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Stochastic Taxation

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# Generic Existence of Equilibria in Finite Horizon Finance Economies with Stochastic Taxation\*

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## Abstract

The paper proves the existence of equilibria in the finite horizon general equilibrium with incomplete markets (GEI) model with insecure property rights. Insecure property rights come in the form of the stochastic taxes imposed on agents' endowments and assets' dividends. This paper finds that under reasonable assumptions, Financial Markets (FM) equilibria exist for most of the stochastic tax rates. Moreover, sufficiently small changes in stochastic taxation preserve the existence and completeness of FM equilibria.

**Keywords:** Stochastic Taxation, GEI, Complete Markets, CCAPM, Property Rights

**JEL Classification:** D5; D9; E13; G12; H20.

## 1. INTRODUCTION

Individuals and corporations are constantly engaged in highly sophisticated legal and illegal tax optimization or even tax evasion. Therefore, taxes

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clearly affect investor-consumer and corporate decision-making process. Also, there is no question that various US tax rates, especially income tax rates, have changed dramatically over the century since these taxes were first imposed, and the direction of those changes seems to have been anything but predictable. Thus, it seems entirely appropriate to regard future taxation as stochastic.

But if taxation is stochastic, then it is clearly a risk factor affecting the value of firms, or the value shareholders derive from dividends. Since this risk cannot be eliminated or substantially reduced by diversification, standard finance theory suggests that it ought to be an asset-pricing risk factor, which ought to affect asset prices and allocations.

The paper proves the existence of equilibria in the finite horizon GEI model with insecure property rights. Insecure property rights come in the form of the stochastic taxes imposed on agents' endowments and assets' dividends.

Surprisingly, there has been very little research done to date on the effects of stochastic taxes on equilibrium asset prices and allocations. The research done so far relies on the CCAPM with identical agents and twice-differentiable utility functions and focuses primarily on resolving the so-called "equity premium puzzle."

Magin (2015) studies the effects of insecure property rights on equilibrium asset prices and allocations in the finite horizon GEI model. Insecure property rights come in the form of the stochastic taxes imposed on the agents' endowments and assets' dividends. The first major finding of the paper is that under reasonable assumptions, an increase in the current dividend tax rate unambiguously reduces current asset prices. The second major finding of the paper is that for a utility function with a low coefficient of relative risk aversion,  $rr < 1$ , an increase in future dividend taxes reduces current prices of tradable assets. At the same time, surprisingly, for a utility function with a high coefficient of relative risk aversion,  $rr > 1$ , an increase in future dividend taxes boosts current prices of tradable assets. Finally, for a utility function with a coefficient of relative risk aversion,  $rr = 1$ , an increase in future dividend taxes leaves current consumption and current prices of tradable assets unchanged. Also, under reasonable assumptions, an increase in the current endowment tax rate unambiguously reduces current asset prices, while an increase in future endowment tax rate boosts current asset prices.

Magin (2014) develops a version of the CCAPM with insecure property rights. Insecure property rights are modeled by introducing a stochastic tax

on the wealth of shareholders, where the after-tax total rate of return on stocks and future consumption are bivariate lognormally distributed. He finds that the current expected equity premium, calculated by Fama and French, using the dividend growth model, can be reconciled with a coefficient of relative risk aversion of 3.76, thus resolving a substantial part of the equity premium puzzle.

Edelstein and Magin (2013) examined and estimated the equity risk premium for securitized real estate (U.S. Real Estate Investment Trusts-REITs). By introducing stochastic taxes for equity REITs shareholders, the analysis demonstrates that the current expected after-tax risk premium for REITs generate a reasonable coefficient of relative risk aversion. Employing a range of plausible stochastic tax burdens, the REITs shareholders' coefficient of relative risk aversion is likely to fall within the interval from 4.3 to 6.3, a value significantly lower than those reported in most of the prior studies.

DeLong and Magin (2009) point to social-democratic political risks, such as heavy taxes on corporate profits or heavy regulatory burdens on corporations that could contribute to the size of the equity risk premium.

Sialm (2009) is an excellent empirical paper showing that aggregate stock valuation levels are related to measures of the aggregate personal tax burden on equity securities. The tax burden is calculated as the ratio of dividend tax per share and taxes on short-term and long-term capital gains per share realized in accordance with historical patterns. That is tax yield calculated. Moreover, the paper finds that stocks paying a greater proportion of their total returns as dividends face significantly heavier tax burdens than stocks paying no dividends. The paper concludes that these results indicate that there is an economically and statistically significant relation between before-tax abnormal asset returns and effective tax rates. Stocks with heavier tax burdens tend to compensate taxable investors by offering higher before-tax returns.

Sialm (2006) points out that personal income tax rates have fluctuated considerably since federal income taxes were permanently introduced in the U.S. in 1913. He develops a generalized version of the Lucas (1978) dynamic general equilibrium tree model of production economy (risky assets being in positive supply) with identical agents and a flat consumption tax that follows a two-state Markov chain. The model is used to analyze the effects of a flat consumption tax on asset prices. He finds that stochastic consumption taxation affects the after-tax returns of risky and safe assets alike. As taxes change, equilibrium bond and stock prices adjust accordingly. However, stock

and long-term bond prices are affected more than T-bills. Under plausible conditions, investors require higher term and equity premia as compensation for the risk introduced by tax changes.

McGrattan and Prescott (2005) developed a dynamic general equilibrium model of production economy to analyze the effects of corporate and personal income taxes on asset prices but these taxes are not stochastic. They find that with the large reduction in individual income tax rates, the increased opportunities to hold equity in nontaxed pension plans, and the increases in intangible and foreign capital, theory correctly predicts a large increase in equity prices between 1962 and 2000, a doubling of the value of equity relative to GDP and a doubling of the price-earnings ratio. They conclude that a corollary of this finding is that there is no equity premium puzzle in the postwar period. However, their paper does not calculate the implied coefficient of relative risk aversion.

While resolving the equity premium puzzle is critically important for confirming the validity of the Lucas-Rubenstein CCAPM with identical agents, the role of insecure property rights (stochastic taxation) in economic theory is much broader. Do Financial Markets (FM) equilibria exist for most of the stochastic tax rates? Do sufficiently small changes in stochastic taxation preserve the existence and completeness of FM equilibria.

This paper finds that under reasonable assumptions, FM equilibria exist for all stochastic tax rates, except for a closed set of measure zero. Also, for any fixed stochastic endowment tax rate, complete FM equilibria exist for all stochastic dividend tax rates, except for a closed set of measure zero. Similarly, for any fixed stochastic dividend tax rate such that the after-tax dividend stream is potentially complete, FM equilibria exist for all stochastic endowment tax rates, except for a closed set of measure zero. Moreover, sufficiently small changes in stochastic taxation preserve the existence and completeness of FM equilibria?

The paper is organized as follows. Section 2 proves generic existence of equilibria in finite horizon FM economies with stochastic taxation of endowments and dividends. Section 3 concludes.

## 2. FINITE HORIZON FM ECONOMIES WITH STOCHASTIC TAXATION OF DIVIDENDS AND ENDOWMENTS

### 2.1. Definitions

First, we need to introduce several basic notions to define finite horizon FM Economies with stochastic taxation of endowments and dividends.

