clientele Hypothesis (DCH) holds, i.e.,

$$\frac{\partial d(\xi,\, \tau_d)}{\partial \tau_d(\xi)} < 0,$$

then

$$\frac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_d(\xi)} < 0.$$

PROOF: See Appendix B.

The economic interpretation of the above result is as follows. Note first that an increase in $\tau_d(\xi)$ affects $\overline{q}(\xi,\, \tau)$ only through stochastic discount factors $\overline{\pi}(\xi',\, \tau)$, $\xi' \in ET^+(\xi)$. Although the sign of a derivative of current equilibrium asset prices $\overline{q}(\xi,\, \tau)$ with respect to current dividend tax rates $\tau_d(\xi)$ may be ambiguous, it is always the same as that of the derivative of the current equilibrium aggregate consumption $\sum_{i \in I} \overline{c}_i(\xi, \tau)$ with respect to current dividend tax rates $\tau_d(\xi)$. Depending on the sign of $\frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\tau)\right]}{\partial \tau_d(\xi)}$, an increase in $\tau_d(\xi)$ might be reducing or boosting $\overline{q}(\xi,\, \tau)$. Therefore, we have two cases to consider here:

Suppose first $\frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\tau)\right]}{\partial \tau_d(\xi)} \leqslant 0$, i.e., the aggregate consumption is a normal good. Then an increase in $\tau_d(\xi)$ reduces $\overline{\pi}(\xi',\, \tau)$, thus decreasing today's price of the future aggregate consumption $\sum_{i \in I} \overline{c}_i(\xi', \tau)$, $\xi' \in ET^+(\xi)$. Since financial assets represent claims on future consumption, the increase in $\tau_d(\xi)$ reduces today's asset prices $\overline{q}(\xi,\, \tau)$.

Suppose now $\frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\tau)\right]}{\partial \tau_d(\xi)} > 0$, i.e., the aggregate consumption is an inferior good. Then an increase in $\tau_d(\xi)$ boosts $\overline{\pi}(\xi',\, \tau)$, thus increasing today's price of the future aggregate consumption $\sum_{i \in I} \overline{c}_i(\xi', \tau)$, $\xi' \in ET^+(\xi)$. Since financial assets represent claims on future consumption, the increase in $\tau_d(\xi)$ boosts today's asset prices $\overline{q}(\xi,\, \tau)$.

While inferior goods have been demonstrated to exist, it is generally believed that they are rare: at any given time and price level, the demand for the vast majority of goods moves in the intuitive direction with respect to changes in after-tax income. So it is reasonable to assume that the aggregate consumption is a normal good. Therefore, it is natural to conclude that under reasonable assumptions, an increase in the current dividend tax rate reduces current asset prices.

COROLLARY 3.3: *Suppose assumptions of the above Theorem 3.2. hold. Assume further that all agents are identical and exhibit CRRA, i.e.,*

$$rr_i(c) = -\left[\frac{u''(c) \cdot c}{u'(c)}\right] = a \; \forall i \in I.$$

Assets' dividends are taxed identically, i.e.,

$$\tau_d(\xi,\, k) = \tau_d(\xi,\, \overline{k})\ \forall\, (\xi,\ k) \in ET \times K.$$

In addition, agents have zero initial endowments, i.e.,

$$e_i(\xi,\, \tau) = 0\ \forall\, (i,\ \xi) \in I \times ET^{8}$$

and

$$\frac{\partial d}{\partial \tau_d} = 0.$$

Then

$$\boxed{E_{q(\xi,\, \tau),\, 1-\tau_d(\xi,\, \overline{k})} = a,}$$

where

$$E_{q(\xi,\, \tau),\, 1-\tau_d(\xi,\, \overline{k})} = \frac{\frac{1}{\overline{q}(\xi,\, \tau)} \frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_d(\xi,\, \overline{k})}}{\frac{1}{(1-\tau_d(\xi,\, \overline{k}))} \frac{\partial(1-\tau_d(\xi,\, \overline{k}))}{\partial \tau_d(\xi,\, \overline{k})}}\ \forall \xi \in ET$$

is the elasticity of asset prices $q(\xi,\, \tau)$ with respect to the economic freedom $1 - \tau_d\left(\xi,\, \overline{k}\right)$ at a node $\xi \in ET$.

PROOF: See Appendix B.

The expression

$$\sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \delta(k)$$

can be interpreted as the country's total GDP at node $\xi \in ET$ and

$$(1 - \tau_d(\xi,\, \overline{k})) = \frac{\sum_{k \in K} (1-\tau_d(\xi,\, \overline{k})) \cdot d(\xi,\, k,\, \tau_d) \cdot \delta(k)}{\sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \delta(k)}$$

can be interpreted as a percentage of the economy's total GDP consumed by the private sector at a node $\xi \in ET$. Hence, $(1 - \tau_d(\xi,\, \overline{k}))$ can be interpreted as the economy's level of economic freedom at a node $\xi \in ET$. Numerically, Magin (2015a) estimated the coefficient of agents' relative risk aversion, $a = 3.76$. So on average, for S&P 500 stocks, a 1% increase of the economy's economic freedom generates a 3.76% increase in share prices.

COROLLARY 3.4: *Suppose assumptions of Theorem 3.2. hold. Then*

$$\boxed{sign \left[\frac{\partial R(\xi',\, \tau)}{\partial \tau_d(\xi)}\right] = sign \left[\frac{\partial ATR(\xi',\, \tau)}{\partial \tau_d(\xi)}\right] = -sign \left[\frac{\partial q(\xi,\, \tau)}{\partial \tau_d(\xi)}\right] \forall\, (\xi,\ \xi') \in ET \times \xi^{+},}$$

[8] We can obtain a similar result without assuming zero initial endowments. It is sufficient to assume $\tau_d(\xi) = \tau_e(\xi)\ \forall \xi \in ET$ instead.

where

$$R(\xi',\ \tau) = \frac{q(\xi',\ \tau)+d(\xi',\ \tau_d)}{q(\xi,\ \tau)}$$

is the total before-tax rate of return of an asset at a node $\xi' \in \xi^+$ *and*

$$ATR(\xi',\ \tau) = \frac{q(\xi',\ \tau)+(1-\tau_d(\xi'))\cdot d(\xi',\ \tau_d)}{q(\xi,\ \tau)}$$

is the total after-tax rate of return of an asset at a node $\xi' \in \xi^+$.

PROOF: See Appendix B.

Empirical findings of Sialm (2006) and (2009) imply that stocks with heavier tax burdens tend to compensate taxable investors by offering higher before-tax returns and equity premia. Corollary 3.4. demonstrates that this is not necessarily the case.

Let us now analyze how a change in a future $\tau_d\left(\overline{\xi}\right)$, $\overline{\xi} \in ET^+(\xi)$ stochastic dividend tax rate will affect current equilibrium asset prices $\overline{q}\,(\xi,\ \tau)$. Again, since a change in $\tau_d\left(\overline{\xi}\right)$ might affect various node prices $\overline{\pi}(\xi',\ \tau)$, $\xi' \in ET^+(\xi)$ and after-tax dividends $(1-\tau_d\,(\xi'))\cdot d\,(\xi',\ \tau_d)$, $\xi' \in ET^+(\xi)$ differently, the net effect of $\tau_d\left(\overline{\xi}\right)$ on $\overline{q}\,(\xi,\ \tau)$ is ambiguous. We need to impose Assumptions D1-D5 from above to remove this ambiguity. We will be able to derive economically intuitive comparative statics of $\overline{q}\,(\xi,\ \tau)$ with respect to $\tau_d\,(\xi')$ results without assuming either CRRA utility functions or identical agents.

