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Infinite Horizon CCAPM with Stochastic Taxation and Monetary Policy

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Abstract

This paper derives the infinite horizon CCAPM with heterogeneous agents, stochastic dividend taxation and monetary policy. I find that under reasonable assumptions on assets' dividends and probability distributions of the future dividend taxes and consumption, the model implies the constant price/after-tax dividend ratios. I also obtain that the higher current and expected dividend tax rates imply lower current asset prices. Finally, contrary to popular belief, monetary policy is neutral, in the long run, with respect to the real equilibrium asset prices.

Keywords: Stochastic Taxation, GEI, Equity Premium, Complete Markets, Comparative Statics, Risk Aversion, CCAPM, Property Rights, Money Supply

JEL Classification: D5; D9; E13; G12; H20.

1. INTRODUCTION

Unlike monetary policy, the stochastic nature of taxation and its effects on equilibrium asset prices and allocations have received surprisingly little attention in the literature. Yet taxes are part of individuals' and corporate budget constraints. Moreover, changes in various tax rates are driven

by an ever-changing political balance of power and the direction of those changes seems to be highly unpredictable and therefore stochastic. Thus, it seems entirely appropriate to regard future taxation as both important and stochastic.

The most recent research on the topic primarily focuses on insecure property rights in the finite horizon GEI model. Magin (2016) studies the comparative statics of asset prices with respect to various current and future tax rates in the finite horizon GEI model. He finds that under reasonable assumptions, an increase in the current or future stochastic dividend or endowment tax rates reduces current asset prices. Magin (2015) finds that under reasonable assumptions, in the finite horizon GEI model, FM equilibria exist for all stochastic tax rates, except for a closed set of measure zero.

The research conducted so far on the role of stochastic taxation in the infinite horizon GEI model relies on the CCAPM with identical agents and focuses primarily on resolving the so-called "Equity Premium Puzzle."[1] Magin (2014) develops a version of the CCAPM with insecure property rights. He finds that the current expected equity premium can be reconciled with a coefficient of relative risk aversion of 3.76, thus resolving a substantial part of the Equity Premium Puzzle in general stocks. Edelstein and Magin (2013) use the CCAPM with insecure property rights developed in Magin (2014) to address the Equity Premium Puzzle in REITs. They find that the current expected equity premium in REITs can be reconciled with a coefficient of relative risk aversion of 4.3-6.3, thus resolving a substantial part of the Equity Premium Puzzle for securitized real estate. Sialm (2009), in an excellent empirical paper, demonstrates that aggregate stock valuation levels are related to measures of the aggregate personal tax burden on equity securities. Sialm (2006) develops a generalized version of the Lucas (1978) dynamic general equilibrium tree model of production economy (risky assets being in positive supply) with identical agents and a flat consumption tax that follows a two-state Markov chain. The model is used to analyze the effects of a flat consumption tax on asset prices. He finds that under plausible conditions, investors require higher term and equity premia as compensation for the risk introduced by tax changes.

This paper derives the infinite horizon CCAPM with heterogeneous agents, stochastic dividend taxation and monetary policy and is a natural development of the emerging literature on the effects of insecure property rights on

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[1]See DeLong and Magin (2009) for a literature review of the Equity Premium Puzzle.

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equilibrium asset prices and allocations. I find that under reasonable assumptions on assets' dividends and probability distributions of the future dividend taxes and consumption, the model implies the constant price/after-tax dividend ratios. I also obtain that the higher current and expected dividend tax rates imply lower current asset prices. Finally, contrary to popular belief, monetary policy is neutral, in the long run, with respect to the real equilibrium asset prices.

The paper is organized as follows. Section 2 derives and analyzes the infinite horizon CCAPM with stochastic taxation. Section 3 derives and analyzes the infinite horizon CCAPM with stochastic taxation and monetary policy. Section 4 concludes.