**DEFINITION:** Let  $(\Omega, \Sigma, \mu)$  be a probability measure space and  $\mathbb{T} = \{0, \dots, \bar{T}\}$ . Consider a family  $\mathbb{F}$  of partitions of  $\Omega$  given by

$$\mathbb{F} = \{\mathcal{F}_{t+T} \mid \mathcal{F}_{t+T} \subset \mathcal{P}(\Omega), T \in \mathbb{T}\},$$

where

$$\begin{cases} \mathcal{F}_t = \Omega, \\ \mathcal{F}_{t+\bar{T}} = \{\{\omega\} \mid \omega \in \Omega\}. \end{cases}$$

Then we say that the partition  $\mathcal{F}_{t+T+1}$  is finer than the partition  $\mathcal{F}_{t+T}$  if

$$\begin{cases} \sigma' \in \mathcal{F}_{t+T} \\ \sigma \in \mathcal{F}_{t+T+1} \end{cases} \implies \begin{matrix} \sigma \subset \sigma' \\ \text{or} \\ \sigma \cap \sigma' = \emptyset \end{matrix}.$$

**DEFINITION:** Consider a family  $\mathbb{F}$  of partitions of  $\Omega$  given by

$$\mathbb{F} = \{\mathcal{F}_{t+T} \mid \mathcal{F}_{t+T} \subset \mathcal{P}(\Omega), T \in \mathbb{T}\},$$

where

$$\begin{cases} \mathcal{F}_t = \Omega, \\ \mathcal{F}_{t+\bar{T}} = \{\{\omega\} \mid \omega \in \Omega\}. \end{cases}$$

Assume that the partition  $\mathcal{F}_{t+T+1}$  is finer than the partition  $\mathcal{F}_{t+T} \forall T \in \mathbb{T}$ . Then we define the event-tree  $ET$  as

$$ET = \{\xi = (t + T, \sigma) \mid T \in \mathbb{T}, \sigma \in \mathcal{F}_{t+T}\}.$$

**DEFINITION:** Let  $ET$  be an event-tree. Then we define function

$$T : ET \longrightarrow \mathbb{N}_+$$

as

$$T(\xi) = T,$$

where

$$\xi = (t + T, \sigma) \in ET.$$

**DEFINITION:** Let  $ET$  be an event-tree. Then we define the initial node  $\xi_0$  of  $ET$  as

$$\xi_0 = (t, \sigma),$$

where  $\sigma = \Omega$ .

**DEFINITION:** Let  $ET$  be an event-tree. If  $\xi_0 \in ET$  is the initial node of  $ET$ , then we define the set  $ET^+$  of non-initial nodes of  $ET$  as

$$ET^+ = ET \setminus \{\xi_0\}.$$

**DEFINITION:** Let  $ET$  be an event-tree. Then we define a terminal node  $\xi_{t+\bar{T}}$  of  $ET$  as

$$\xi_{t+\bar{T}} = (t + \bar{T}, \sigma),$$

where  $\sigma \in \mathcal{F}_{t+\bar{T}}$ .

**DEFINITION:** Let  $ET$  be an event-tree. Then we define the set  $ET_{t+\bar{T}}$  of all terminal nodes of  $ET$  as

$$ET_{t+\bar{T}} = \{(t + \bar{T}, \sigma) \mid \sigma \in \mathcal{F}_{t+\bar{T}}\}.$$
<sup>1</sup>

**DEFINITION:** Let  $ET$  be an event-tree. Then we define the set  $ET^-$  of all non-terminal nodes of  $ET$  as

$$ET^- = ET \setminus ET_{t+\bar{T}}.$$
<sup>2</sup>

**DEFINITION:** Let  $ET$  be an event-tree. Then  $\forall \xi \in ET^-$  such that  $\xi = (t + T, \sigma)$  we define the set of all immediate successors of  $\xi$  as

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<sup>1</sup>If  $|ET| = \infty$ , then  $ET_{t+\bar{T}} = \emptyset$ .

<sup>2</sup>If  $|ET| = \infty$ , then  $ET^- = ET$ .

$$\begin{aligned}\xi^+ &= \{\xi' \in ET \mid \xi' = (t + T(\xi) + 1, \sigma'), \sigma' \subset \sigma\} \\ &= \{\xi' \in ET \mid \xi' = (t + T + 1, \sigma'), \sigma' \subset \sigma\}.\end{aligned}$$

**DEFINITION:** Let  $ET$  be an event-tree. Then  $\forall \xi \in ET^-$  define the number of all immediate successors of  $\xi$  called the branching number at  $\xi$  as

$$b(\xi) = |\xi^+|.$$

**DEFINITION:** Let  $ET$  be an event-tree. Then we define a binary relation  $\geq$  on  $ET$  as follows. Suppose that

$$\begin{cases} \xi = (t + T, \sigma), \\ \xi' = (t + T', \sigma'). \end{cases}$$

Then

$$\xi' \geq \xi \iff \begin{cases} T(\xi') \geq T(\xi), \\ \sigma' \subset \sigma. \end{cases} \iff \begin{cases} T' \geq T, \\ \sigma' \subset \sigma. \end{cases}$$

We say that  $\xi'$  succeeds  $\xi$ .

**DEFINITION:** Let  $ET$  be an event tree. Then  $\forall \xi \in ET$  define the subtree  $ET(\xi)$  starting at  $\xi$  or the set of all successors of  $\xi$  as

$$ET(\xi) = \{\xi' \in ET \mid \xi' \geq \xi\}.$$

**DEFINITION:** Let  $ET$  be an event tree. Then we define a binary relation  $>$  on  $ET$  as follows. Suppose that

$$\begin{cases} \xi = (t + T, \sigma), \\ \xi' = (t + T', \sigma'). \end{cases}$$

Then

$$\xi' > \xi \iff \begin{cases} T(\xi') > T(\xi), \\ \sigma' \subset \sigma. \end{cases} \iff \begin{cases} T' > T, \\ \sigma' \subset \sigma. \end{cases}$$

We say that  $\xi'$  strictly succeeds  $\xi$ .

Clearly,

$$\xi' > \xi \iff \begin{cases} \xi' \geq \xi, \\ \xi' \neq \xi. \end{cases}$$



**DEFINITION:** Let  $ET$  be an event-tree. Then  $\forall \xi \in ET$  define the set of all strict successors of  $\xi$  as

$$ET^+(\xi) = \{\xi' \in ET(\xi) \mid \xi' > \xi\}.$$

**DEFINITION:** Let  $ET$  be an event-tree. Then  $\forall \xi \in ET$  define the set of all non-terminal successors of  $\xi$  as

$$ET^-(\xi) = \{\xi' \in ET(\xi) \mid \xi' \in ET^-\}.$$

Suppose there is an event-tree  $ET$  and a set  $I$  of finitely living investors-consumers who trade a set  $L$  of commodities on spot markets and a set  $K$  of assets on financial markets.

**DEFINITION:** We define the before-tax individual endowment  $e_i$  of agent  $i \in I$  as

$$e_i = \{e_i(\xi, l)\}_{(\xi, l) \in ET \times L} \in \mathbb{R}_+^{|ET \times L|}.$$

Set also

$$e_i(\xi) = \{e_i(\xi, l)\}_{l \in L} \in \mathbb{R}_+^{|L|} \quad \forall (\xi, i) \in ET \times I.$$

**DEFINITION:** We define the matrix of before-tax individual endowments  $e$  as

$$e = \{e_i\}_{i \in I} \in \mathbb{R}_+^{|ET \times L \times I|}.$$

**DEFINITION:** We define the consumption of agent  $i \in I$  as

$$c_i = \{c_i(\xi, l)\}_{(\xi, l) \in ET \times L} \in \mathbb{R}_+^{|ET \times L|}.$$

Set also

$$c_i(\xi) = \{c_i(\xi, l)\}_{l \in L} \in \mathbb{R}_+^{|L|} \quad \forall (\xi, i) \in ET \times I.$$

**DEFINITION:** We define the vector of spot prices as

$$p = \{p(\xi, l)\}_{(\xi, l) \in ET \times L} \in \mathbb{R}^{|ET \times L|}$$

such that

$$p(\xi, 1) = 1 \quad \forall \xi \in ET.$$

Set also

$$p(\xi) = \{p(\xi, l)\}_{l \in L} \in \mathbb{R}^{|L|} \quad \forall \xi \in ET.$$

We are now ready to discuss the asset structure of our model.

**DEFINITION:** Let  $\xi(k) \in ET$  be the node of issue for an asset  $k \in K$ . Define the set  $\zeta$  of all nodes of issue of existing financial contracts

$$\zeta = \{\xi(k) \mid k \in K\}.$$

**DEFINITION:** We define the matrix of before-tax dividends  $d$  paid in units of good 1 as

$$d = \{d(\xi, k)\}_{(\xi, k) \in ET \times K} \in \mathbb{R}^{|ET \times K|},$$

where

$$d(\xi, k) = 0 \quad \forall \xi \in ET \setminus ET^+(\xi(k)) \quad \forall k \in K,$$

i.e., an asset  $k \in K$  issued at node  $\xi(k) \in ET$  pays no dividends prior to or at node  $\xi(k) \in ET$ .<sup>3</sup>

Set also

$$d(\xi) = \{d(\xi, k)\}_{k \in K} \in \mathbb{R}^{|K|} \quad \forall \xi \in ET.$$

**DEFINITION:** We define the space of asset dividends as

$$\mathcal{D} = \mathbb{R}^{|ET \times K|}.$$

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<sup>3</sup>The model could be generalized for the case, where dividends are paid in bundles of all  $|L|$  goods, not just in units of good 1. See Duffie and Shafer (1986), for example. In that case  $d = \{d(\xi, k, l)\}_{(\xi, k, l) \in ET \times K \times L} \in \mathbb{R}^{|ET \times K \times L|}$ .