THEOREM 3.5: *Suppose assumptions D1-D5 hold and*

$$\frac{\partial d(\xi,\ \ k,\ \tau_d)}{\partial \tau_d(\xi,\ \ k)} < 0\ \forall\,(\xi,\ k) \in ET \times K.$$

Let

$$\left(\{(\overline{c}_i,\ \overline{z}_i)\}_{i\in I}\,,\ \overline{q}\right) \in \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

be a regular equilibrium in which markets are complete for the CCAPM with stochastic taxation $\overline{\tau} = (\overline{\tau}_e,\ \overline{\tau}_d)$. *Let* ξ *be the initial node of the event tree* ET. *Then an open neighborhood* $O_{\overline{\tau}}$ *of* $\overline{\tau}$ *and a function*

$$h(\tau) = \left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I}\,,\ \overline{q}(\tau),\ \overline{\pi}(\tau)\right)\ \forall \tau \in O_{\overline{\tau}},$$

obtained in the previous Lemma 3.1. are s.t. $\exists \overline{i} \in I$ *with*

$$\boxed{sign\left[\frac{\partial \overline{q}(\xi,\ k,\ \tau)}{\partial \tau_d\left(\overline{\xi},\ k\right)}\right] = sign\left[[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \ \tau))] - B(\overline{\xi},\ \ k,\ \ \tau)\right]},$$

where

$$B(\overline{\xi},\ k,\ \tau) = \frac{1}{\frac{(1-\tau_d(\overline{\xi},\ k))\cdot d(\overline{\xi},\ k,\ \tau_d)\cdot \delta(k)}{\sum\limits_{i\in I}(1-\tau_{e_i}(\overline{\xi}))\cdot e_i(\overline{\xi},\ \tau_{e_i})+\sum\limits_{k\in K}(1-\tau_d(\overline{\xi},\ k))\cdot d(\overline{\xi},\ k,\ \tau_d)\cdot \delta(k)}} \geqslant 1$$

$\forall \left(\overline{\xi},\ k\right) \in ET^{+}(\xi) \times K$. *Moreover, if* $|K| = 1$ *and* $e_i(\overline{\xi},\ \tau_{e_i}) = 0\ \forall i \in I$, *then* $B(\overline{\xi},\ k,\ \tau) = 1$.

PROOF: See Appendix B.

The economic interpretation of the above result is as follows. Fix $\overline{\xi} \in ET^{+}(\xi)$. Note first that an increase in $\tau_d\left(\overline{\xi},\ k\right)$ affects $\overline{q}\,(\xi,\ k,\ \tau)$ through both stochastic discount factors $\overline{\pi}(\xi',\ \tau)$ and after-tax dividends $(1 - \tau_d\,(\xi',\ k)) \cdot d\,(\xi',\ k\ ,\tau_d)$, $\xi' \in ET^{+}(\xi)$. Under the assumptions of the theorem, an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates two effects which are working in opposite directions. On the one hand, an increase in $\tau_d\left(\overline{\xi},\ k\right)$ boosts the stochastic discount factor $\overline{\pi}(\overline{\xi},\ \tau)$, thus boosting asset prices $\overline{q}\,(\xi,\ k,\ \tau)$. On the other hand, an increase in $\tau_d\left(\overline{\xi},\ k\right)$ reduces after-tax dividends $\left(1 - \tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus reducing asset prices $\overline{q}\,(\xi,\ k,\ \tau)$. The net effect of the increase in $\tau_d\left(\overline{\xi},\ k\right)$ on $\overline{q}\,(\xi,\ k,\ \tau)$ is ambiguous and is determined by the value of the coefficient of relative risk aversion $rr_{\overline{i}}(\sum_{i \in I}\overline{c}_i(\overline{\xi},\ \tau))]$ at the node $\overline{\xi}$. We have three cases to consider here:

If $rr_{\overline{i}}(\sum_{i \in I}\overline{c}_i(\overline{\xi},\ \tau))] > B(\overline{\xi},\ k,\ \tau)$, i.e., the coefficient of relative risk aversion $rr_{\overline{i}}(\sum_{i \in I}\overline{c}_i(\overline{\xi},\ \tau))]$ is high, then an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates such a strong boosting effect on the stochastic discount factor $\overline{\pi}(\overline{\xi},\ \tau)$ that it dominates the reducing effect it has on after-tax dividends $\left(1 - \tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus boosting asset prices $\overline{q}\,(\xi,\ k,\ \tau)$.

If $rr_{\overline{i}}(\sum_{i \in I}\overline{c}_i(\overline{\xi},\ \tau))] < B(\overline{\xi},\ k,\ \tau)$, i.e., the coefficient of relative risk aversion a is low, then an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates such a weak boosting effect on the stochastic discount factor $\overline{\pi}(\overline{\xi},\ \tau)$ that it is dominated by the reducing effect it has on after-tax dividends $\left(1 - \tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus reducing asset prices $\overline{q}\,(\xi,\ k,\ \tau)$.

Finally, if $rr_{\overline{i}}(\sum_{i \in I}\overline{c}_i(\overline{\xi},\ \tau))] = B(\overline{\xi},\ k,\ \tau)$, then an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates such a boosting effect on the stochastic discount factor $\overline{\pi}(\overline{\xi},\ \tau)$ that it is completely canceled out by the reducing effect it has on after-tax dividends $\left(1 - \tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus leaving asset prices $\overline{q}\,(\xi,\ k,\ \tau)$ unchanged.

4. Comparative Statics of Equilibria with Respect to the Endowment Tax τ_{e_i}

For the rest of this section we will assume

E1 (Assets): *Assets are in positive supply, i.e.,*

$$\sum_{i \in I} z_i = \delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}^{|K|}_{++}.$$

E2 (Preferences): *Agents' preferences* $\succeq_i$ *on* $\mathbb{R}^{|ET|}_{+} \times \mathbb{R}^{|ET|}_{+}$ *are given by the utility function*

$$U_i(c_i,\ G) = \sum_{\xi \in ET} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi)) + v_i(G(\xi))],$$

where $u_i \in C^2$ *such that* $u_i'(\cdot) > 0$ *and* $u_i''(\cdot) < 0\ \forall i \in I$.

E3 (Consumption): $\forall\,(\xi,\ \xi') \in ET \times ET^{+}(\xi)\ \exists \overline{i} \in I$ *such that the equilibrium consumption* $\overline{c}_{\overline{i}}(\tau)$ *is such that*

$$\frac{u_{\overline{i}}'\left(\overline{c}_{\overline{i}}(\xi',\ \tau)\right)}{u_{\overline{i}}'\left(\overline{c}_{\overline{i}}(\xi,\ \tau)\right)} = \frac{u_{\overline{i}}'\left(\sum_{i \in I}\overline{c}_i(\xi',\ \tau)\right)}{u_{\overline{i}}'\left(\sum_{i \in I}\overline{c}_i(\xi,\ \tau)\right)}.$$

In case of the CRRA utility function, it means that agent's $\bar{i}$ consumption is growing at the same rate as the aggregate consumption, i.e.,

$$\frac{\overline{c}_{\bar{i}}(\xi',\,\tau)}{\overline{c}_{\bar{i}}(\xi,\,\tau)}=\frac{\displaystyle\sum_{i\in I}\overline{c}_i(\xi',\,\tau_{e_i})}{\displaystyle\sum_{i\in I}\overline{c}_i(\xi,\,\tau_{e_i})}.$$

This assumption makes sense. This is exactly assumption D3. See explanation on p. 8.

E4 (Endowments): *Agents' endowments e_i of consumption are differentiable with respect to the endowment tax τ_{e_i}, i.e.,*

$$\exists\frac{\partial e_i}{\partial\tau_{e_i}}=\left\{\frac{\partial e_i\left(\xi',\,\tau_{e_i}\right)}{\partial\tau_{e_i}(\xi)}\right\}_{(\xi',\,\xi)\in ET\times ET}\quad\forall i\in I.$$

Moreover, endowments $e_i\left(\xi',\ \tau_{e_i}\right)$ are unaffected by any other tax rates, except $\tau_{e_i}\left(\xi'\right)\ \forall\left(\xi',\ i\right)\in ET\times I$, i.e.,

$$\frac{\partial e_i\left(\xi',\,\tau_{e_i}\right)}{\partial\tau_{e_i}(\xi)}=0\text{ for }\left(\xi',\ i\right)\neq\left(\xi,\ j\right).$$

E5 (Endowment Tax): *Endowment tax rates τ_{e_i} are unaffected by each other*

$$\frac{\partial\tau_{e_i}(\xi')}{\partial\tau_{e_j}(\xi)}=0\text{ for }\left(\xi',\ i\right)\neq\left(\xi,\ j\right)$$

and

$$\frac{\partial\tau_d}{\partial\tau_{e_i}}=\frac{\partial d}{\partial\tau_{e_i}}=0\ \forall i\in I.$$

This assumption also makes sense. It means that dividend tax rates τ_d and dividends d are unaffected by endowment tax rates τ_{e_i}.

The analysis of how a change in the stochastic endowment tax τ_{e_i} of an agent $i\in I$ will affect equilibrium asset prices $\overline{q}\left(\tau\right)$ is much easier than the effects of τ_d on $\overline{q}\left(\tau\right)$, since, under reasonable assumptions, increases in τ_{e_i} affect $\overline{q}\left(\tau\right)$ only through stochastic discount factors $\overline{\pi}(\tau)\in\mathbb{R}_{++}^{|ET|}$.