2. The CCAPM with Stochastic Dividend Taxation

We start our analysis by stating the following lemma:

LEMMA (Rubinstein (1976)): *Let random variables x and y be bivariate normally distributed with expectations*

$$\left(E\left[x\right] ,\ E\left[y\right] \right) =\left(\mu _{x},\ \mu _{y}\right)$$

and variance-covariance matrix

$$V=\left(
\begin{array}{cc}
\sigma _{x}^{2} & \rho \sigma _{x}\sigma _{y} \\
\rho \sigma _{x}\sigma _{y} & \sigma _{y}^{2}
\end{array}
\right) ,$$

where

$$\rho =\frac{COV[x,\ y]}{\sqrt{VAR[x]VAR[y]}}$$

is the correlation coefficient between random variables x and y. That is, random variables x and y have joint density function

$$f\left(x,y\right) =\frac{1}{2\pi \sigma _{x}\sigma _{y}\sqrt{1-\rho ^{2}}}e^{-\frac{1}{2\left(1-\rho ^{2}\right) }\left[\frac{\left(x-\mu _{x}\right) ^{2}}{\sigma _{x}^{2}}-2\rho \frac{\left(x-\mu _{x}\right) \left(y-\mu _{y}\right) }{\sigma _{x}\sigma _{y}}+\frac{\left(y-\mu _{y}\right) ^{2}}{\sigma _{y}^{2}}\right] }.$$

Then

$$\int\limits_{a}^{\infty}\int\limits_{-\infty}^{\infty} e^x f(x,y)dxdy = e^{\mu_x+\frac{\sigma_x^2}{2}} \cdot N(\frac{-a+\mu_y}{\sigma_y} + \rho\sigma_x), \quad \bigstar$$

$$\int\limits_{-\infty}^{\infty}\int\limits_{a}^{\infty} e^y f(x,y)dxdy = e^{\mu_y+\frac{\sigma_y^2}{2}} \cdot N(\frac{-a+\mu_x}{\sigma_x} + \rho\sigma_y) \quad \bigstar\bigstar$$

and

$$\int\limits_{-\infty}^{\infty}\int\limits_{a}^{\infty} e^{x+y} f(x,y)dxdy = e^{\mu_x+\mu_y+\frac{(\sigma_x^2+2\rho\sigma_x\sigma_y+\sigma_y^2)}{2}} \cdot N(\frac{-a+\mu_x}{\sigma_x} + \rho\sigma_y + \sigma_x). \quad \bigstar\bigstar\bigstar$$

We will first use the Lemma to derive the infinite horizon CCAPM with stochastic taxation:

THEOREM 1: *Let K be the set of financial assets and I be the set of agents. Consider an economy with $|K| < \infty$ financial assets and $|I| < \infty$ agents, where the total supply of each asset is equal to 1 and agents maximize their utility function*

$$U_i(c_i, G) = E\left[\sum_{T=0}^{\infty} b_i^T \left(u_i(c_{it+T}) + v_i(G_{t+T})\right)\right] \forall i \in I,$$

where u_i is a CRRA utility function, such that $u_i(c) = \frac{c^{1-\lambda_i}}{1-\lambda_i}$, subject to

$$c_{it+T} + \sum_{k=1}^{n} z_{i\ kt+T+1}p_{kt+T} = \sum_{k=1}^{n} z_{i\ kt+T}\left(p_{kt+T} + (1 - \tau_{t+T}^d)d_{kt+T}\right)$$

$\forall (i,\ T) \in I \times \{0, ..., \infty\}$, where c_{it} is the consumption of an agent $i \in I$ at period t, $z_{i\ kt}$ is the number of shares of an asset $k \in K$ held by an agent $i \in I$ at period t, d_{kt} is the dividend per share of an asset $k \in K$ at period t, τ_t^d is the stochastic dividend tax rate at period t, G_t is the government spending at period t given by

$$G_{t+T} = \sum_{k=1}^{n} \tau_{t+T}^d d_{kt+T}.$$

The Transversality Condition (TC) holds for all assets and agents, i.e.,

$$\lim_{T \longrightarrow \infty} E\left[b_i^T \cdot \frac{u_i'(c_{it+T})}{u_i'(c_{it})} \cdot p_{kt+T}\right] = 0 \ \forall \, (k, \ i) \in K \times I,$$

where $\{c_{it+T}\}_{T=1}^{\infty}$ is the equilibrium consumption for an agent $i \in I$. Assume further

$$\frac{c_{ti+T}}{c_{it}} = \frac{\sum_{k=1}^{n}(1-\tau_{t+T}^d)\cdot d_{kt+T}}{\sum_{k=1}^{n}(1-\tau_t^d)\cdot d_{kt}} = \frac{(1-\tau_{t+T}^d)\cdot d_{kt+T}}{(1-\tau_t^d)\cdot d_{kt}} \ \forall \, (k, \ i, \ T) \in K \times I \times \{0, ..., \infty\},$$

i.e., all after-tax dividends are growing at the same rate and individuals' consumption is growing at the same rate as total dividends.[2]