**DEFINITION:** We define the set of all actively traded financial contracts  $K(\xi) \subset K$  at node  $\xi \in ET$  as

$$K(\xi) = \underbrace{\{k \in K \mid \xi \in ET(\xi(k))\}}_{(1)}, \underbrace{\{\exists \xi' \in ET^+(\xi) \text{ s.t. } d(\tau_d(\xi', k)) \neq 0\}}_{(2)} \subset K.$$

That is, the set of financial contracts which are actively traded at node  $\xi \in ET$  consists of those financial contracts which (1) have been issued prior to or at node  $\xi \in ET$  and (2) whose expiration node strictly follows node  $\xi \in ET$ , since there is no point in trading a security which never yields a dividend.

**DEFINITION:** We define the matrix of asset prices as

$$q = \{q(\xi, k)\}_{(\xi, k) \in ET \times K} \in \mathbb{R}^{|ET \times K|},$$

where

$$q(\xi, k) = 0 \quad \forall \xi \in ET \text{ s.t. } k \in K \setminus K(\xi), \quad \forall k \in K,$$

i.e.,  $q(\xi, k) = 0$  if an asset  $k \in K$  is not actively traded at node  $\xi \in ET$ .<sup>4</sup>

Set also

$$q(\xi) = \{q(\xi, k)\}_{k \in K} \in \mathbb{R}^{|K|},$$

**DEFINITION:** We define the space of asset prices as

$$\mathcal{Q} = \mathbb{R}^{|ET \times K|}.$$

**DEFINITION:** We define an asset portfolio  $z_i$  held by agent  $i \in I$  as follows

$$z_i = \{z_i(\xi, k)\}_{(\xi, k) \in ET \times K} \in \mathbb{R}^{|ET \times K|},$$

where

$$z_i(\xi, k) = 0 \quad \forall \xi \in ET \text{ s.t. } k \in K \setminus K(\xi), \quad \forall k \in K,$$

i.e.,  $z_i(\xi, k) = 0$  if an asset  $k \in K$  is not actively traded at node  $\xi \in ET$ .<sup>4</sup>

Set also

$$z_i(\xi) = \{z_i(\xi, k)\}_{k \in K} \in \mathbb{R}^{|K|}.$$

**DEFINITION:** We define the portfolio space as

$$\mathcal{Z} = \mathbb{R}^{|ET \times K|}.$$

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<sup>4</sup>If  $|ET| < \infty$ , then  $ET_{t+\bar{T}} \neq \emptyset$  and  $K(\xi) = \emptyset \quad \forall \xi \in ET_{t+\bar{T}}$ , since there is no point in trading a security which will never yield a dividend.

We need to introduce now several definitions to incorporate stochastic taxation  $\tau = (\tau_e, \tau_d)$  imposed on agents' endowments and assets' dividends and used to finance government spending  $G$  into the General Equilibrium Theory of Financial Markets.

**DEFINITION:** We define the individual endowment tax  $\tau_{e_i}$  imposed on the individual endowment  $e_i$  of agent  $i \in I$  as

$$\tau_{e_i} = \{\tau_{e_i}(\xi, l)\}_{(\xi, l) \in ET \times L} \in [0, 1]^{|ET \times L|}.$$

Set also

$$\tau_{e_i}(\xi) = \{\tau_{e_i}(\xi, l)\}_{l \in L} \in [0, 1]^{|L|} \quad \forall (\xi, i) \in ET \times I.$$

**DEFINITION:** We define the matrix of individual endowment taxes  $\tau_e$  as

$$\tau_e = \{\tau_{e_i}\}_{i \in I} \in [0, 1]^{|ET \times L \times I|}.$$

It is reasonable to assume that the sizes of individual endowments  $e_i$  are decreasing functions  $e_i(\tau_{e_i})$  of individual endowment tax rates  $\tau_{e_i}$ .<sup>4</sup> Moreover, since future endowment tax rates are uncertain, it is reasonable to view the taxation of individual endowments as stochastic. So set

$$\begin{aligned} e_i &= e_i(\tau_{e_i}) \quad \forall i \in I, \\ e &= \{e_i(\tau_{e_i})\}_{i \in I}. \end{aligned}$$

**DEFINITION:** We define the after-tax individual endowment  $(1 - \tau_{e_i}) \cdot e_i(\tau_{e_i})$  of agent  $i \in I$  as

$$(1 - \tau_{e_i}) e_i(\tau_{e_i}) = \{(1 - \tau_{e_i}(\xi, l)) \cdot e_i(\tau_{e_i}(\xi, l))\}_{(\xi, l) \in ET \times L} \in \mathbb{R}_+^{|ET \times L|}.$$

Set also

$$(1 - \tau_{e_i}(\xi)) e_i(\tau_{e_i}(\xi)) = \{(1 - \tau_{e_i}(\xi, l)) \cdot e_i(\tau_{e_i}(\xi, l))\}_{l \in L} \in \mathbb{R}_+^{|L|} \quad \forall \xi \in ET.$$

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<sup>4</sup>See Kawano (2013), for example, for a review of the Dividend Clientele Hypothesis.

**DEFINITION:** We define the matrix of after-tax individual endowments  $(1 - \tau) e(\tau_e)$  as

$$(1 - \tau_e) e(\tau_e) = \{(1 - \tau_{e_i}) e_i(\tau_{e_i})\}_{i \in I} \in \mathbb{R}_+^{|ET \times L \times I|}.$$

**DEFINITION:** We define the dividend tax  $\tau_d$  imposed on assets' dividends  $d$  as

$$\tau_d = \{\tau_d(\xi, k)\}_{(\xi, k) \in ET \times K} \in [0, 1]^{|ET \times K|}.$$

Set also

$$\tau_d(\xi) = \{\tau_d(\xi, k)\}_{k \in K} \in [0, 1]^{|K|} \forall (\xi, k) \in ET \times K.$$

Consistent with the Dividend Clientele Hypothesis (DCH), it is reasonable to assume that assets' dividends are decreasing functions  $d(\tau_d)$  of dividend tax rates  $\tau_d$ .<sup>5</sup> Moreover, since future dividend tax rates are uncertain, it is reasonable to view the taxation of dividends as stochastic. So set

$$d(\xi, k) = d(\tau_d(\xi, k)) \forall (\xi, k) \in ET \times K.$$

**DEFINITION:** We define after-tax dividends as

$$(1 - \tau_d) d(\tau_d) = \{(1 - \tau_d(\xi, k)) \cdot d(\tau_d(\xi, k))\}_{(\xi, k) \in ET \times K} \in \mathbb{R}^{|ET \times K|}.$$

Set also

$$(1 - \tau_d(\xi)) d(\tau_d(\xi)) = \{(1 - \tau_d(\xi, k)) \cdot d(\tau_d(\xi, k))\}_{k \in K} \in \mathbb{R}^{|K|} \forall \xi \in ET.$$

**DEFINITION:** Let  $\zeta$  be the set of all nodes of issue of  $|K|$  existing financial contracts and  $d$  be the  $|ET \times K|$  matrix of dividends. Then we call the pair

$$\mathcal{A}(\tau_d) = (\zeta, (1 - \tau_d) \cdot d(\tau_d))$$

the financial structure.

We are now ready to define a finite horizon FM Economy with stochastic taxation.

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<sup>5</sup>See Kawano (2013), for example, for a review of the Dividend Clientele Hypothesis.