Let us first analyze how a change in the current $\tau_{e_i}\left(\xi\right)$ endowment tax rate of an agent $i\in I$ will affect current equilibrium asset prices $\overline{q}\left(\xi,\ \tau\right)$. We will start with the following theorem:

THEOREM 4.1: *Suppose assumptions E1-E5 hold. Let*

$$\left(\left\{\left(\overline{c}_i,\ \overline{z}_i\right)\right\}_{i\in I},\overline{q}\right)\in\left(\mathbb{R}_+^{|ET\times I|}\times\mathcal{Z}^{|I|}\right)\times Q$$

be a regular equilibrium in which markets are complete for the CCAPM with stochastic taxation $\overline{\tau}=(\overline{\tau}_e,\ \overline{\tau}_d)$. Let ξ be the initial node of the event tree ET. Then an open neighborhood $O_{\overline{\tau}}$ of $\overline{\tau}$ and a function

$$h(\tau)=\left(\left\{\left(\overline{c}_i(\tau),\ \overline{z}_i(\tau)\right)\right\}_{i\in I},\overline{q}(\tau),\overline{\pi}(\tau)\right)\ \forall\tau\in O_{\overline{\tau}},$$

obtained in the previous Lemma 3.1. are s.t. $\exists\bar{i}\in I$ with

$$\boxed{\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_{e_{\overline{i}}}(\xi)} = \overline{\pi}(\xi',\tau)\cdot\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))\right]\cdot g_{\sum\limits_{i\in I}\overline{c}_i(\xi,\tau)}} \tag{6}$$

$\forall\xi' \in ET^{+}(\xi)$,

$$\boxed{\frac{\partial\overline{q}(\xi,\,\tau)}{\partial\tau_{e_{\overline{i}}}(\xi)} = \overline{q}\,(\xi,\ \tau)\cdot\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))\right]\cdot g_{\sum\limits_{i\in I}\overline{c}_i(\xi,\tau)}} \tag{7}$$

and

$$\boxed{sign\left[\frac{\partial\overline{q}(\xi,\,\tau)}{\partial\tau_{e_{\overline{i}}}(\xi)}\right] = sign\left[\frac{\partial\left[\sum\limits_{i\in I}\overline{c}_i(\xi,\,\tau)\right]}{\partial\tau_{e_{\overline{i}}}(\xi)}\right],} \tag{8}$$

where

$$rr_i(c) = -\left[\frac{u_i"(c)\cdot c}{u_i'(c)}\right]$$

is the coefficient of relative risk aversion of an agent $i \in I$ and

$$g_{\sum\limits_{i\in I}\overline{c}_i(\xi,\,\tau)} = \frac{1}{\left[\sum\limits_{i\in I}\overline{c}_i(\xi,\,\tau)\right]}\cdot\frac{\partial\left[\sum\limits_{i\in I}\overline{c}_i(\xi,\,\tau)\right]}{\partial\tau_{e_{\overline{i}}}(\xi)}.$$

PROOF: It is similar to the proof of Theorem 3.2.

The economic interpretation of the above result is the same as for Theorem 3.2.

COROLLARY 4.2: *Suppose assumptions of the above Theorem 4.1. hold. Assume further that all agents are identical and exhibit CRRA, i.e.,*

$$rr_i(c) = -\left[\frac{u"(c)\cdot c}{u'(c)}\right] = a\ \forall i \in I.$$

Agents' endowments are taxed identically, i.e.,

$$\tau_{e_i}(\xi) = \tau_{e_{\overline{i}}}(\xi)\ \forall\,(\xi,\ i) \in ET \times I.$$

In addition, assets pay zero dividends, i.e.,

$$d(\xi,\,k,\,\tau_d) = 0\ \forall\,(\xi,\ k) \in ET \times K^{9}$$

[9] We can obtain a similar result without assuming that assets pay zero dividends. It is sufficient to assume $\tau_d(\xi) = \tau_e(\xi)$ $\forall\xi \in ET$ instead.

and

$$\frac{\partial e}{\partial \tau_e} = 0.$$

Then

$$\boxed{E_{q(\xi,\,\tau),\,1-\tau_e(\xi)} = a,}$$

where

$$E_{q(\xi,\,\tau),\,1-\tau_{e_{\overline{i}}}(\xi)} = \frac{\frac{1}{\overline{q}(\xi,\,\tau)}\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_{\overline{i}}}(\xi)}}{\frac{1}{(1-\tau_{e_{\overline{i}}}(\xi))}\frac{\partial(1-\tau_{e_{\overline{i}}}(\xi))}{\partial \tau_{e_{\overline{i}}}(\xi)}}\ \forall \xi \in ET$$

is the elasticity of asset prices $q(\xi,\,\tau)$ with respect to the economic freedom $1-\tau_{e_{\overline{i}}}(\xi)$ at a node $\xi \in ET$.
PROOF: It is similar to the proof of Corollary 3.3.

Let us now analyze how a change in a future $\tau_{e_i}\left(\overline{\xi}\right)$, $\overline{\xi} \in ET^+(\xi)$ stochastic endowment tax rate of an agent $i \in I$ will affect current equilibrium asset prices $\overline{q}\,(\xi,\ \tau)$. Since a change in $\tau_{e_i}\left(\overline{\xi}\right)$ might affect various node prices $\overline{\pi}(\xi',\,\tau)$, $\xi' \in ET^+(\xi)$ differently, the net effect of $\tau_{e_i}\left(\overline{\xi}\right)$ on $\overline{q}\,(\xi,\ \tau)$ is ambiguous. Unlike the comparative statics of $\overline{q}\,(\xi,\ \tau)$ with respect to $\tau_{e_i}\,(\xi)$, however, it does not appear to be possible to derive economically intuitive comparative statics of $\overline{q}\,(\xi,\ \tau)$ with respect to $\tau_{e_i}\left(\overline{\xi}\right)$ results without either assuming CRRA utility functions or identical agents.

Suppose now that agents exhibit CRRA but are not necessarily identical. Our goal here is to determine the sign of

$$\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_i}\left(\overline{\xi}\right)} = \left\{\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_{e_i}\left(\overline{\xi}\right)}\right\}_{k\in K} \in \mathbb{R}^{|K|}\ \forall \overline{\xi} \in ET^+(\xi).$$

THEOREM 4.3: *Suppose assumptions E1-E5 hold. Let*

$$\left(\left\{(\overline{c}_i,\ \overline{z}_i)\right\}_{i\in I},\,\overline{q}\right) \in \left(\mathbb{R}_+^{|ET\times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

be a regular equilibrium in which markets are complete for the CCAPM with stochastic taxation $\overline{\tau} = (\overline{\tau}_e,$ $\overline{\tau}_d)$. Let ξ be the initial node of the event tree ET. Then an open neighborhood $O_{\overline{\tau}}$ of $\overline{\tau}$ and a function

$$h(\tau) = \left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\,\overline{q}(\tau),\,\overline{\pi}(\tau)\right)\ \forall \tau \in O_{\overline{\tau}},$$

obtained in the previous Lemma 3.1. are s.t. $\exists \overline{i} \in I$ with

$$\boxed{\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_{\overline{i}}}\left(\overline{\xi}\right)} = -\left[rr_{\overline{i}}\big(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau)\big)\right]\cdot g_{\sum\limits_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}\cdot\overline{\pi}(\overline{\xi},\,\tau)\cdot\left(1-\tau_d\left(\overline{\xi}\right)\right)\cdot d\left(\overline{\xi},\ \tau_d\right),}$$

where

$$g_{\sum\limits_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)} = \frac{-e_{\overline{i}}(\overline{\xi},\,\tau_{e_{\overline{i}}})+\frac{\partial e_{\overline{i}}(\overline{\xi},\,\tau_{e_{\overline{i}}})}{\partial \tau_{e_{\overline{i}}}(\overline{\xi})}\cdot\left(1-\tau_{e_i}(\overline{\xi})\right)}{\sum\limits_{i\in I}\left(1-\tau_{e_i}(\overline{\xi})\right)\cdot e_i(\overline{\xi},\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)}$$

and

$$\boxed{sign\left[\frac{\partial\overline{q}(\xi,\,\tau)}{\partial\tau_{e_{\overline{i}}}(\overline{\xi})}\right]=-sign\left[\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)\right]}{\partial\tau_{e_{\overline{i}}}(\overline{\xi})}\right]}$$

$\forall\overline{\xi}\in ET^+(\xi)$.