$$E\left[b_i\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+T+1}}{(1-\tau_t^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right] = e^{\mu_c+\frac{1}{2}\cdot\sigma_c^2} < 1 \, [3] \ \forall \, (k, \ i, \ T) \in K\times I\times\{0, ..., \infty\},$$

and

$$\ln\left(b_i\left(\frac{c_{it+T+1}}{c_{it+T}}\right)^{1-\lambda_i}\right) \sim N(\mu_c, \ \sigma_c) \ \forall \, (i, \ T) \in I \times \{0, ..., \infty\}$$

with

$$COV\left[\ln\left(c_{it+T_1}\right), \ \ln\left(c_{it+T_2}\right)\right] = 0 \ \forall \, (\, T_1, \ T_2) \in \{1, ..., \infty\} \times \{1, ..., \infty\},$$
$$T_1 \neq T_2.$$

Then

$$\boxed{p_{kt} = \frac{e^{\mu_c+\frac{1}{2}\sigma_c^2}}{1-e^{\mu_c+\frac{1}{2}\sigma_c^2}} \cdot (1-\tau_t^d) \cdot d_{kt} \ \forall k \in K.}$$

PROOF: See Appendix.

Let us now introduce monetary policy into our analysis.

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[2] Since our modified CCAPM describes a production economy, the total dividend $\sum_{k=1}^{n} d_{kt+T}$ represents total output, i.e. GDP.

[3] See Magin (2014) for calculations.

3. The CCAPM with Stochastic Dividend Taxation and Monetary Policy

We will now use the Lemma to derive the infinite horizon CCAPM with Stochastic Taxation and monetary policy:

THEOREM 2: *Let K be the set of financial assets and I be the set of agents. Consider an economy with $|K| < \infty$ financial assets and $|I| < \infty$ agents, where the total supply of each asset is equal to 1 and agents maximize their utility function*

$$U_i(c_i, G) = E\left[\sum_{T=0}^{\infty} b_i^T \left(u_i(c_{it+T}) + v_i(G_{t+T})\right)\right],$$

where $0 < b_i < 1$ and u_i is a CRRA utility function, such that $u_i(c) = \frac{c^{1-\lambda_i}}{1-\lambda_i}$, subject to

$$c_{it+T} + \sum_{k=1}^{n} p_{kt+T} z_{ikt+T+1} + \frac{M_{it+T+1}}{p_{t+T}} = \sum_{k=1}^{n}(p_{kt+T}+(1-\tau_{t+T}^d)d_{kt+T})z_{ikt+T} + \frac{M_{it+T}}{p_{t+T}}$$

$\forall (i, T) \in I \times \{0,...,\infty\}$, where M_{it} is the quantity of money held by an agent $i \in I$ at period t, $M_t = \sum_{i\in I} M_{it}$ is the total supply of money, G_t is the government spending at period t given by

$$G_{t+T} = \tau_{t+T}^d \cdot \sum_{k=1}^{n} d_{kt+T} + \underbrace{\left(\frac{M_{t+T+1} - M_{t+T}}{p_{t+T}}\right)}_{Seigniorage}.$$

The Transversality Condition (TC) holds for all assets and agents, i.e.,

$$\lim_{T \longrightarrow \infty} E\left[b_i^T \cdot \frac{u_i'(c_{it+T})}{u_i'(c_{it})} \cdot p_{kt+T}\right] = 0 \ \forall (k, i) \in K \times I,$$

where $\{c_{it+T}\}_{T=1}^{\infty}$ is the equilibrium consumption for an agent $i \in I$. Assume further

$$\frac{c_{ti+T}}{c_{it}} = \frac{(1-\tau_{t+T}^d) \cdot \sum_{k=1}^{n} d_{kt+T} - \left[\frac{M_{i+T+1}-M_{t+T}}{p_{t+T}}\right]}{(1-\tau_t^d) \cdot \sum_{k=1}^{n} d_{kt} - \left[\frac{M_{t+1}-M_t}{p_t}\right]} \ \forall (i, T) \in I \times \{0,...,\infty\},$$

i.e., individuals' consumption is growing at the same rate as the total output $\sum_{k=1}^{n} d_{kt+T}$ (GDP) consumed by the private sector,

$$\frac{\tau_{t+T}^{d} \cdot \sum_{k=1}^{n} d_{kt+T} + \left(\frac{M_{t+T+1} - M_{t+T}}{p_{t+T}} \right)}{\sum_{k=1}^{n} d_{kt+T}} = \overline{g} \ \forall T \in \{0, ..., \infty\},$$