**DEFINITION:** We denote by  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  with

$$\mathcal{A}(\tau_d) = (\zeta, (1 - \tau_d) \cdot d(\tau_d))$$

a finite horizon FM Economy with stochastic taxation

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|},$$

where

$$|ET| < \infty, |I| < \infty, |L| < \infty, |K| < \infty,$$

agents' preferences  $\succeq_i$  are given by the utility function

$$U_i : \mathbb{R}_+^{|ET \times L|} \times \mathbb{R}_+^{|ET \times L|} \longrightarrow \mathbb{R}$$

such that

$$U_i(c_i, G) = \sum_{(\xi, l) \in ET \times L} \text{Pr}(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi, l)) + v_i(G(\xi, l))] \quad \forall i \in I,$$

where the government spending

$$G = \{G(\xi, l)\}_{(\xi, l) \in ET \times L} \in \mathbb{R}_+^{|ET \times L|}$$

is given by

$$\left\{ \begin{array}{l} G(\xi, l) = \\ \sum_{i \in I} \tau_{e_i}(\xi, l) \cdot e_i(\xi, l) \quad \forall (\xi, l) \in ET \times [L \setminus \{1\}] \\ \left[ \sum_{i \in I} \tau_{e_i}(\xi, l) \cdot e_i(\xi, l) + \sum_{k \in K} \bar{z}(k) \cdot \tau_d(\xi, k) \cdot d(\xi, k) \right] \quad \forall (\xi, l) \in ET \times \{1\} \end{array} \right.$$

and  $\bar{z}(k)$  is the total number of outstanding shares of asset  $k \in K$ .

Following Magill and Quinzii (1996), define matrix  $W(q(\tau), (1 - \tau_d) \cdot d(\tau_d))$

$$\forall (q(\tau), (1 - \tau_d) \cdot d(\tau_d)) \in Q \times \mathbb{R}^{|ET \times K|},$$

which will significantly simplify writing of agents' budget constraints.

**DEFINITION:** Let  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  be a finite horizon FM Economy with stochastic taxation

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}.$$

Define the  $ET \times ET$  payoff matrix matrix  $W(q(\tau), (1 - \tau_d) \cdot d(\tau_d))$

$$\forall (q(\tau), (1 - \tau_d) \cdot d(\tau_d)) \in Q \times \mathbb{R}^{|ET \times K|}$$

as

$$\begin{aligned} W_{\xi, \xi^+}(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) &= q(\xi^+) + (1 - \tau_d(\xi^+)) \cdot d(\tau_d(\xi^+)), \\ W_{\xi, \xi}(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) &= -q(\xi), \\ W_{\xi, \xi'}(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) &= 0 \quad \forall \xi' \notin \xi^+, \xi' \neq \xi. \end{aligned}$$

Matrix  $W(q(\tau), (1 - \tau_d) \cdot d(\tau_d))$

$ K $ Columns for $\xi_0$	$ K $ Columns for $\xi^-$	$ K $ Columns for $\xi$	$ K $ Columns for $\xi^+$
$-q(\xi_0, \tau)$	$0$	$0$	$0$
$q(\xi_0^+, \tau) + (1 - \tau_d(\xi_0^+)) \cdot d(\tau_d(\xi_0^+))$	$\dots$	$\dots$	$0$
$0$	$\dots$	$0$	$0$
$0$	$q(\xi, \tau) + (1 - \tau_d(\xi)) \cdot d(\tau_d(\xi))$	$-q(\xi, \tau)$	$0$
$0$	$\dots$	$0$	$\dots$
$0$	$0$	$q(\xi^+, \tau) + (1 - \tau_d(\xi^+)) \cdot d(\tau_d(\xi^+))$	$\dots$
$0$	$0$	$0$	$\dots$



**DEFINITION:** Let  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  be a finite horizon FM Economy with stochastic taxation

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}.$$

Then define the budget set of agent  $i \in I$  as follows

$$\begin{aligned} & B(p, q, (1 - \tau_{e_i}) \cdot e_i(\tau_{e_i}), \mathcal{A}(\tau_d)) = \\ & \left\{ c_i \in E_+ \mid \begin{array}{l} \exists z_i \in \mathcal{Z} \text{ such that } p(\xi) \cdot c_i(\xi) + q(\xi) \cdot z_i(\xi) \leq \\ \leq p(\xi) \cdot (1 - \tau_{e_i}(\xi)) \cdot e_i(\xi) + (q(\xi) + (1 - \tau_d(\xi)) \cdot d(\xi)) \cdot z_i(\xi^-) \quad \forall \xi \in ET \end{array} \right\} = \\ & = \\ & \{c_i \in E_+ \mid \exists z_i \in \mathcal{Z} \text{ such that } p \cdot c_i - p \cdot (1 - \tau_{e_i}) \cdot e_i(\tau_{e_i}) = W(q(\tau_e, \tau_d), (1 - \tau_d) \cdot d(\tau_d)) \cdot z_i\}. \end{aligned}$$

**DEFINITION:** An FM equilibrium for an FM economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  with stochastic taxation

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

is a pair

$$(\{\bar{c}_i(\tau), \bar{z}_i(\tau)\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \times \mathbb{R}_+^{|ET \times L|} \times Q$$

such that

$$\begin{aligned} & (\bar{c}_i(\tau), \bar{z}_i(\tau)) \in \\ & \in \arg \max \{U_i(c_i, G) \mid (c_i, z_i) \in B(\bar{p}(\tau), \bar{q}(\tau), (1 - \tau_{e_i}) \cdot e_i(\tau_{e_i}), \mathcal{A}(\tau_d))\} \\ & \quad \forall i \in I, \\ & \sum_{i \in I} \bar{c}_i(\tau) = \sum_{i \in I} (1 - \tau_{e_i}) \cdot e_i(\tau_e) \end{aligned}$$

and

$$\sum_{i \in I} \bar{z}_i(\tau) = 0.$$

**DEFINITION:** Let  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  be an FM economy with stochastic taxation

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}.$$

Given

$$(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) \in Q \times \mathbb{R}^{|ET \times K|},$$

we say that the No Arbitrage Condition (NAC) holds if

$$\text{rank}[W(q(\tau), (1 - \tau_d) \cdot d(\tau_d))] \leq |ET| - 1$$

or, equivalently, there exists a price vector

$$\pi(\tau) = \{\pi(\xi, \tau)\}_{\xi \in ET} \in \mathbb{R}_{++}^{|ET|}$$

such that

$$\pi(\tau) \cdot W(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) = 0,$$

i.e.,

$$q(\xi, \tau) = \sum_{\xi' \in ET^+(\xi)} \frac{\bar{\pi}(\xi', \tau)}{\bar{\pi}(\xi, \tau)} (1 - \tau_d(\xi')) \cdot d(\tau_d(\xi')) \quad \forall \xi \in ET.$$

**DEFINITION:** Let  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  be an FM economy with stochastic taxation

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}.$$

Given

$$(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) \in Q \times \mathbb{R}^{|ET \times K|},$$

the FM are complete if

$$\text{rank}[W(q(\tau), (1 - \tau_d) \cdot d(\tau_d))] = |ET| - 1$$

or, equivalently, there exists a unique price vector

$$\pi(\tau) = \{\pi(\xi, \tau)\}_{\xi \in ET} \in \mathbb{R}_{++}^{|ET|}$$

such that

$$\pi(\tau) \cdot W(q(\tau), (1 - \tau_d) \cdot d(\tau_d)) = 0,$$

i.e.,

$$q(\xi, \tau) = \sum_{\xi' \in ET^+(\xi)} \frac{\bar{\pi}(\xi', \tau)}{\bar{\pi}(\xi, \tau)} (1 - \tau_d(\xi')) \cdot d(\tau_d(\xi')) \quad \forall \xi \in ET.$$

## 2.2. Generic Existence and Completeness of FM Equilibria in Finite Horizon FM Economies with Stochastic Taxation of Dividends and Endowments

First, we are going to establish the existence of an FM equilibrium with short-lived securities. Our strategy here is to use separability of agents' utility functions and the fact that government spending is an exogenous variable.