PROOF: See Appendix B.

The economic interpretation of the above result is as follows. Fix $\overline{\xi}\in ET^+(\xi)$. An increase in $\tau_{e_i}\left(\overline{\xi}\right)$ boosts stochastic discount factors $\overline{\pi}(\overline{\xi},\,\tau)$, $\xi'\in ET^+(\xi)$ without affecting after-tax dividends $\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus boosting asset prices $\overline{q}\left(\xi,\ k,\ \tau\right)$.

5. Conclusion

This paper studies comparative statics of equilibria in the finite horizon CCAPM with respect to changes in stochastic tax rates imposed on agents' endowments and dividends. We show that under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate unambiguously reduces current asset prices. The paper also finds that there exists a bound $\overline{B}$ such that for a coefficient of relative risk aversion less than $\overline{B}$, an increase in a future dividend tax rate reduces current price of tradable assets. At the same time, for a coefficient of relative risk aversion greater than $\overline{B}$, an increase in a future dividend tax rate boosts current price of tradable assets. Finally, for a coefficient of relative risk aversion equal to $\overline{B}$, an increase in a future dividend tax rate leaves current price of tradable assets unchanged. As a special case, under additional assumptions, $\overline{B}$ is equal to 1. Also, under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices, while an increase in a future endowment tax rate boosts current asset prices.

Appendix A: Additional Definitions

We need to introduce several additional notions to define the finite horizon CCAPM with stochastic taxation $\tau=(\tau_e,\ \tau_d)$ imposed on agents' endowments and assets' dividends and used to finance spending on the public good G.

Suppose there is an event tree ET and a set I of finitely living investors-consumers who trade 1 commodity on a spot market and a set K of assets on financial markets, such that

$$max\left[|ET|\,,\ \ |I|\,,\ \ |K|\right]<\infty.$$

First, we discuss the asset structure of our model.

DEFINITION: *Let $\xi(k)\in ET$ be the node of issue for an asset $k\in K$. Define the set ζ of all nodes of issue of existing financial contracts as*

$$\zeta=\{\xi(k)\mid k\in K\}.$$

DEFINITION: *Let*

$$d\left(\xi,\ k,\ \tau_d\right)\in\mathbb{R}$$

be the real before-tax dividend of asset $k\in K$ paid in units of the consumption good at node $\xi\in ET$, where

$$d(\xi,\,k,\,\tau_d)=0\;\;\forall\xi\in ET\setminus ET^{+}(\xi(k))\;\forall k\in K,$$

i.e., an asset $k\in K$ *issued at node* $\xi(k)\in ET$ *pays no dividends prior to or at node* $\xi(k)\in ET$*,*

$$d(\tau_d)=\{d\,(\xi,\;\tau_d)\}_{\xi\in ET}\in\mathbb{R}^{|ET\times K|}$$

be the matrix of real before-tax dividends of the $|K|$ *assets paid in units of the consumption good.*

Next, $\forall(q,\,(1-\tau_d)\cdot d(\tau_d))\in\mathbb{R}^{|ET\times K|}\times\mathbb{R}^{|ET\times K|}$ we will define the Payoff matrix $W(q,\,(1-\tau_d)\cdot d(\tau_d))$, which will significantly simplify writing of agents' budget constraints.[10]

DEFINITION: $\forall(q,\,(1-\tau_d)\cdot d(\tau_d))\in\mathbb{R}^{|ET\times K|}\times\mathbb{R}^{|ET\times K|}$ *define the* $|ET|\times[\,|ET^{-}|\cdot|K|\,]$ *Payoff matrix* $W(q,\,(1-\tau_d)\cdot d(\tau_d))$ *as*

$$\begin{gathered}W_{\xi,\,\xi^{+}}(q,\,(1-\tau_d)\cdot d(\tau_d))=q(\xi^{+})+\left(1-\tau_d(\xi^{+})\right)\cdot d(\xi^{+},\,\tau_d)),\\ W_{\xi,\,\xi}(q,\,(1-\tau_d)\cdot d(\tau_d))=-q(\xi),\\ W_{\xi,\,\xi'}(q,\,(1-\tau_d)\cdot d(\tau_d))=0\;\forall\xi'\notin\xi^{+},\,\xi'\neq\xi.\end{gathered}$$

MATRIX $\mathbf{W}(\mathbf{q},(\mathbf{1}-\boldsymbol{\tau}_{\mathbf{d}})\cdot\mathbf{d}(\boldsymbol{\tau}_{\mathbf{d}}))$

$\lvert K\rvert$ Columns for ξ_0		$\lvert K\rvert$ Columns for ξ^{-}	$\lvert K\rvert$ Columns for ξ		
$-q(\xi_0)$	0	0	0	0	ξ_0
$q(\xi_0^{+})+$ $+\left(1-\tau_d(\xi_0^{+})\right)\cdot$ $\cdot d(\xi_0^{+},\,\tau_d)$	…	…	0	0	ξ_0^{+}
0	…	…	0	0	
0	0	$q(\xi)+$ $+(1-\tau_d(\xi))\cdot$ $\cdot d(\xi,\,\tau_d)$	$-q(\xi)$	0	ξ
0	…	…	0	…	
0	0	0	$q(\xi^{+})+$ $+(1-\tau_d(\xi^{+}))\cdot$ $\cdot d(\xi^{+},\,\tau_d)$	…	ξ^{+}
0	0	0	0	…	

[10] See Magill and Quinzii (1996) for the original definition of the Payoff matrix $W(q,\,d)$ without stochastic taxation.

Appendix B: Proofs
PROOF OF LEMMA 3.1: We know that the total excess demand function

$$ED: \qquad \underbrace{Q \times \left[\mathbb{R}_{++}^{|ET \times I|} \times \mathbb{R}^{|ET \times K|}\right]}_{Open\ Subset\ of\ \left[\mathbb{R}^{|ET\times K|}\right] \times \left[\mathbb{R}^{|ET\times I|} \times \mathbb{R}^{|ET\times K|}\right]} \qquad \longrightarrow \left[\mathbb{R}^{|ET|} \times \mathbb{R}^{|ET \times K|}\right]$$

given by

$$ED(q,\ \ (1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)) =$$
$$\left[\underbrace{ED_C(q,\ \ (1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d))}_{Excess\ Demand\ for\ Commodities}\right] \times \left[\underbrace{ED_A(q,\ \ (1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d))}_{Excess\ Demand\ for\ Assets}\right].$$

is such that

$$ED \in C^{\infty}(Q \times \left[\mathbb{R}_{++}^{|ET \times I|} \times \mathbb{R}^{|ET \times K|}\right]).^{11}$$

Let

$$(\{(\overline{c}_i,\ \overline{z}_i)\}_{i \in I}\,,\ \overline{q}) \in \left(\mathbb{R}_{+}^{|ET \times I|} \times \mathcal{Z}^{|I|}\right) \times Q$$

be a regular equilibrium in which markets are complete for the CCAPM with stochastic taxation $\overline{\tau} = (\overline{\tau}_e,\ \overline{\tau}_d)$. Therefore,

$$ED(\overline{q},\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ (1-\overline{\tau}_d)\cdot d(\overline{\tau}_d)) = 0$$

and

$$det\left[D_q ED(\overline{q},\ \ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ (1-\overline{\tau}_d)\cdot d(\overline{\tau}_d))\right] \neq 0.$$

The Classical Implicit Function Theorem (IFT) states:
Let $X \times A$ *be an open subset* of $\mathbb{R}^n \times \mathbb{R}^m$, *and function* $f : X \times A \longrightarrow \mathbb{R}^n$ *be s.t.* $f \in C^{\infty}(X \times A)$. *Let* $\overline{y} = f(\overline{x},\ \overline{a})$ *and* $\exists D_x^{-1} f(\overline{x},\ \overline{a})$. *Then* $\exists$ *open neighborhoods* $U_{\overline{x}}$ *of* $\overline{x}$ *and* $W_{\overline{a}}$ *of* $\overline{a}$ *s.t.* $\exists$ *a unique function*

$$\begin{gathered} g : W_{\overline{a}} \longrightarrow U_{\overline{x}},\ s.t. \\ g(\overline{a}) = \overline{x}, \\ f(g(a),\ a) = \overline{y}\ \forall a \in W_{\overline{a}}, \\ g \in C^{\infty}(W_{\overline{a}}), \\ \frac{\partial g}{\partial a} = -\left[\frac{\partial f}{\partial x}\right]^{-1} \frac{\partial f}{\partial a}. \end{gathered}$$