i.e., the percentage of the total output $\sum_{k=1}^{n} d_{kt+T}$ (GDP) consumed by the government is constant over time,

$$\frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}} = \frac{d_{kt+T}}{d_{kt}} \ \forall (k, \ i, \ T) \in K \times I \times \{0, ..., \infty\},$$

i.e., all dividends are growing at the same rate,

$$E\left[b_i \left(\frac{d_{kt+T+1}}{d_{kt+T}} \right)^{1-\lambda_i} \right] = e^{\mu_c + \frac{1}{2} \cdot \sigma_c^2} < 1^4 \ \forall (k, \ i, \ T) \in K \times I \times \{0, ..., \infty\}$$

and random variables

$$x_{t+T} = \ln\left(1 - \tau_{t+T+1} \right),$$
$$y_{it+T} = \ln\!\big(b_i \big(\tfrac{c_{ti+T}}{c_{it}} \big)^{1-\lambda_i} \big)$$

are bivariate normally distributed with expectations

$$\left(E\left[\ln\left(1 - \tau_{t+T} \right) \right], \ E\left[\ln\!\big(b_i \big(\tfrac{c_{ti+T}}{c_{it}} \big)^{1-\lambda_i} \big) \right] \right) = (\mu_\tau, \ \mu_c)$$

and the variance-covariance matrix

$$V = \begin{pmatrix} \sigma_\tau^2 & 0 \\ 0 & \sigma_c^2 \end{pmatrix}^4 \ \forall (i, \ T) \in I \times \{0, ..., \infty\}$$

with

$$COV\left[\ln\left(c_{it+T_1} \right), \ \ln\left(c_{it+T_2} \right) \right] = 0 \ \forall (\ T_1, \ T_2) \in \{1, ..., \infty\} \times \{1, ..., \infty\},$$
$$T_1 \neq T_2.$$

Then

$$\boxed{ p_{kt} = e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2} \cdot \frac{e^{\mu_c + \frac{1}{2}\sigma_c^2}}{1 - e^{\mu_c + \frac{1}{2}\sigma_c^2}} \cdot d_{kt} \ \forall k \in K. }$$

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[4]See Magin (2014) for calculations.

4. Conclusion

This paper derives the infinite horizon CCAPM with heterogeneous agents, stochastic dividend taxation and monetary policy and is a natural development of the emerging literature on the effects of insecure property rights on equilibrium asset prices and allocations. I find that under reasonable assumptions on assets' dividends and probability distributions of the future dividend taxes and consumption, the model implies the constant price/after-tax dividend ratios. I also obtain that the higher current and expected dividend tax rates imply lower current asset prices. Finally, contrary to popular belief, monetary policy is neutral, in the long run, with respect to the real equilibrium asset prices.

Appendix

PROOF OF THEOREM 1: We have that

$$c_{it+T} = \sum_{k=1}^{n} z_{i\ kt+T} \left(p_{kt+T} + (1 - \tau_{t+T}^d)d_{kt+T} \right) - \sum_{k=1}^{n} z_{i\ kt+T+1} p_{kt+T}$$

$\forall (i,\ T) \in I \times \{0, ..., \infty\}$.
Substituting budget constraint into the utility function, we obtain

$$U_i(c_i,\ G) =$$

$$E \left[\sum_{T=0}^{\infty} b_i^T \left(u_i(\underbrace{\sum_{k=1}^{n} z_{i\ kt+T} \left(p_{kt+T} + (1 - \tau_{t+T}^d)d_{kt+T} \right) - \sum_{k=1}^{n} z_{i\ kt+T+1} p_{kt+T}}_{c_{it+T}}) + v_i(G_{t+T}) \right) \right].$$

Differentiating U_i with respect to $z_{i\ kt+1}$ and setting $\frac{\partial U_i}{\partial z_{i\ kt+1}}$ to 0, we obtain

$$-p_{kt} \cdot u_i'(c_{it}) + b_i \cdot E \left[u_i'(c_{it+1}) \left(p_{kt+1} + (1 - \tau_{t+1}^d)d_{kt+1} \right) \right] = 0.$$

Therefore,

$$p_{kt} = E \left[b_i \left(\frac{c_{ti+T}}{c_{it}} \right)^{-\lambda_i} \left(p_{kt+1} + (1 - \tau_{t+1}^d) \cdot d_{kt+1} \right) \right] \forall (k,\ i) \in K \times I.$$

