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Abstract

Tax rates on personal, corporate and investment incomes have varied significantly throughout U.S. history. Although the current tax rates are the lowest in postwar U.S. history, the current political and economic environment, combined with the borrowing constraints that the U.S. government is implicitly facing, make the long-term sustainability of current tax rates highly doubtful, thus creating significant tax rate uncertainty. Individuals, corporations and investors worldwide are constantly looking for ways to reduce the risk of excessive taxation. This paper designs tax rate swaps to hedge against this tax rate uncertainty.

Keywords: Tax Rate Swaps, Stochastic Tax

JEL Classification: G1; G2

I. Introduction

Tax rates on personal income, corporate income and investment income have varied considerably throughout U.S. history.[1] While the current tax rates are definitely the lowest in postwar U.S. history, the current political and economic environment, combined with the borrowing constraints that the U.S. government is potentially facing, make the long-term sustainability of current tax rates highly doubtful, thus creating significant tax rate uncertainty.[2] This uncertainty, in turn, may cause economic agents to postpone potentially profitable investment decisions. As a result, individuals, corporations and investors worldwide are constantly looking for ways to reduce the risk of excessive taxation. This paper designs tax rate swaps to hedge against this tax rate uncertainty. My major finding is that holding a portfolio consisting of one share of an underlying asset and a tax rate swap contract written on the stochastic tax burden imposed on the dividend stream of this asset is equivalent to holding an asset with the same dividend stream as the underlying asset but subject to the constant tax rate. Thus, this portfolio is

[1] See Appendix I.

[2] The effect of stochastic taxation on equilibrium prices and allocations is a largely unexplored area of finance. For pioneering work see Sialm (2004), Magin (2009), Edelstein and Magin (2012).

a hedge that completely eliminates tax rate uncertainty. The remainder of the paper is organized as follows. Section II develops the model of tax rate swaps. Section III concludes.

II. **Model**

I will start with basic definitions.

$DEFINITION$: *A swap contract is is an agreement between two parties to exchange cash flows in the future.*

$DEFINITION$: *A tax rate swap contract is an agreement between two parties written at period* t*, where one party (the holder) agrees to receive cash flow equal to* $\tau_{t+T} \cdot d_{t+T}$ *at period* $t+T$ *for all* $T = 1, ..., \overline{T}$*, where* τ_{t+T} *is the current tax rate on the holder's taxable income* d_{t+T} *at period* $t+T$ *in exchange for cash flow equal to* $\overline{\tau} \cdot d_{t+T}$ *for all* $T = 1, ..., \overline{T}$*, where* $\overline{\tau}$ *is the predetermined fixed tax rate on the same taxable income* d_{t+T}*.*

Therefore, the holder of this contract receives net cash flow equal to

$$[\tau_{t+T} - \overline{\tau}] \cdot d_{t+T}$$

for all $T = 1, ..., \overline{T}$.

We denote the period t price of this swap contract by

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{t+T},\, \overline{T}).$$

PROPOSITION 1 : *Consider an economy with* N *identical agents and* 2 *assets. The representative investor maximizes*

$$u(c_t) + E[\sum_{T=1}^{\overline{T}} b^T u(c_{t+T})],$$

subject to

$$z_{1t+T}(p_{1t+T} + (1 - \tau_{t+T})d_{1t+T}) + z_{2t+T}(p_{2t+T} + d_{2t+T}) = c_{t+T} + \sum_{k=1}^{2} p_{kt+T} z_{kt+T+1}$$

for all $T = 0, ..., \overline{T}$,

where $u(\cdot)$ *is the individual's one-period utility function such that* $u'(\cdot) > 0$ *and* $u''(\cdot) < 0$, p_{kt+T} *is the price per share of asset* k *at period* $t+T$, d_{kt+T} *is the dividend per share of asset* k *at period* $t+T$, τ_{t+T} *is the stochastic tax imposed on the dividend of asset* 1 *at period* $t+T$, z_{kt+T} *is the number of shares of asset* k *that the representative investor holds at period* $t+T$. *Asset* 1 *is an arbitrary dividend-paying asset. The total supply of asset* 1 *is* N *shares. Asset* 2 *is a tax rate swap contract paying its holder*