Therefore, we can conclude by the IFT that $\exists$ open neighborhoods U of $\overline{q} \in Q$ and W of $((1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ (1-\overline{\tau}$
$\mathbb{R}_{++}^{|ET \times I|} \times \mathbb{R}^{|ET \times K|}$ and a function

[11] See Lemma 11.5 on p. 96 of Magill and Quinzii (1996).

$$g : W \longrightarrow U, \text{ s.t.}$$
$$g(\,(1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ (1-\overline{\tau}_d)\cdot d(\overline{\tau}_d)) = \overline{q},$$
$$ED(g(\,(1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)),\ (1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)) = 0$$

$\forall\,((1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)) \in W$ and

$$g \in C^{\infty}\,(W).$$

By the assumption D4 and since $e_i \in C^1\left([0,1]^{|ET|}\right)\ \forall i \in I$, we know that the function

$$f : [0,\ 1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|} \longrightarrow \mathbb{R}^{|ET\times I|} \times \mathbb{R}^{|ET\times K|}$$

defined as

$$f(\tau_e, \tau_d) = ((1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d))$$

is such that

$$f \in C^1\left([0,\ 1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|}\right).$$

Therefore, since $W \subset \mathbb{R}^{|ET\times I|}_{++} \times \mathbb{R}^{|ET\times K|}$ is an open set, the set

$$O_{\overline{\tau}} = f^{-1}(W)$$

is an open set as well. Since

$$f : [0,\ 1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|} \longrightarrow f([0,\ 1]^{|ET\times I|} \times [0,\ 1]^{|ET\times K|})$$

is onto, we can conclude that

$$f(O_{\overline{\tau}}) = f\left(f^{-1}(W)\right) = W.$$

Thus,

$$g(f(O_{\overline{\tau}})) = g\,(W).$$

Hence, the function

$$[g \circ f] : O_{\overline{\tau}} \longrightarrow g\,(W)$$

defined as

$$[g \circ f]\,(\tau) = g\,(f(\tau)) = \overline{q}(\tau)\ \forall \tau \in O_{\overline{\tau}}$$

is onto and

$$[g \circ f] \in C^1\,(O_{\overline{\tau}}).$$

Using Lemma 11.5 on p. 96 of Magill and Quinzii (1996) we can conclude that the individual demand function for consumption

$$\overline{c}_i : Q \times \left[\mathbb{R}^{|ET|}_{++} \times \mathbb{R}^{|ET \times K|}\right] \longrightarrow \mathbb{R}^{|ET|}_{+}$$

given by

$$\overline{c}_i(q,\ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d)) = \arg\max \{U_i(c_i) \mid c_i \in B(q,\ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d))\}$$

and the individual demand function for assets

$$\overline{z}_i : Q \times \left[\mathbb{R}^{|ET|}_{++} \times \mathbb{R}^{|ET \times K|}\right] \longrightarrow \mathbb{R}^{|ET|}$$

given by

$$\begin{gathered} \overline{z}_i(q,\ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d)) = \\ \{z_i \in \mathcal{Z} \mid \overline{c}_i(q,\ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d)) - (1-\tau_{e_i}) \cdot e(\tau_{e_i}) \\ = W(q,\ (1-\tau_d) \cdot d(\tau_d)) \cdot z_i\} \end{gathered}$$

are such that

$$\overline{c}_i,\ \overline{z}_i \in C^\infty(Q \times \left[\mathbb{R}^{|ET|}_{++} \times \mathbb{R}^{|ET \times K|}\right]\ )\ \forall i \in I.$$

Hence, the individual equilibrium consumption of agent $i \in I$

$$\overline{c}_i : O_{\overline{\tau}} \longrightarrow \overline{c}_i(O_{\overline{\tau}})$$

defined with some abuse of notations as

$$\overline{c}_i(\tau) = \overline{c}_i(\overline{q}(\tau),\ (1-\tau_e) \cdot e(\tau_e),\ (1-\tau_d) \cdot d(\tau_d))\ \forall \tau \in O_{\overline{\tau}}$$

and the individual equilibrium portfolio of agent $i \in I$

$$\overline{z}_i : O_{\overline{\tau}} \longrightarrow \overline{c}_i(O_{\overline{\tau}})$$

defined with some abuse of notations as

$$\overline{z}_i(\tau) = \overline{z}_i(\overline{q}(\tau),\ (1-\tau_e) \cdot e(\tau_e),\ (1-\tau_d) \cdot d(\tau_d))\ \forall \tau \in O_{\overline{\tau}}$$

are such that

$$\overline{c}_i,\ \overline{z}_i \in C^1\left(O_{\overline{\tau}}\right)\ \forall i \in I.$$

So

$$\left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i \in I},\ \overline{q}(\tau)\right) \in \left(\mathbb{R}^{|ET \times I|}_{+} \times \mathcal{Z}^{|I|}\right) \times Q$$

is an equilibrium for the CCAPM with stochastic taxation $\forall \tau \in O_{\overline{\tau}}$. Moreover, as Magin (2015b) demonstrated, we can always make $O_{\overline{\tau}}$ sufficiently small so that this FM equilibrium is complete $\forall \tau \in O_{\overline{\tau}}$.

Therefore, the function

$$h : O_{\overline{\tau}} \longrightarrow \left(\mathbb{R}_{+}^{|ET \times I|} \times \mathcal{Z}^{|I|}\right) \times Q \times \mathbb{R}_{++}^{|ET|}$$

defined as

$$h(\tau) = \left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ \overline{q}(\tau),\ \overline{\pi}(\tau)\right)$$

is well-defined and $\overline{c}_i,\ \overline{z}_i \in C^1\left(O_{\overline{\tau}}\right)\ \forall i \in I$. ■

PROOF OF THEOREM 3.2: Let us set up Lagrangian

$$\pounds_i = U_i(c_i,\, G) - \lambda_i \cdot [c_i - (1-\tau_{e_i}) \cdot e(\tau_{e_i}) - W(q,\, (1-\tau_d)\cdot d(\tau_d)) \cdot z_i]\ \forall i \in I,$$

where $\lambda_i \in \mathbb{R}^{|ET|}$ is the Lagrangian multiplier. Taking first-order conditions we obtain

$$D\pounds_i = \begin{cases} \frac{\partial \pounds_i}{\partial c_i} = DU_i(c_i) - \lambda_i = 0 \\ \frac{\partial \pounds_i}{\partial z_i} = \lambda_i \cdot W(q,\, (1-\tau_d)\cdot d(\tau_d)) = 0 \\ \frac{\partial \pounds_i}{\partial \lambda_i} = -c_i + (1-\tau_{e_i}) \cdot e(\tau_{e_i}) + W(q,\, (1-\tau_d)\cdot d(\tau_d)) \cdot z_i = 0 \end{cases}$$

Therefore, substituting equilibrium consumption of agent $i \in I$ back into first-order conditions and taking into consideration that markets are complete we obtain

$$\boxed{\overline{\pi}(\xi',\, \tau) = \overline{\pi}_i(\xi',\, \tau) = \frac{\lambda_i(\xi')}{\lambda_i(\xi)} = b_i^{T(\xi')} \cdot \frac{u_i'(\overline{c}_i(\xi',\, \tau))}{u_i'(\overline{c}_i(\xi,\, \tau))} \cdot \Pr(\xi')\ \forall (\xi',\, i) \in ET^{+}(\xi) \times I.} \tag{1}$$

and

$$\boxed{\overline{q}\,(\xi,\ \tau) = \sum_{\xi' \in ET^{+}(\xi)} \underbrace{b_i^{T(\xi')} \cdot \frac{u_i'\left(\overline{c}_i(\xi',\ \tau)\right)}{u_i'\left(\overline{c}_i(\xi,\ \tau)\right)} \cdot \Pr(\xi')}_{\overline{\pi}(\xi',\, \tau)} \cdot \left(1 - \tau_d\left(\xi'\right)\right) \cdot d\left(\xi',\ \tau_d\right)\ \forall \xi \in ET.} \tag{2}$$