By repeated substitution and using the Transversality Condition, we get

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$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^T \left(\frac{c_{ti+T}}{c_{it}}\right)^{-\lambda_i} (1 - \tau_{t+T}^d) \cdot d_{kt+T}\right] \forall (k, \ i) \in K \times I. \qquad (1)$$

By assumption of the Theorem, we have that

$$\frac{c_{ti+T}}{c_{it}} = \frac{\sum_{k=1}^{n} (1-\tau_{t+T}^d) \cdot d_{kt+T}}{\sum_{k=1}^{n} (1-\tau_t^d) \cdot d_{kt}} = \frac{(1-\tau_{t+T}^d) \cdot d_{kt+T}}{(1-\tau_t^d) \cdot d_{kt}} \ \forall (k, \ i, \ T) \in K \times I \times \{0, ..., \infty\}.$$

Hence,

$$\frac{c_{ti+T}}{c_{it}} = \frac{(1-\tau_{t+T}^d) \cdot d_{kt+T}}{(1-\tau_t^d) \cdot d_{kt}} \ \forall (k, \ i, \ T) \in K \times I \times \{0, ..., \infty\}.$$

Substituting the expression for $\frac{c_{ti+T}}{c_{it}}$ into the previous equation (1), we obtain

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^T \left(\frac{(1-\tau_{t+T}^d) \cdot d_{kt+T}}{(1-\tau_t^d) \cdot d_{kt}}\right)^{-\lambda_i} (1 - \tau_{t+T}^d) \cdot d_{kt+T}\right] \forall (k, \ i) \in K \times I.$$

Therefore,

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^T \left(\frac{(1-\tau_{t+T}^d) \cdot d_{kt+T}}{(1-\tau_t^d) \cdot d_{kt}}\right)^{1-\lambda_i}\right] \cdot (1 - \tau_t^d) \cdot d_{kt} \ \forall (k, \ i) \in K \times I.$$

Moreover,

$$\ln\left(b_i \left(\frac{(1-\tau_{t+T}^d) \cdot d_{kt+T+1}}{(1-\tau_t^d) \cdot d_{kt+T}}\right)^{1-\lambda_i}\right) = \ln(b_i(\tfrac{c_{ti+T+1}}{c_{it+T}})^{1-\lambda_i}) \sim N(\mu_c, \ \sigma_c)$$

$\forall (k, \ i, \ T) \in K \times I \times \{0, ..., \infty\}.$
Also,

$$b_i^{\overline{T}} \left(\frac{(1-\tau_{t+T}^d) \cdot d_{kt+\overline{T}}}{(1-\tau_t^d) \cdot d_{kt}}\right)^{1-\lambda_i} = \prod_{T=0}^{\overline{T}-1}\left[b_i \left(\frac{(1-\tau_{t+T}^d) \cdot d_{kt+T+1}}{(1-\tau_t^d) \cdot d_{kt+T}}\right)^{1-\lambda_i}\right]$$

$\forall \left(k, \ i, \ \overline{T}\right) \in K \times I \times \{1, ..., \infty\}.$
Taking logarithms of both sides, we obtain

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$$\ln\left(b_i^{\overline{T}}\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right) = \ln\left(\prod_{T=0}^{\overline{T}-1}\left[b_i\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+T+1}}{(1-\tau_{t+T}^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right]\right) =$$

$$= \sum_{T=0}^{\overline{T}-1}\ln\left(b_i\left(\frac{(1-\tau_{t+T+1}^d)\cdot d_{kt+T+1}}{(1-\tau_{t+T}^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right) \forall\left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}.$$

Hence,

$$\ln\left(b_i^{\overline{T}}\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right) = \sum_{T=0}^{\overline{T}-1}\ln\left(b_i\left(\frac{(1-\tau_{t+T+1}^d)\cdot d_{kt+T+1}}{(1-\tau_{t+T}^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right)$$

$\forall\left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}.$
Clearly,

$$E\left[\ln\left(b_i^{\overline{T}}\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right)\right] =$$

$$\sum_{T=0}^{\overline{T}-1}E\left[\ln\left(b_i\left(\frac{(1-\tau_{t+T+1}^d)\cdot d_{kt+T+1}}{(1-\tau_{t+T}^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right)\right] = \overline{T}\cdot\mu_c$$