**THEOREM 2.2.1. (Existence of an FM Equilibrium with Short-Lived Securities):** *Let  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  be a finite horizon FM Economy with stochastic taxation*

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

such that

1. The number of agents  $|I| < \infty$ ,
2. Individual endowments  $(1 - \tau_{e_i}) \cdot e_i(\tau_{e_i}) \in \mathbb{R}_{++}^{|ET \times L|} \forall i \in I$ ,
3. Agents' preferences  $\succeq_i$  on  $\mathbb{R}_+^{|ET \times L|} \times \mathbb{R}_+^{|ET \times L|}$  are given by the utility function

$$U_i(c_i, G) = \sum_{(\xi, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi, l)) + v_i(G(\xi, l))] \forall i \in I.$$

where  $u_i$  is continuous, strongly monotone and strictly quazi-concave  $\forall i \in I$ ,

4. Financial structure  $\mathcal{A}(\tau_d)$  is composed solely of Short-Lived Securities. Then the economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  has an FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right).$$

**PROOF:** See appendix.

Next, I am going to establish the existence of an FM equilibrium with long-lived securities. Our strategy here is again to use separability of agents' utility functions and the fact that government spending  $G$  is an exogenous variable. The main result here is that under reasonable assumptions, FM equilibria exist for all stochastic tax rates  $\tau_d$  and  $\tau_e$  except for a closed set of measure zero. Moreover, sufficiently small changes in stochastic tax rates  $\tau_d$  and  $\tau_e$  preserve the existence and completeness of FM equilibria.

We need several new definitions to establish the existence of an FM equilibrium with long-lived securities.

**DEFINITION (Subtree Inner product):** Given a vector of node prices

$$\pi = \{\pi(\xi) \mid \xi \in ET\}$$

and a vector of income

$$y = \{y(\xi) \mid \xi \in ET\},$$

we define

$$\pi \bullet_{\xi} y = \sum_{\xi' \in ET(\xi)} \pi(\xi') y(\xi') \quad \forall \xi \in ET.$$

**DEFINITION (Successor Box-Product):** Given a vector of node prices

$$\pi = \{\pi(\xi) \mid \xi \in ET\}$$

and a vector of income

$$y = \{y(\xi) \mid \xi \in ET\},$$

we define the successor box-product of  $\pi$  and  $y$  as

$$\pi \square_{\xi^+} y = \left\{ \pi \bullet_{\xi'} y \right\}_{\xi' \in \xi^+} \in \mathbb{R}^{b(\xi)}.$$

**DEFINITION:** A dividend stream  $d \in \mathbb{R}^{|ET \times K|}$  is potentially complete if there exists a vector of node prices

$$\pi = \{\pi(\xi) \mid \xi \in ET\} \in \mathbb{R}^{|ET|}$$

such that

$$\text{rank} \left[ \pi \square_{\xi^+} d \right] = b(\xi) \quad \forall \xi \in ET^-.$$

**DEFINITION:** Let  $U \subset \mathbb{R}^m$  be an open set and

$$f : U \longrightarrow \mathbb{R}^n$$

be such that

$$f \in C^r$$

and

$$r \geq 0.$$

Then  $f$  is a  $C^r$  diffeomorphism if it is a homeomorphism and  $f^{-1} \in C^r$ .

**DEFINITION:** Let  $U \subset \mathbb{R}^m$  be an open set and

$$f : U \longrightarrow \mathbb{R}^n$$

be such that

$$f \in C^r$$

and

$$r \geq 0.$$

Then  $f$  is a local  $C^r$  diffeomorphism at  $x \in X$  if  $\exists$  an open neighborhood  $O \subset U$  of  $x$  such that

$$f|_O : O \longrightarrow f(O)$$

is a  $C^r$  diffeomorphism.

**DEFINITION:** Let  $U \subset \mathbb{R}^m$  be an open set and

$$f : U \longrightarrow \mathbb{R}^n$$

be such that

$$f \in C^r$$

and

$$r \geq 0.$$

Then  $f$  is a local  $C^r$  diffeomorphism on  $X$  if  $f$  is a local  $C^r$  diffeomorphism  $\forall x \in X$ .

**DEFINITION:** Let  $U \subset \mathbb{R}^m$  be an open set and

$$f : U \longrightarrow \mathbb{R}^n$$

be such that

$$f \in C^1.$$

Then we define the set of critical points  $\mathbf{CP}_f$  of  $f$  as

$$\mathbf{CP}_f = \{x \in U \mid \det [Df(x)] = 0\}.$$

**DEFINITION:** Let  $U \subset \mathbb{R}^m$  be an open set and

$$f : U \longrightarrow \mathbb{R}^n$$

be such that

$$f \in C^1.$$

Then we define the set of critical values  $\mathbf{CV}_f$  of  $f$  as

$$\mathbf{CV}_f = \{y \in \mathbb{R}^n \mid y = f(x), x \in \mathbf{CP}_f\}.$$

**DEFINITION:** Let  $U \subset \mathbb{R}^m$  be an open set and

$$f : U \longrightarrow \mathbb{R}^n$$

be such that

$$f \in C^1.$$

Then we define the set of regular values  $\mathbf{RV}_f$  of  $f$  as

$$\mathbf{RV}_f = \mathbb{R}^n \setminus \mathbf{CV}_f.$$

**THEOREM 2.2.2. (Generic Existence and Completeness of an FM Equilibrium with Long-Lived Securities):** *Let  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  be a finite horizon FM economy with stochastic taxation*

$$\tau = (\tau_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

such that

1. The number of agents  $|I| < \infty$ ,
2. Agents' endowments  $(1 - \tau_e) \cdot e(\tau_e) \in \mathbb{R}_{++}^{|ET \times L \times I|} \forall \tau_e \in [0, 1]^{|ET \times L \times I|}$ ,
3. Agents' preferences  $\succeq_i$  on  $\mathbb{R}_+^{|ET \times L|} \times \mathbb{R}_+^{|ET \times L|}$  are given by the utility function

$$U_i(c_i, G) = \sum_{(\xi, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi, l)) + v_i(G(\xi, l))] \forall i \in I \text{ and}$$

- 3.1.  $U_i \in C^\infty \forall i \in I$ ,
- 3.2.  $DU_i(c) \gg 0 \forall c \in \mathbb{R}_+^{|ET \times L|} \forall i \in I$ ,
- 3.3.  $h' \cdot D^2 U_i(c) \cdot h < 0 \forall h \neq 0, h \cdot DU_i(c) = 0 \forall i \in I$ ,
- 3.4.  $\left\{ c \in \mathbb{R}_{++}^{|ET \times L|} \mid U_i(c) \geq U_i(\bar{c}) \right\}$  is closed in  $\mathbb{R}^{|ET \times L|} \forall (\bar{c}, i) \in \mathbb{R}_{++}^{|ET \times L|} \times I$ .

I.

A) Let  $K(\xi) \geq b(\xi) \forall \xi \in ET^-$ . Suppose function

$$f : [0, 1]^{|ET \times K|} \longrightarrow \mathbb{R}^{|ET \times K|}$$

defined as

$$f(\tau_d) = (1 - \tau_d) \cdot d(\tau_d)$$

be such that

$$\mathbf{T} = [0, 1]^{|ET \times K|} \setminus \mathbf{CP}_f$$

is an open set,  $\mathbf{CP}_f$  is the set of critical points of  $f$  and

$$m[\mathbf{T}] = m\left[[0, 1]^{|ET \times K|}\right] = 1.$$

Then for fixed  $\bar{\tau}_e \in [0, 1]^{|ET \times L \times I|}$  there exists an open set

$$O \subset [0, 1]^{|ET \times K|}$$

such that

$$m [O] = m \left[ [0, 1]^{|ET \times K|} \right] = 1$$

and  $\forall \tau_d \in O$  the economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  has an FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right),$$

where

$$\tau = (\bar{\tau}_e, \tau_d) \in [0, 1]^{|ET \times L \times I|} \times O$$

in which FM markets are complete.

B) Let  $K(\xi) \geq b(\xi) \forall \xi \in ET^-$ . Suppose function

$$f : [0, 1]^{|ET \times L \times I|} \longrightarrow \mathbb{R}_{++}^{|ET \times L \times I|}$$

defined as

$$f(\tau_e) = (1 - \tau_e) \cdot e(\tau_e)$$

be such that

$$\mathbf{T} = [0, 1]^{|ET \times L \times I|} \setminus \mathbf{CP}_f$$

is an open set,  $\mathbf{CP}_f$  is the set of critical points of  $f$  and

$$m [\mathbf{T}] = m \left[ [0, 1]^{|ET \times L \times I|} \right] = 1.$$

Then for fixed  $\bar{\tau}_d \in [0, 1]^{|ET \times K|}$  such that the dividend stream  $(1 - \bar{\tau}_d) \cdot d(\bar{\tau}_d)$  is potentially complete, there exists an open set

$$O \subset [0, 1]^{|ET \times L \times I|}$$

such that



$$m[O] = m\left[[0, 1]^{|ET \times L \times I|}\right] = 1$$

and  $\forall \tau_e \in O$  the economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  has an FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left(\mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}^{|ET \times L|} \times Q\right),$$

where

$$\tau = (\tau_e, \tau_d) \in O \times [0, 1]^{|ET \times K|}$$

in which markets are complete.