By the assumption D3 $\exists \bar{i} \in I$ *such that*

$$\frac{u_{\bar{i}}'\left(\overline{c}_{\bar{i}}(\xi',\, \tau)\right)}{u_{\bar{i}}'\left(\overline{c}_{\bar{i}}(\xi,\, \tau)\right)} = \frac{u_{\bar{i}}'\left(\sum\limits_{i\in I} \overline{c}_i(\xi',\, \tau)\right)}{u_{\bar{i}}'\left(\sum\limits_{i\in I} \overline{c}_i(\xi,\, \tau)\right)} = \frac{u_{\bar{i}}'\left(\sum\limits_{i\in I}\left(1-\tau_{e_i}(\xi')\right)\cdot e_i(\tau_{e_i}(\xi')) + \sum\limits_{k\in K}(1-\tau_d(\xi',\, k))\cdot d(\xi',\, k,\, \tau_d)\cdot \delta(k)\right)}{u_{\bar{i}}'\left(\sum\limits_{i\in I}\left(1-\tau_{e_i}(\xi)\right)\cdot e_i(\tau_{e_i}(\xi)) + \sum\limits_{k\in K}(1-\tau_d(\xi,\, k))\cdot d(\xi,\, k,\, \tau_d)\cdot \delta(k)\right)}$$

$\forall\,(\xi,\ \xi') \in ET \times ET$. Therefore,

$$\overline{\pi}(\xi',\,\tau) = b_{\bar{i}}^{T(\xi')} \cdot \frac{u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\,\tau)\right)}{u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)} \cdot \Pr(\xi').$$

Differentiating $\overline{\pi}(\xi',\,\tau)$ with respect to $\tau_d(\xi)$ and keeping in mind D3 we obtain

$$\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_d(\xi)} = -b_{\bar{i}}^{T(\xi')} \cdot \frac{u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\,\tau)\right)}{\left[u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)\right]^2} \cdot u''_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right) \cdot \frac{\partial\left[\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right]}{\partial\tau_d(\xi)} \cdot \Pr(\xi') =$$

$$\underbrace{b_{\bar{i}}^{T(\xi')} \cdot \frac{u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\;\tau)\right)}{u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right)} \cdot \Pr(\xi')}_{\overline{\pi}(\xi',\tau)} \cdot \underbrace{\left[-\frac{u''_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right)\cdot\left[\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right]}{u'_{\bar{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right)}\right]}_{rr_{\bar{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))} \cdot \underbrace{\frac{1}{\left[\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right]} \cdot \frac{\partial\left[\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right]}{\partial\tau_d(\xi)}}_{g_{\sum_{i\in I}\overline{c}_i(\xi,\,\tau)}}$$

$$= \overline{\pi}(\xi',\,\tau) \cdot \left[rr_{\bar{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))\right] \cdot g_{\sum_{i\in I}\overline{c}_i(\xi,\,\tau)}.$$

Therefore,

$$\boxed{\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_d(\xi)} = \overline{\pi}(\xi',\tau) \cdot \left[rr_{\bar{i}}(\sum_{i\in I}\overline{c}_i(\xi,\tau))\right] \cdot g_{\sum_{i\in I}\overline{c}_i(\xi,\tau)}.} \tag{3}$$

We know that

$$\overline{q}\,(\xi,\;\tau) = \sum_{\xi'\in ET^+(\xi)} \overline{\pi}(\xi',\tau)\cdot(1-\tau_d\,(\xi'))\cdot d\,(\xi',\;\tau_d)\,. \tag{2}$$

Now, differentiating equation (2) with respect to $\tau_d(\xi)$ we obtain

$$\frac{\partial\overline{q}(\xi,\,\tau)}{\partial\tau_d(\xi)} = \underbrace{\sum_{\xi'\in ET^+(\xi)} \frac{\partial\overline{\pi}(\xi',\;\tau)}{\partial\tau_d\,(\xi)}\cdot(1-\tau_d\,(\xi'))\cdot d\,(\xi',\;\tau_d)}_{\textit{Changes in the Stochastic Discount } \overline{\pi}(\xi',\,\tau) \textit{ Factor for } \xi'} +$$

$$+\underbrace{\sum_{\xi'\in ET^+(\xi)} \overline{\pi}(\xi',\;\tau)\cdot\left[\frac{\partial d\,(\xi',\;\tau_d)}{\partial\tau_d\,(\xi)}\cdot(1-\tau_d\,(\xi')) - d\,(\xi',\;\tau_d)\cdot\frac{\partial\tau_d\,(\xi')}{\partial\tau_d\,(\xi)}\right]}_{\textit{Changes in After-tax Dividends } (1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}$$

$\forall \xi \in ET$.

Substituting (3) into the previous equation and taking into consideration that by assumptions D4 and D5

$$\frac{\partial \tau_d(\xi')}{\partial \tau_d(\xi)} = \frac{\partial d(\xi',\, \tau_d)}{\partial \tau_d(\xi)} = 0 \ \forall\, (\xi,\ \xi') \in ET \times [ET \setminus \{\xi\}]$$

we get

$$\frac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_d(\xi)} = \sum_{\xi' \in ET^+(\xi)} \left[rr_{\overline{i}}(\sum_{i \in I} \overline{c}_i(\xi, \tau)) \right] \cdot g_{\sum_{i \in I} \overline{c}_i(\xi,\tau)} \cdot$$
$$\cdot \overline{\pi}(\xi',\, \tau) \cdot (1 - \tau_d\,(\xi')) \cdot d\,(\xi',\ \tau_d) =$$
$$= \left[rr_{\overline{i}}(\sum_{i \in I} \overline{c}_i(\xi, \tau)) \right] \cdot g_{\sum_{i \in I} \overline{c}_i(\xi,\tau)} \cdot \left[\underbrace{\sum_{\xi' \in ET^+(\xi)} \overline{\pi}(\xi', \tau) \cdot (1 - \tau_d\,(\xi')) \cdot d\,(\xi',\ \tau_d)}_{\overline{q}(\xi,\, \tau)} \right] =$$
$$= \overline{q}\,(\xi,\ \tau) \cdot \left[rr_{\overline{i}}(\sum_{i \in I} \overline{c}_i(\xi, \tau)) \right] \cdot g_{\sum_{i \in I} \overline{c}_i(\xi,\tau)}.$$

Therefore,

$$\boxed{\frac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_d(\xi)} = \overline{q}\,(\xi,\ \tau) \cdot \left[rr_{\overline{i}}(\sum_{i \in I} \overline{c}_i(\xi, \tau)) \right] \cdot g_{\sum_{i \in I} \overline{c}_i(\xi,\tau)}} \tag{4}$$

and

$$\boxed{sign\left[\frac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_d(\xi)}\right] = sign\left[\frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\tau)\right]}{\partial \tau_d(\xi)}\right]. \ \blacksquare} \tag{5}$$

PROOF OF COROLLARY 3.3: By the assumption D1, the total supply of assets is given by $\delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}^{|K|}_{++}$. Then, since all agents are identical, we have

$$\overline{c}_i(\xi,\, \tau) = (1 - \tau_e(\xi)) \cdot e(\xi,\, \tau_e) + \sum_{k \in K} (1 - \tau_d(\xi,\, k)) \cdot d(\xi,\, k,\, \tau_d) \cdot \frac{\delta(k)}{|I|}$$

$\forall (\xi,\, i) \in ET \times I$.

All agents have zero initial endowments, i.e.,

$$e_i(\xi,\, \tau_{e_i}) = 0 \;\forall\, (i,\; \xi) \in I \times ET.$$

Also, all assets' dividends are taxed identically, i.e.,

$$\tau_d(\xi,\, k) = \tau_d(\xi,\, \overline{k}) \;\forall\, (\xi,\; k) \in ET \times K.$$

Thus, we have that

$$\overline{c}_i(\xi,\, \tau) = (1 - \tau_d(\xi,\, \overline{k})) \cdot \sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \tfrac{\delta(k)}{|I|} \;\forall (\xi,\, i) \in ET \times I.$$

Therefore,

$$\sum_{i \in I} \overline{c}_i(\xi,\, \tau) = (1 - \tau_d(\xi,\, \overline{k})) \cdot \sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \delta(k).$$

Also, by the assumption of the Corollary

$$\tfrac{\partial d(\xi',\, \tau_d)}{\partial \tau_d(\xi)} = 0 \;\forall (\xi,\, \xi') \in ET \times ET.$$

Therefore,

$$\frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\, \tau) \right]}{\partial \tau_d(\xi,\, \overline{k})} = -\sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \delta(k) \;\forall (\xi,\, i) \in ET \times I.$$