$\forall\left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}$
and, since by assumption of the Theorem

$$COV\left[\ln\left(c_{it+T_1}\right),\ \ln\left(c_{it+T_2}\right)\right] = 0 \ \forall\left(\ T_1,\ T_2\right) \in \{1,...,\infty\} \times \{1,...,\infty\},$$
$$T_1 \neq T_2.$$

we have that

$$VAR\left[\ln\left(b_i^{\overline{T}}\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right)\right] =$$

$$\sum_{T=0}^{\overline{T}-1}VAR\left[\ln\left(b_i\left(\frac{(1-\tau_{t+T+1}^d)\cdot d_{kt+T+1}}{(1-\tau_{t+T}^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right)\right] = \overline{T}\cdot\sigma_c^2$$

$\forall\left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}.$
Therefore,

$$\ln\left(b_i^T\left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right) \sim N(\overline{T}\cdot\mu_c, \sqrt{\overline{T}}\cdot\sigma_c)$$

$\forall \left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}$.

Fix an arbitrary $\left(k,\ i,\ \overline{T}\right) \in K \times I \times \{0,...,\infty\}$. Let

$$x = \ln\left(b_i^{\overline{T}} \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right).$$

Using ★ from the previous Lemma, we obtain

$$E\left[b_i^{\overline{T}} \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right] = E\left[e^x\right] = e^{\mu_x + \frac{\sigma_x^2}{2}} = e^{\overline{T}\cdot\mu_c + \frac{1}{2}\overline{T}\cdot\sigma_c^2}$$

$\forall \left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}$.

Thus,

$$E\left[b_i^{\overline{T}} \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+\overline{T}}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right] = e^{\overline{T}\cdot\mu_c + \frac{1}{2}\overline{T}\cdot\sigma_c^2} \ \ \forall \left(k,\ i,\ \overline{T}\right) \in K \times I \times \{1,...,\infty\}.$$

Hence, summing over $\forall T \in \{1,...,\infty\}$, we obtain

$$E\left[\sum_{T=1}^{\infty} b_i^{T} \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+T}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right] = \sum_{T=1}^{\infty} E\left[b_i^{T} \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+T}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right] =$$

$$= \sum_{T=1}^{\infty} e^{T\cdot\mu_c + \frac{1}{2}T\cdot\sigma_c^2} \ \forall \left(k,\ i\right) \in K \times I.$$

Taking into consideration that by assumption

$$E\left[b_i \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+T+1}}{(1-\tau_t^d)\cdot d_{kt+T}}\right)^{1-\lambda_i}\right] = e^{\mu_c + \frac{1}{2}\cdot\sigma_c^2} < 1$$

$\forall \left(k,\ i,\ T\right) \in K \times I \times \{0,...,\infty\}$ and summing over $\forall T \in \{1,...,\infty\}$, we obtain

$$\sum_{T=1}^{\infty} e^{T\cdot\mu_c + \frac{1}{2}T\cdot\sigma_c^2} = \frac{e^{\mu_c + \frac{1}{2}\sigma_c^2}}{1-e^{\mu_c + \frac{1}{2}\sigma_c^2}}.$$

Therefore,

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^{T} \left(\frac{(1-\tau_{t+T}^d)\cdot d_{kt+T}}{(1-\tau_t^d)\cdot d_{kt}}\right)^{1-\lambda_i}\right] \cdot (1-\tau_t^d) \cdot d_{kt} = \frac{e^{\mu_c + \frac{1}{2}\sigma_c^2}}{1-e^{\mu_c + \frac{1}{2}\sigma_c^2}} \cdot (1-\tau_t^d) \cdot d_{kt}.$$

So

$$p_{kt} = \frac{e^{\mu_c + \frac{1}{2}\sigma_c^2}}{1 - e^{\mu_c + \frac{1}{2}\sigma_c^2}} \cdot (1 - \tau_t^d) \cdot d_{kt} \ \forall k \in K. \quad \blacksquare$$

PROOF OF THEOREM 2: We have that

$$c_{it+T} =$$

$$\sum_{k=1}^{n} (p_{kt+T} + (1 - \tau_{t+T}^d)d_{kt+T})z_{ikt+T} - \sum_{k=1}^{n} p_{kt+T}z_{ikt+T+1} - \left(\frac{M_{it+T+1} - M_{it+T}}{p_{t+T}}\right)$$