C) Let  $\exists \xi \in ET^-$  such that  $K(\xi) < b(\xi)$ . Suppose function

$$f : [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|} \longrightarrow \mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}$$

defined as

$$f(\tau_e, \tau_d) = ((1 - \tau_e) \cdot e(\tau_e), (1 - \tau_d) \cdot d(\tau_d))$$

be such that

$$\mathbf{T} = \left( [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|} \right) \setminus \mathbf{CP}_f$$

is an open set,  $\mathbf{CP}_f$  is the set of critical points of  $f$  and

$$m[\mathbf{T}] = m\left[[0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}\right] = 1.$$

Then there exists an open set

$$O \subset [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

such that

$$m[O] = m\left[[0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}\right] = 1$$

and

$$\forall \tau = (\tau_e, \tau_d) \in O$$

the economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$  has an FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right).$$

**PROOF:** See Appendix.

It turns out that under reasonable assumptions on asset prices  $q$  and dividends  $d$ , sufficiently small changes in stochastic taxation  $\tau = (\tau_e, \tau_d)$  preserve completeness of FM equilibria.

**THEOREM 2.2.3. (Local Completeness of FM Equilibria):** *Let*

$$(\{(\bar{c}_i(\bar{\tau}), \bar{z}_i(\bar{\tau}))\}_{i \in I}, (\bar{p}(\bar{\tau}), \bar{q}(\bar{\tau}))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}_+^{|ET \times L|} \times Q \right)$$

*be an FM equilibrium in which markets are complete for the FM economy  $\mathcal{E}(ET, (1 - \bar{\tau}_e) \cdot e(\bar{\tau}_e), \succeq, \mathcal{A}(\bar{\tau}_d))$  with stochastic taxation*

$$\bar{\tau} = (\bar{\tau}_e, \bar{\tau}_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|},$$

*i.e.,*

$$\text{rank}[W(\bar{q}(\bar{\tau}_e, \bar{\tau}_d), (1 - \bar{\tau}_d) \cdot d(\bar{\tau}_d))] = |ET| - 1.$$

*Let also*

$$\begin{aligned} \bar{q} &: [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|} \longrightarrow Q, \\ d &: [0, 1]^{|ET \times K|} \longrightarrow \mathbb{R}^{|ET \times K|} \end{aligned}$$

*be continuous functions of  $\tau = (\tau_e, \tau_d)$  and  $\tau_d$  respectively, such that NAC holds for all tax rates, i.e.,*

$$\begin{aligned} \text{rank}[W(\bar{q}(\tau), (1 - \tau_d) \cdot d(\tau_d))] &\leq |ET| - 1 \\ \forall \tau &\in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}. \end{aligned}$$

*Then there exists an open neighborhood*

$$O_{\bar{\tau}} \subset [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

*of  $\bar{\tau} = (\bar{\tau}_e, \bar{\tau}_d)$  such that*

$$\text{rank}[W(\bar{q}(\tau_e, \tau_d), (1 - \tau_d) \cdot d(\tau_d))] = |ET| - 1 \quad \forall \tau = (\tau_e, \tau_d) \in O_{\bar{\tau}},$$

*i.e.*, FM are complete  $\forall \tau = (\tau_e, \tau_d) \in O_{\bar{\tau}}$ .

**PROOF:** See appendix.

The above results show that under reasonable assumptions sufficiently small changes in stochastic taxation  $\tau$  preserve the existence and completeness of FM equilibria.

#### 4. CONCLUSION

The paper proves the existence of equilibria in the finite horizon GEI model with insecure property rights. Insecure property rights come in the form of the stochastic taxes imposed on agents' endowments and assets' dividends. The major finding of this paper is that under reasonable assumptions, FM equilibria exist for most of the stochastic tax rates. Also, for any fixed stochastic endowment tax rate, complete FM equilibria exist for most of the stochastic dividend tax rates. Similarly, for any fixed stochastic dividend tax rate such that the after-tax dividend stream is potentially complete, complete FM equilibria exist for most of the stochastic endowment tax rates. Moreover, sufficiently small changes in stochastic taxation preserve the existence and completeness of FM equilibria.

#### 5. APPENDIX

**PROOF OF THEOREM 2.2.1. (Existence of an FM Equilibrium with Short-Lived Securities):** Consider an economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq', \mathcal{A}(\tau_d))$  satisfying assumptions of the Theorem except that

3'. Agents' preferences  $\succeq'_i$  on  $\mathbb{R}_+^{|ET \times L|}$  are given by the utility function

$$U_i(c_i) = \sum_{(\xi, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot u_i(c_i(\xi, l)) \quad \forall i \in I.$$

where  $u_i$  is continuous, strongly monotone and strictly quasi-concave  $\forall i \in I$ . We can conclude by Proposition 25.1 of Magill and Quinzii (1996)<sup>6</sup> that the FM economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq', \mathcal{A}(\tau_d))$  has an FM equilibrium

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<sup>6</sup>See also Proposition 1 of Geanakoplos and Polemarchakis (1986).

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right).$$

Clearly, using separability of agents' utility functions and the fact that government spending  $G$  is an exogenous variable, we can conclude

$$\begin{aligned} & (\bar{c}_i(\tau), \bar{z}_i(\tau)) \in \\ & \in \arg \max \{U_i(c_i) \mid (c_i, z_i) \in B(\bar{p}(\tau), \bar{q}(\tau), (1-\tau) \cdot e_i(\tau), \mathcal{A}(\tau))\} = \\ & \arg \max \{U_i(c_i, G) \mid (c_i, z_i) \in B(\bar{p}(\tau), \bar{q}(\tau), (1-\tau) \cdot e_i(\tau), \mathcal{A}(\tau))\} \forall i \in I. \end{aligned}$$

So

$$\begin{aligned} & (\bar{c}_i(\tau), \bar{z}_i(\tau)) \in \\ & \in \arg \max \{U_i(c_i, G) \mid (c_i, z_i) \in B(\bar{p}(\tau), \bar{q}(\tau), (1-\tau) \cdot e_i, \mathcal{A}(\tau))\} \forall i \in I. \end{aligned}$$

Therefore,  $(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau)))$  is also an equilibrium for  $\mathcal{E}(ET, (1-\tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$ . ■

**PROOF OF THEOREM 2.2.2. (Generic Existence and Completeness of an FM Equilibrium with Long-Lived Securities):**

A) Consider an economy  $\mathcal{E}(ET, (1-\tau_e) \cdot e(\tau_e), \succeq', \mathcal{A}(\tau_d))$  satisfying assumptions of the Theorem except that

3'. Agents' preferences  $\succeq'_i$  on  $\mathbb{R}_+^{|ET \times L|}$  are given by the utility function

$$U_i(c_i) = \sum_{(\xi, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot u_i(c_i(\xi, l)) \forall i \in I.$$

Fix  $\bar{\tau}_e \in [0, 1]^{|ET \times L \times I|}$ . Then  $(1-\bar{\tau}_e) \cdot \bar{e}(\bar{\tau}_e) \in \mathbb{R}_{++}^{|ET \times L \times I|}$ . We can conclude by Magill and Shafer (1990)<sup>7</sup> that there exists an open set

$$D \subset \mathbb{R}^{|ET \times \mathbb{K}|}$$

such that

$$m[\mathbb{R}^{|ET \times \mathbb{K}|} \setminus D] = 0$$

and

$$\forall ((1-\tau_d) \cdot d(\tau_d)) \in D$$

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<sup>7</sup>See also Magill and Shafer (1991) and Magill and Quinzii (1996).

the FM economy  $\mathcal{E}(ET, (1-\bar{\tau}_e) \cdot \bar{e}(\bar{\tau}_e), \succeq', \mathcal{A}(\tau_d))$  has an FM equilibrium  $(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left(\mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}^{|ET \times L|} \times Q\right)$ .