So we can conclude that

$$g_{\sum_{i \in I} \overline{c}_i(\xi,\tau)} = \frac{1}{\left[\sum_{i \in I} \overline{c}_i(\xi,\, \tau) \right]} \cdot \frac{\partial \left[\sum_{i \in I} \overline{c}_i(\xi,\, \tau) \right]}{\partial \tau_d(\xi)} = \frac{-\sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \delta(k)}{(1 - \tau_d(\xi,\, \overline{k})) \cdot \sum_{k \in K} d(\xi,\, k,\, \tau_d) \cdot \delta(k)} = -\frac{1}{(1 - \tau_d(\xi,\, \overline{k}))} \;\forall \xi \in ET.$$

Therefore, by (4) we obtain

$$\tfrac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_d(\xi)} = -\overline{q}\,(\xi,\; \tau) \cdot rr_i(\sum_{i \in I} \overline{c}_i(\xi,\, \tau)) \cdot \tfrac{1}{(1 - \tau_d(\xi,\, \overline{k}))} \;\forall (\xi,\, i) \in ET \times I.$$

Since all identical agents exhibit CRRA, i.e.,

$$rr_i(c) = -\left[\tfrac{u_i^{"}(c) \cdot c}{u_i'(c)} \right] = a \;\forall i \in I,$$

we have that

$$\tfrac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_d(\xi)} = -\overline{q}\,(\xi,\; \tau) \cdot a \cdot \tfrac{1}{(1 - \tau_d(\xi,\, \overline{k}))} \;\forall \xi \in ET.$$

Hence,

$$E_{q(\xi,\,\tau),\,1-\tau_d(\xi,\,\overline{k})} = \frac{\frac{1}{\overline{q}(\xi,\,\tau)}\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi,\,\overline{k})}}{\frac{1}{(1-\tau_d(\xi,\,\overline{k}))}\frac{\partial(1-\tau_d(\xi,\,\overline{k}))}{\partial \tau_d(\xi,\,\overline{k})}} = \frac{-a\cdot\frac{1}{(1-\tau_d(\xi,\,\overline{k}))}}{-\frac{1}{(1-\tau_d(\xi,\,\overline{k}))}} = a\ \forall \xi \in ET.$$

So

$$\boxed{E_{q(\xi,\,\tau),\,1-\tau_d(\xi,\,\overline{k})} = a\ \forall \xi \in ET.}\ \blacksquare$$

PROOF OF COROLLARY 3.4: We know that

$$R(\xi',\,\tau) = \frac{q(\xi',\,\tau)+d(\xi',\,\tau_d)}{q(\xi,\,\tau)},$$

where $\xi' \in \xi^+$. Differentiating $R(\xi',\,\tau)$ with respect to $\tau_d(\xi)$ we obtain

$$\frac{\partial R(\xi',\,\tau)}{\partial \tau_d(\xi)} = -\frac{q(\xi',\,\tau)+d(\xi',\,\tau_d)}{q^2(\xi,\,\tau)}\cdot\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)}.$$

We also know that

$$ATR(\xi',\,\tau) = \frac{q(\xi',\,\tau)+(1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}{q(\xi,\,\tau)},$$

where $\xi' \in \xi^+$. Differentiating $ATR(\xi',\,\tau)$ with respect to $\tau_d(\xi)$ we obtain

$$\frac{\partial ATR(\xi',\,\tau)}{\partial \tau_d(\xi)} = -\frac{q(\xi',\,\tau)+(1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}{q^2(\xi,\,\tau)}\cdot\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)}.$$

Therefore,

$$\boxed{sign\left[\frac{\partial R(\xi',\,\tau)}{\partial \tau_d(\xi)}\right] = sign\left[\frac{\partial ATR(\xi',\,\tau)}{\partial \tau_d(\xi)}\right] = -sign\left[\frac{\partial q(\xi,\,\tau)}{\partial \tau_d(\xi)}\right]\ \forall\,(\xi,\ \xi') \in ET \times \xi^+.}\ \blacksquare$$

PROOF OF THEOREM 3.5: By the assumption D3 $\exists \overline{i} \in I$ such that

$$\frac{u'_{\overline{i}}\left(\overline{c}_{\overline{i}}(\xi',\,\tau)\right)}{u'_{\overline{i}}\left(\overline{c}_{\overline{i}}(\xi,\,\tau)\right)} = \frac{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\,\tau)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)} = \frac{u'_{\overline{i}}\left(\sum_{i\in I}\left(1-\tau_{e_i}(\xi')\right)\cdot e_i(\tau_{e_i}(\xi'))+\sum_{k\in K}(1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)\cdot\delta(k)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\left(1-\tau_{e_i}(\xi)\right)\cdot e_i(\tau_{e_i}(\xi))+\sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)\right)}$$

$\forall\,(\xi,\ \xi') \in ET \times ET$. Therefore,

$$\overline{\pi}(\xi',\,\tau) = b_{\overline{i}}^{T(\xi')}\cdot\frac{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\,\tau)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)}\cdot \Pr(\xi').$$

$\forall \xi' \in ET^+(\xi)$.
Fix $\overline{\xi} \in ET^+(\xi)$. Then, clearly, keeping in mind D3 we obtain

$$\frac{\partial \overline{\pi}(\xi',\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)} = 0\ \forall \xi' \in ET^+(\xi) \setminus \left\{\overline{\xi}\right\}$$

and

$$\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_d(\overline{\xi})} = b_{\overline{i}}^{T(\overline{\xi})}\cdot\frac{u_{\overline{i}}''\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)\right)}{u_{\overline{i}}'\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)}\cdot\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)\right]}{\partial\tau_d(\overline{\xi})}\cdot\Pr(\overline{\xi}) =$$

$$\underbrace{-b_{\overline{i}}^{T(\overline{\xi})}\cdot\frac{u_{\overline{i}}'\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau)\right)}{u_{\overline{i}}'\left(\sum_{i\in I}\overline{c}_i(\xi,\;\tau)\right)}\cdot\Pr(\overline{\xi})}_{\overline{\pi}(\overline{\xi},\,\tau)}\cdot\underbrace{\left[-\frac{u_{\overline{i}}''\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau)\right)\cdot\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau)\right]}{u_{\overline{i}}'\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau)\right)}\right]}_{rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau))}\cdot\underbrace{\frac{1}{\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau)\right]}\cdot\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau)\right]}{\partial\tau_d\left(\overline{\xi}\right)}}_{g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}}$$

$$= -\overline{\pi}(\overline{\xi},\,\tau)\cdot\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\;\tau))\right]\cdot g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}.$$

Therefore,

$$\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_d(\overline{\xi})} = -\overline{\pi}(\overline{\xi},\,\tau)\cdot\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau))\right]\cdot g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}.\ \bigstar$$

We know that

$$\overline{q}\left(\xi,\;\tau\right) = \sum_{\xi'\in ET^+(\xi)}\overline{\pi}(\xi',\tau)\cdot\left(1-\tau_d\left(\xi'\right)\right)\cdot d\left(\xi',\;\tau_d\right). \tag{2}$$

Now, differentiating equation (2) with respect to $\tau_d\left(\overline{\xi}\right)$ we obtain

$$\frac{\partial\overline{q}(\xi,\,\tau)}{\partial\tau_d(\overline{\xi})} = \underbrace{\sum_{\xi'\in ET^+(\xi)}\frac{\partial\overline{\pi}(\xi',\;\tau)}{\partial\tau_d\left(\overline{\xi}\right)}\cdot\left(1-\tau_d\left(\xi'\right)\right)\cdot d\left(\xi',\;\tau_d\right)}_{\textit{Changes in the Stochastic Discount } \overline{\pi}(\xi',\,\tau) \textit{ Factor for } \xi'} +$$

$$\underbrace{+\sum_{\xi'\in ET^+(\xi)}\overline{\pi}(\xi',\;\tau)\cdot\left[\frac{\partial d\left(\xi',\;\tau_d\right)}{\partial\tau_d\left(\overline{\xi}\right)}\cdot\left(1-\tau_d\left(\xi'\right)\right)-d\left(\xi',\;\tau_d\right)\cdot\frac{\partial\tau_d\left(\xi'\right)}{\partial\tau_d\left(\overline{\xi}\right)}\right]}_{\textit{Changes in After-tax Dividends } (1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}$$

$\forall\xi\in ET$.