$\forall (i, \ T) \in I \times \{0, ..., \infty\}$.
Substituting budget constraint into the utility function, we obtain

$$U_i(c_i, \ G) =$$

$$E\left[\sum_{T=0}^{\infty} b_i^T \left(u_i(\underbrace{\sum_{k=1}^{n} z_{ikt+T}\left(p_{kt+T} + (1 - \tau_{t+T}^d)d_{kt+T}\right) - \sum_{k=1}^{n} z_{ikt+T+1}p_{kt+T} - \left(\frac{M_{it+T+1} - M_{it+T}}{p_{t+T}}\right)}_{c_{it+T}}) + v_i\right.$$

Differentiating U_i with respect to z_{ikt+1} and setting $\frac{\partial U_i}{\partial z_{ikt+1}}$ to 0, we obtain

$$-p_{kt} \cdot u_i'(c_{it}) + b_i \cdot E\left[u_i'(c_{it+1})\left(p_{kt+1} + (1 - \tau_{t+1}^d)d_{kt+1}\right)\right] = 0$$

$\forall (k, \ i) \in K \times I$.
Therefore,

$$p_{kt} = E\left[b_i \left(\frac{c_{ti+T}}{c_{it}}\right)^{-\lambda_i}\left(p_{kt+1} + (1 - \tau_{t+1}^d) \cdot d_{kt+1}\right)\right] \ \forall (k, \ i) \in K \times I.$$

By repeated substitution and using the Transversality Condition, we get

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^T \left(\frac{c_{ti+T}}{c_{it}}\right)^{-\lambda_i}(1 - \tau_{t+T}^d) \cdot d_{kt+T}\right] \ \forall (k, \ i) \in K \times I. \quad (2)$$

By assumption of the Theorem, we have that

$$\frac{c_{ti+T}}{c_{it}} = \frac{(1 - \tau_{t+T}^d) \cdot \sum_{k=1}^{n} d_{kt+T} - \left[\frac{M_{i+T+1} - M_{t+T}}{p_{t+T}}\right]}{(1 - \tau_t^d) \cdot \sum_{k=1}^{n} d_{kt} - \left[\frac{M_{t+1} - M_t}{p_t}\right]} \ \forall (i, \ T) \in I \times \{0, ..., \infty\}.$$

Also, by assumption of the Theorem, we have that

$$\frac{\overbrace{\tau^d_{t+T} \cdot \sum_{k=1}^{n} d_{kt+T} + \underbrace{\left(\frac{M_{t+T+1} - M_{t+T}}{p_{t+T}}\right)}_{Seigniorage}}^{Toyal\ Government\ Revenue}}{\underbrace{\sum_{k=1}^{n} d_{kt+T}}_{Total\ Output}} = \overline{g}\ \forall T \in \{1, ..., \infty\}.$$

Therefore,

$$\frac{c_{ti+T}}{c_{it}} = \frac{(1-\tau^d_{t+T}) \cdot \sum_{k=1}^{n} d_{kt+T} - \left[\frac{M_{i+T+1} - M_{t+T}}{p_{t+T}}\right]}{(1-\tau^d_{t}) \cdot \sum_{k=1}^{n} d_{kt} - \left[\frac{M_{t+1} - M_t}{p_t}\right]} = \frac{\sum_{k=1}^{n} d_{kt+T} - \overline{g} \cdot \sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt} - \overline{g} \cdot \sum_{k=1}^{n} d_{kt}} = \frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}}$$

$\forall (i,\ T) \in I \times \{0, ..., \infty\}.$
In addition, by assumption of the Theorem, we have that

$$\frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}} = \frac{d_{kt+T}}{d_{kt}}\ \forall k \in K.$$

Hence,

$$\frac{c_{ti+T}}{c_{it}} = \frac{d_{kt+T}}{d_{kt}}\ \forall (k,\ i) \in K \times I.$$

Substituting the expression for $\frac{c_{ti+T}}{c_{it}}$ into the previous equation (2), we obtain

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^T \left(\frac{d_{kt+T}}{d_{kt}}\right)^{-\lambda_i} \cdot (1 - \tau^d_{t+T}) \cdot d_{kt+T}\right]\ \forall (k,\ i) \in K \times I.$$

Therefore,

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b_i^T \underbrace{\left(\frac{d_{kt+T}}{d_{kt}}\right)^{1-\lambda_i}}_{e^{y_{it+T}}} \cdot \underbrace{(1 - \tau^d_{t+T})}_{e^{x_{t+T}}}\right] \cdot d_{kt}\ \forall (k,\ i) \in K \times I.$$