Now, using separability of agents' utility functions and the fact that government spending  $G$  is an exogenous variable, we can conclude that

$$\forall((1 - \tau_d) \cdot d(\tau_d)) \in D$$

the FM economy

$$\mathcal{E}_{(1-\bar{\tau}_e) \cdot \bar{e}(\bar{\tau}_e), (1-\tau_d) \cdot d(\tau_d)} = \mathcal{E}(ET, (1 - \bar{\tau}_e) \cdot \bar{e}(\bar{\tau}_e), \succeq, \mathcal{A}(\tau_d))$$

has the same FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left(\mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}^{|ET \times L|} \times Q\right).$$

Since function  $f : [0, 1]^{|ET \times K|} \longrightarrow \mathbb{R}^{|ET \times K|}$  defined as

$$f(\tau_d) = (1 - \tau_d) \cdot d(\tau_d)$$

is continuous on  $[0, 1]^{|ET \times K|}$ , we can conclude that

$$O = f^{-1}(D) \subset [0, 1]^{|ET \times K|}$$

is an open set. Therefore,

$$\forall \tau_d \in O \subset [0, 1]^{|ET \times K|}$$

the economy  $\mathcal{E}_{(1-\bar{\tau}_e) \cdot \bar{e}(\bar{\tau}_e), (1-\tau_d) \cdot d(\tau_d)}$  has an FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left(\mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}^{|ET \times L|} \times Q\right).$$

It is left to show that

$$m[O] = m([0, 1]^{|ET \times K|}) = 1.$$

Since

$$\mathbf{T} = [0, 1]^{|ET \times K|} \setminus \mathbf{CP}_f,$$

where  $\mathbf{CP}_f$  is the set of critical points of  $f$ , we can conclude by the Inverse Function Theorem that

$$f : \mathbf{T} \longrightarrow \mathbb{R}^{|\mathbb{E}T \times K|}$$

is a local  $C^1$  diffeomorphism. Therefore,  $\forall \tau \in \mathbf{T} \exists B_\tau \subset \mathcal{B}$ , such that  $\tau \in B_\tau$ , where  $\mathcal{B} \subset \mathcal{P}(\mathbb{R}^{|\mathbb{E}T \times K|})$  is a base for  $(\mathbb{R}^{|\mathbb{E}T \times K|}, |\cdot|)$  and

$$f_\tau : B_\tau \longrightarrow f_\tau(B_\tau)$$

is a  $C^1$  diffeomorphism.

Moreover, since  $(\mathbb{R}^{|\mathbb{E}T \times K|}, |\cdot|)$  is a second countable topological space, we can conclude that  $\mathcal{B}$  is a countable base. Then

$$\begin{aligned} \mathbf{T} &= \bigcup_{n=1}^{\infty} B_{\tau_n} = \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}))] = \\ &= \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \setminus D)] \cup \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \cap D)], \end{aligned}$$

where  $\{B_{\tau_n}\}_{n=1}^{\infty} \subset \mathcal{B}$ . Therefore,

$$m[\mathbf{T}] = m \left[ \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \setminus D)] \right] + m \left[ \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \cap D)] \right]. \quad (1)$$

We also know that

$$f(B_{\tau_n}) \setminus D \subset f(B_{\tau_n}) \quad \forall n,$$

where  $f(B_{\tau_n})$  is an open set and

$$m[f(B_{\tau_n}) \setminus D] \leq m[\mathbb{R}^{|\mathbb{E}T \times K|} \setminus D] = 0 \quad \forall n. \quad (2)$$

Lemma 1.1. on p. 68 of Hirsch (1994) states: *Let  $U \subset \mathbb{R}^n$  be an open set and  $g : U \longrightarrow \mathbb{R}^n$  is a  $C^1$  map. If  $A \subset U$  is such that  $m[A] = 0$ , then  $m[g(A)] = 0$ .*

Set

$$\begin{aligned} U &= f(B_{\tau_n}), \\ g &= f_{\tau_n}^{-1} \end{aligned}$$

and

$$A = f(B_{\tau_n}) \setminus D \subset U.$$

Clearly,  $U$  is an open set in  $\mathbb{R}^{|ET \times K|}$ ,  $g : U \rightarrow \mathbb{R}^{|ET \times K|}$  is a  $C^1$  map and by (2)  $m[A] = 0$ . Therefore, we can conclude by Lemma 1.1. of Hirsch (1994)

$$m \left[ \underbrace{g(f(B_{\tau_n}) \setminus D)}_A \right] = 0 \quad \forall n$$

and so

$$m [f_{\tau_n}^{-1}(f(B_{\tau_n}) \setminus D)] = 0 \quad \forall n.$$

Thus, we can conclude by (1)

$$m[\mathbf{T}] = m \left[ \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \cap D)] \right] \leq m[f^{-1}(D)]$$

and so

$$m[f^{-1}(D)] = m[\mathbf{T}] = m[0, 1]^{|ET \times K|} = 1. \quad \blacksquare$$

B) The proof is similar to A.

C) Consider an economy  $\mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq', \mathcal{A}(\tau_d))$  satisfying assumptions of the Theorem except that

3'. Agents' preferences  $\succeq'_i$  on  $\mathbb{R}_+^{|ET \times L|}$  are given by the utility function

$$U_i(c_i) = \sum_{(\xi, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot u_i(c_i(\xi, l)) \quad \forall i \in I.$$

We can conclude by the Theorem 1 on p. 204 of Duffie and Shafer (1986)<sup>8</sup> that there exists an open set

$$ED \subset \mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}$$

such that

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<sup>8</sup>See also Magill and Shafer (1991) and Magill and Quinzii (1996).

$$m \left[ \left( \mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|} \right) \setminus ED \right] = 0$$

and

$$\forall ((1 - \tau_e) \cdot e(\tau_e), (1 - \tau_d) \cdot d(\tau_d)) \in ED$$

the FM economy  $\mathcal{E}(ET, (1 - \tau) \cdot e, \succeq', \mathcal{A}(\tau))$  has an FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right).$$

Now, using separability of agents' utility functions and the fact that government spending  $G$  is an exogenous variable, we can conclude that

$$\forall ((1 - \tau_e) \cdot e(\tau_e), (1 - \tau_d) \cdot d(\tau_d)) \in ED$$

the FM economy

$$\mathcal{E}_{(1-\tau_e) \cdot e(\tau_e), (1-\tau_d) \cdot d(\tau_d)} = \mathcal{E}(ET, (1 - \tau_e) \cdot e(\tau_e), \succeq, \mathcal{A}(\tau_d))$$

has the same FM equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right).$$

Since function  $f : [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|} \longrightarrow \mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}$  defined as

$$f(\tau_e, \tau_d) = ((1 - \tau_e) \cdot e(\tau_e), (1 - \tau_d) \cdot d(\tau_d))$$

is continuous on  $[0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$ , we can conclude that

$$O = f^{-1}(ED) \subset [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

is an open set. Therefore,

$$\forall \tau = (\tau_e, \tau_d) \in O \subset [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

the economy  $\mathcal{E}_{(1-\tau_e) \cdot e(\tau_e), (1-\tau_d) \cdot d(\tau_d)}$  has FM an equilibrium

$$(\{(\bar{c}_i(\tau), \bar{z}_i(\tau))\}_{i \in I}, (\bar{p}(\tau), \bar{q}(\tau))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}^{|ET \times L|} \times Q \right).$$



It is left to show that

$$m[O] = m\left[[0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}\right] = 1.$$

Since

$$\mathbf{T} = \left([0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}\right) \setminus \mathbf{CP}_f,$$

where  $\mathbf{CP}_f$  is the set of critical points of  $f$ , we can conclude by the Inverse Function Theorem that

$$f : \mathbf{T} \longrightarrow \mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}$$

is a local  $C^1$  diffeomorphism. Therefore,  $\forall \tau \in \mathbf{T} \exists B_\tau \subset \mathcal{B}$ , such that  $\tau \in B_\tau$ , where  $\mathcal{B} \subset \mathcal{P}\left(\mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}\right)$  is a base for  $(\mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}, |\cdot|)$  and

$$f_\tau : B_\tau \longrightarrow f_\tau(B_\tau)$$

is a  $C^1$  diffeomorphism.