Substituting $\bigstar$ into the previous equation and taking into consideration that by assumptions D3, D4 and D5

$$\frac{\partial \overline{\pi}(\xi',\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)} = 0\ \forall \xi' \in ET^{+}(\xi) \setminus \{\overline{\xi}\},$$
$$\frac{\partial \tau_d(\xi')}{\partial \tau_d(\xi)} = \frac{\partial d(\xi',\,\tau_d)}{\partial \tau_d(\xi)} = 0\ \forall\,(\xi,\ \xi') \in ET \times [ET \setminus \{\xi\}]$$

we get

$$\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)} = -\left[rr_{\overline{i}}\Big(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau)\Big)\right]\cdot g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}\cdot$$
$$\cdot\overline{\pi}(\overline{\xi},\,\tau)\cdot\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$$
$$+\overline{\pi}(\overline{\xi},\,\tau)\cdot\left[\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)-d\left(\overline{\xi},\ k,\ \tau_d\right)\right].$$

Let

$$\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)} > 0.$$

Then, since $\frac{\partial d\left(\overline{\xi},\ k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\ k\right)} < 0$ we have that

$$\left[rr_{\overline{i}}\Big(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau)\Big)\right] > \frac{d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{-g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)}$$

$$= \frac{d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{\frac{\left[d(\overline{\xi},\,k,\,\tau_d)-\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)}\cdot(1-\tau_d(\overline{\xi},\,k))\right]\cdot\delta(k)}{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)} = \frac{1}{\left[\frac{\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi})\right)\cdot e_i(\overline{\xi},\,\tau_{e_i})+\sum_{k\in K}\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}\right]}.$$

Therefore,

$$\left[rr_{\overline{i}}\Big(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau)\Big)\right] > \frac{1}{\left[\frac{\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi})\right)\cdot e_i(\overline{\xi},\,\tau_{e_i})+\sum_{k\in K}\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}\right]} > 1.$$

Set

$$B(\overline{\xi},\,k,\,\tau) = \frac{1}{\left[\frac{\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi})\right)\cdot e_i(\overline{\xi},\,\tau_{e_i})+\sum_{k\in K}\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}\right]}.$$

Thus,

$$\boxed{sign\left[\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)}\right] = sign\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau)) - B(\overline{\xi},k,\tau)\right]}$$

$\forall \overline{\xi} \in ET^{+}(\xi)$. Moreover,

$$B(\overline{\xi},\,k,\,\tau) \geqslant 1.\blacksquare$$

PROOF OF THEOREM 4.3: By the assumption E3 $\exists \overline{i} \in I$ such that

$$\frac{u'_{\overline{i}}\left(\overline{c}_{\overline{i}}(\xi',\,\tau)\right)}{u'_{\overline{i}}\left(\overline{c}_{\overline{i}}(\xi,\,\tau)\right)} = \frac{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\,\tau)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)} = \frac{u'_{\overline{i}}\left(\sum_{i\in I}\left(1-\tau_{e_i}(\xi')\right)\cdot e_i(\tau_{e_i}(\xi'))+\sum_{k\in K}(1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)\cdot\delta(k)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\left(1-\tau_{e_i}(\xi)\right)\cdot e_i(\tau_{e_i}(\xi))+\sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)\right)}$$

$\forall\,(\xi,\ \xi') \in ET \times ET$. Therefore,

$$\overline{\pi}(\xi',\,\tau) = b_{\overline{i}}^{T(\xi')}\cdot\frac{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi',\,\tau)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)}\cdot \Pr(\xi').$$

$\forall \xi' \in ET^{+}(\xi)$.
Fix $\overline{\xi} \in ET^{+}(\xi)$. Then, clearly, keeping in mind E3 we obtain

$$\frac{\partial \overline{\pi}(\xi',\,\tau)}{\partial \tau_{e_i}\left(\overline{\xi}\right)} = 0\ \forall \xi' \in ET^{+}(\xi)\setminus\left\{\overline{\xi}\right\}$$

and

$$\frac{\partial \overline{\pi}(\overline{\xi},\,\tau)}{\partial \tau_{e_{\overline{i}}}\left(\overline{\xi}\right)} = b_{\overline{i}}^{T(\overline{\xi})}\cdot\frac{u''_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\,\tau)\right)}\cdot\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)\right]}{\partial \tau_{e_{\overline{i}}}\left(\overline{\xi}\right)}\cdot \Pr(\overline{\xi}) =$$

$$\underbrace{-b_{\overline{i}}^{T(\overline{\xi})}\cdot\frac{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau)\right)}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\xi,\ \tau)\right)}\cdot \Pr(\overline{\xi})}_{\overline{\pi}(\overline{\xi},\,\tau)}\cdot\underbrace{\left[-\frac{u''_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau)\right)\cdot\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau)\right]}{u'_{\overline{i}}\left(\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau)\right)}\right]}_{rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau))}\cdot\underbrace{\left[\frac{1}{\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau)}\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau)\right]}{\partial \tau_{e_{\overline{i}}}\left(\overline{\xi}\right)}\right]}_{g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}}$$

$$= -\overline{\pi}(\overline{\xi},\,\tau)\cdot\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\ \tau))\right]\cdot g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}.$$

Therefore,

$$\frac{\partial \overline{\pi}(\overline{\xi},\,\tau)}{\partial \tau_{e_i}(\overline{\xi})} = -\overline{\pi}(\overline{\xi},\,\tau)\cdot\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau))\right]\cdot g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}. \ \blacklozenge$$

We know that

$$\overline{q}\,(\xi,\ \tau) = \sum_{\xi'\in ET^+(\xi)} \overline{\pi}(\xi',\tau)\cdot(1-\tau_d\,(\xi'))\cdot d\,(\xi',\ \tau_d)\,. \tag{2}$$

Now, differentiating equation (2) with respect to $\tau_{e_i}\left(\overline{\xi}\right)$ we obtain

$$\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_{\overline{i}}}(\overline{\xi})} = \underbrace{\sum_{\xi'\in ET^+(\xi)} \frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_{e_{\overline{i}}}\left(\overline{\xi}\right)}\cdot(1-\tau_d\,(\xi'))\cdot d\,(\xi',\ \tau_d)\ +}_{Changes\ in\ the\ Stochastic\ Discount\ \overline{\pi}(\xi',\,\tau)\ Factor\ for\ \xi'}$$

$$+\underbrace{\sum_{\xi'\in ET^+(\xi)} \overline{\pi}(\xi',\ \tau)\cdot\left[\frac{\partial d\,(\xi',\ \tau_d)}{\partial \tau_{e_{\overline{i}}}\left(\overline{\xi}\right)}\cdot(1-\tau_d\,(\xi'))-d\,(\xi',\ \tau_d)\cdot\frac{\partial \tau_d\,(\xi',\ \tau_d)}{\partial \tau_{e_i}\left(\overline{\xi}\right)}\right]}_{Changes\ in\ After-tax\ Dividends\ (1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}$$

$\forall\overline{\xi}\in ET^+(\xi)$.

Substituting $\blacklozenge$ into the previous equation and taking into consideration that by assumptions E4 and E5

$$\frac{\partial \tau_d}{\partial \tau_{e_{\overline{i}}}} = \frac{\partial d}{\partial \tau_{e_{\overline{i}}}} = 0,$$

we get

$$\boxed{\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_{\overline{i}}}(\overline{\xi})} = -\left[rr_{\overline{i}}(\sum_{i\in I}\overline{c}_i(\overline{\xi},\tau))\right]\cdot g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)}\cdot\overline{\pi}(\overline{\xi},\,\tau)\cdot(1-\tau_d\,(\xi'))\cdot d\,(\xi',\ \tau_d),}$$

where

$$g_{\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)} = \frac{-e_{\overline{i}}(\overline{\xi},\,\tau_{e_{\overline{i}}})+\frac{\partial e_{\overline{i}}(\overline{\xi},\,\tau_{e_{\overline{i}}})}{\partial \tau_{e_{\overline{i}}}(\overline{\xi})}\cdot\left(1-\tau_{e_{\overline{i}}}(\overline{\xi})\right)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi})\right)\cdot e_i(\overline{\xi},\,\tau_{e_i})+\sum_{k\in K}\delta(k)\cdot\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)}.$$

Therefore,

$$\boxed{sign\left[\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_{\overline{i}}}(\overline{\xi})}\right] = -sign\left[\frac{\partial\left[\sum_{i\in I}\overline{c}_i(\overline{\xi},\,\tau)\right]}{\partial \tau_{e_{\overline{i}}}(\overline{\xi})}\right]}$$

$\forall \bar{\xi} \in ET^{+}(\xi)$. ■

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