Then

$$p_{kt} = E\left[\sum_{T=1}^{\infty} \underbrace{b_i^T \left(\frac{d_{kt+T}}{d_{kt}}\right)^{1-\lambda_i}}_{e^{y_{it+T}}} \cdot \underbrace{(1 - \tau_{t+T}^d)}_{e^{x_{t+T}}}\right] \cdot d_{kt}$$

$$= \sum_{T=1}^{\infty} E\left[\underbrace{b_i^T \left(\frac{d_{kt+T}}{d_{kt}}\right)^{1-\lambda_i}}_{e^{y_{it+T}}} \cdot \underbrace{(1 - \tau_{t+T}^d)}_{e^{x_{t+T}}}\right] \cdot d_{kt} \; \forall \, (k, \; i) \in K \times I$$

and so

$$p_{kt} = \sum_{T=1}^{\infty} E\left[\underbrace{b_i^T \left(\frac{d_{kt+T}}{d_{kt}}\right)^{1-\lambda_i}}_{e^{y_{it+T}}} \cdot \underbrace{(1 - \tau_{t+T}^d)}_{e^{x_{t+T}}}\right] \cdot d_{kt} \; \forall \, (k, \; i) \in K \times I.$$

We also know from the previous Lemma that if random variables x_{t+T} and $y_{y_{it+T}}$ are bivariate normally distributed with expectations

$$(E\left[\underbrace{\ln(1 - \tau_{t+T})}_{x_{t+T}}\right], \; E\left[\underbrace{\ln(b_i(\frac{c_{ti+T}}{c_{it}})^{1-\lambda_i})}_{y_{y_{it+T}}}\right]) = (\mu_\tau, \; \mu_c)$$

and the variance-covariance matrix

$$V = \begin{pmatrix} \sigma_\tau^2 & 0 \\ 0 & \sigma_c^2 \end{pmatrix},$$

then by ★★★ we obtain

$$E\left[\underbrace{b_i^T \left(\frac{d_{kt+T}}{d_{kt}}\right)^{1-\lambda_i}}_{e^{y_{it+T}}} \cdot \underbrace{(1 - \tau_{t+T}^d)}_{e^{x_{t+T}}}\right] = \left[e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2}\right] \cdot \left[e^{T \cdot \mu_c + \frac{1}{2} T \cdot \sigma_c^2}\right]$$

$\forall\,(k,\ i,\ T) \in K \times I \times \{0,...,\infty\}$.
Therefore,

$$p_{kt} = \sum_{T=1}^{\infty} E \left[\underbrace{b_i^T \left(\frac{d_{kt+T}}{d_{kt}} \right)^{1-\lambda_i}}_{e^{y_{it+T}}} \cdot \underbrace{(1 - \tau_{t+T}^d)}_{e^{x_{t+T}}} \right] \cdot d_{kt} =$$

$$= e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2} \cdot \sum_{T=1}^{\infty} \left[e^{T \cdot \mu_c + \frac{1}{2}T \cdot \sigma_c^2} \right] \cdot d_{kt} \ \forall\,(k,\ i) \in K \times I.$$

Hence,

$$p_{kt} = e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2} \cdot \left[\sum_{T=1}^{\infty} e^{T \cdot \mu_c + \frac{1}{2}T \cdot \sigma_c^2} \right] \cdot d_{kt} \ \forall k \in K.$$

Taking into consideration that by assumption

$$E \left[b_i \left(\frac{d_{kt+T+1}}{d_{kt+T}} \right)^{1-\lambda_i} \right] = e^{\mu_c + \frac{1}{2} \cdot \sigma_c^2} < 1$$

$\forall\,(k,\ i,\ T) \in K \times I \times \{0,...,\infty\}$ and summing over $\forall T \in \{1,...,\infty\}$, we obtain

$$\sum_{T=1}^{\infty} e^{T \cdot \mu_c + \frac{1}{2}T \cdot \sigma_c^2} = \frac{e^{\mu_c + \frac{1}{2}\sigma_c^2}}{1 - e^{\mu_c + \frac{1}{2}\sigma_c^2}}.$$

Therefore,

$$\boxed{p_{kt} = e^{\mu_\tau + \frac{1}{2}\sigma_\tau^2} \cdot \frac{e^{\mu_c + \frac{1}{2}\sigma_c^2}}{1 - e^{\mu_c + \frac{1}{2}\sigma_c^2}} \cdot d_{kt} \ \forall k \in K.} \quad \blacksquare$$

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