Moreover, since  $(\mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}, |\cdot|)$  is a second countable topological space, we can conclude that  $\mathcal{B}$  is a countable base. Then

$$\begin{aligned} \mathbf{T} &= \bigcup_{n=1}^{\infty} B_{\tau_n} = \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}))] = \\ &= \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \setminus ED)] \cup \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \cap ED)], \end{aligned}$$

where  $\{B_{\tau_n}\}_{n=1}^{\infty} \subset \mathcal{B}$ . Therefore,

$$m[\mathbf{T}] = m\left[\bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \setminus ED)]\right] + m\left[\bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \cap ED)]\right]. \quad (3)$$

We also know that

$$f(B_{\tau_n}) \setminus ED \subset f(B_{\tau_n}) \quad \forall n,$$

where  $f(B_{\tau_n})$  is an open set and

$$m[f(B_{\tau_n}) \setminus ED] \leq m \left[ \left( \mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|} \right) \setminus ED \right] = 0 \quad \forall n. \quad (4)$$

Lemma 1.1. on p. 68 of Hirsch (1994) states: *Let  $U \subset \mathbb{R}^n$  be an open set and  $g : U \rightarrow \mathbb{R}^n$  is a  $C^1$  map. If  $A \subset U$  is such that  $m[A] = 0$ , then  $m[g(A)] = 0$ .*

Set

$$\begin{aligned} U &= f(B_{\tau_n}), \\ g &= f_{\tau_n}^{-1} \end{aligned}$$

and

$$A = f(B_{\tau_n}) \setminus ED \subset U.$$

Clearly,  $U$  is an open set in  $\mathbb{R}^{|ET \times K|}$ ,  $g : U \rightarrow \mathbb{R}^{|ET \times K|}$  is a  $C^1$  map and by (4)  $m[A] = 0$ . Therefore, we can conclude by Lemma 1.1. of Hirsch (1994)

$$m \left[ \underbrace{g(f(B_{\tau_n}) \setminus ED)}_A \right] = 0 \quad \forall n$$

and so

$$m[f_{\tau_n}^{-1}(f(B_{\tau_n}) \setminus ED)] = 0 \quad \forall n.$$

Thus, we can conclude by (3)

$$m[\mathbf{T}] = m \left[ \bigcup_{n=1}^{\infty} [f_{\tau_n}^{-1}(f(B_{\tau_n}) \cap ED)] \right] \leq m[f^{-1}(ED)]$$

and so

$$m[f^{-1}(ED)] = m[\mathbf{T}] = m \left[ [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|} \right] = 1. \quad \blacksquare$$

**PROOF OF THEOREM 2.2.3. (Local Completeness of FM Equilibria):** Let

$$(\{\bar{c}_i(\bar{\tau}), \bar{z}_i(\bar{\tau})\}_{i \in I}, (\bar{p}(\bar{\tau}), \bar{q}(\bar{\tau}))) \in \left( \mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|} \right) \times \left( \mathbb{R}_+^{|ET \times L|} \times Q \right)$$

be an FM equilibrium in which markets are complete for an FM economy  $E(ET, (1 - \bar{\tau}_e) \cdot e(\bar{\tau}_e), \succeq, A(\bar{\tau}_d))$  with stochastic taxation

$$\bar{\tau} = (\bar{\tau}_e, \bar{\tau}_d) \in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}.$$

Since

$$\text{rank}[W(\bar{q}(\bar{\tau}_e, \bar{\tau}_d), (1 - \bar{\tau}_d) \cdot d(\bar{\tau}_d))] = |ET| - 1,$$

we know that there exists a  $(|ET| - 1) \times (|ET| - 1)$  minor  $M(\bar{\tau}_e, \bar{\tau}_d)$  of  $W(\bar{q}(\bar{\tau}_e, \bar{\tau}_d), (1 - \bar{\tau}_d) \cdot d(\bar{\tau}_d))$  such that

$$M(\bar{\tau}_e, \bar{\tau}_d) \in \mathbb{R} \setminus \{0\}.$$

Moreover, since

$$M : [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|} \longrightarrow \mathbb{R},$$

is a continuous function of  $\tau = (\tau_e, \tau_d)$  and  $\mathbb{R} \setminus \{0\}$  is an open set, we can conclude that there exists an open neighborhood

$$O_{\bar{\tau}} \subset [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}$$

of  $\bar{\tau} = (\bar{\tau}_e, \bar{\tau}_d)$  such that

$$M(\tau_e, \tau_d) \in \mathbb{R} \setminus \{0\} \quad \forall \tau = (\tau_e, \tau_d) \in O_{\bar{\tau}}.$$

Thus,

$$\text{rank}[W(\bar{q}(\tau_e, \tau_d), (1 - \tau_d) \cdot d(\tau_d))] \geq |ET| - 1 \quad \forall \tau = (\tau_e, \tau_d) \in O_{\bar{\tau}}.$$

But since NAC holds for all tax rates we know that

$$\begin{aligned} \text{rank}[W(\bar{q}(\tau_e, \tau_d), (1 - \tau_d) \cdot d(\tau_d))] &\leq |ET| - 1 \\ \forall \tau &\in [0, 1]^{|ET \times L \times I|} \times [0, 1]^{|ET \times K|}. \end{aligned}$$

Therefore,

$$\text{rank}[W(\bar{q}(\tau_e, \tau_d), (1 - \tau_d) \cdot d(\tau_d))] = |ET| - 1 \quad \forall \tau = (\tau_e, \tau_d) \in O_{\bar{\tau}},$$

i.e., FM are complete  $\forall \tau = (\tau_e, \tau_d) \in O_{\bar{\tau}}$ . ■

## References

- DeLong, J.B. and K. Magin. The U.S. Equity Return Premium: Past, Present, and Future. *Journal of Economic Perspectives*, 2009, 23:1, 193-208.
- Duffie, D. and W. Shafer. Equilibrium in Incomplete Markets: II. Generic Existence in Stochastic Economies. *Journal of Mathematical Economics* 1986, Volume 15: 199-216.
- Edelstein, R. and K. Magin. The Equity Risk Premium for Securitized Real Estate –The Case for U.S. Real Estate Investment Trusts. *The Journal of Real Estate Research*, 2013, Vol. 35:4.
- Geanakoplos. J. and H. Polemarchakis. Existence, Regularity, and Constrained Suboptimality of Competitive Allocations when the Asset Market is Incomplete, in W. Heller, R. Starr, and D. Starrett, eds., *Essays in Honor of Kenneth Arrow*. Cambridge University Press, 1986, Vol. 3, pp. 65-95.
- Hirsch, M. W., 1994. *Differential Topology*. Berlin: Springer-Verlag.
- Lucas, R. E. Asset Prices in an Exchange Economy. *Econometrica*, 1978, 46:6, 1429-45.
- Magill, M. and M. Quinzii, 1996. *Theory of Incomplete Markets*. Volume I. MIT Press.
- Magill, M. and W. Shafer, 1991. Incomplete markets, *Handbook of Mathematical Economics*, in: W. Hildenbrand & H. Sonnenschein (ed.), *Handbook of Mathematical Economics*, edition 1, volume 4, chapter 30, pages 1523-1614 Elsevier.
- Magill, M. and W. Shafer. Characterization of Generically Complete Real Asset Structures. *Journal of Mathematical Economics* 1990, Volume 19: 167-194.
- Magin, K., *Finite Horizon GEI, Stochastic Taxation and Comparative Statics*, The Center for Risk Management Research Working Paper 2015.
- Magin, K., *Equity Risk Premium and Insecure Property Rights*, *Economic Theory Bulletin*, 2014, DOI 10.1007/s40505-014-0043-7.
- McGrattan E. R. and E.C., Prescott. Taxes, Regulations, and the Value of U.S. and U.K. Corporations. *Review of Economic Studies*, 72(3): 767–796, July 2005.
- Sialm, C. Tax Changes and Asset Pricing. *American Economic Review*, 2009, 99:4, 1356-1383.
- Sialm, C. Stochastic Taxation and Asset Pricing in Dynamic General Equilibrium. *Journal of Economic Dynamics and Control*, 2006, 30, 511-540.