$$[\tau_{t+T} - \overline{\tau}] \cdot d_{1t+T}$$

for all $T = 1, ..., \overline{T}$. *The total supply of asset* 2 *is* N *identical tax rate swap contracts. Clearly,*

$$d_{2t+T} = [\tau_{t+T} - \overline{\tau}] \cdot d_{1t+T}$$

and

$$p_{2t} = swap_t(\tau_{t+T}, \, \overline{\tau}, \, d_{1t+T}, \, \overline{T})\text{.}$$

Assume that $\tau_t = \overline{\tau}$.

Then the equilibrium asset prices at period t *are given by*

$$p_{1t} = E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'((1-\overline{\tau})d_{1t+T})}{u'((1-\overline{\tau})d_{1t})}[1 - \tau_{t+T}]d_{1t+T}\right],$$

$$swap_t(\tau_{t+T}, \, \overline{\tau}, \, d_{1t+T}, \, \overline{T}) = E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'((1-\overline{\tau})d_{1t+T})}{u'((1-\overline{\tau})d_{1t})}[\tau_{t+T} - \overline{\tau}]d_{1t+T}\right].$$

Moreover, in equilibrium, the representative agent will hold a portfolio consisting of 1 *share of asset* 1 *and* 1 *tax rate swap contract written on the stochastic tax burden imposed on the dividend stream of asset* 1*. Therefore, the equilibrium price of the portfolio that the representative agent will hold is*

$$p_{1t} + swap_t = E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'((1-\overline{\tau})d_{1t+T})}{u'((1-\overline{\tau})d_{1t})}[1 - \overline{\tau}]d_{1t+T}\right].$$

PROOF : See Appendix II.

The intuition behind Proposition 1 is clear: the lower is the effective tax rate $\overline{\tau}$ that the holder of the tax rate swap contract is guaranteed to pay, the more valuable is the swap. Moreover, holding the portfolio consisting of 1 share of asset 1 and 1 tax rate swap contract written on the stochastic tax burden imposed on the dividend stream of asset 1 is equivalent to holding an asset with the same dividend stream as asset 1 but subject to the constant tax rate $\overline{\tau}$.

$PROPOSITION\ 2:$ *Suppose all the assumptions of Proposition 1 from above hold and* $u(c) = \frac{c^{1-\lambda}}{1-\lambda}$. *Then the equilibrium asset prices at period* t *are given by*

$$p_{1t} = E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{d_{1t+T}}{d_{1t}}\right)^{-\lambda} [1 - \tau_{t+T}] d_{1t+T}\right],$$

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) = E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{d_{1t+T}}{d_{1t}}\right)^{-\lambda} [\tau_{t+T} - \overline{\tau}] d_{1t+T}\right],$$

$$\frac{\partial swap_t}{\partial \overline{\tau}} = -E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{d_{1t+T}}{d_{1t}}\right)^{-\lambda} d_{1t+T}\right] < 0$$

and *therefore the equilibrium price of the portfolio that the representative agent will hold is*

$$p_{1t} + swap_t = E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{d_{1t+T}}{d_{1t}}\right)^{-\lambda} [1 - \overline{\tau}] d_{1t+T}\right].$$

$PROOF:$ See Appendix II.

$PROPOSITION\ 3:$ *Suppose all the assumptions of Proposition 1 hold and* $d_{t+T} = d$ *for all* $T = 0, ..., \overline{T}$. *Then the equilibrium asset prices at period* t *are given by*

$$p_{1t} = E\left[\sum_{T=1}^{\overline{T}} b^T [1 - \tau_{t+T}]\right] d,$$

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) = \sum_{T=1}^{\overline{T}} b^T E\left[\tau_{t+T}\right] d - \tfrac{b}{1-b}\overline{\tau} d,$$

$$\tfrac{\partial swap_t}{\partial \overline{\tau}} = -\tfrac{b}{1-b} d < 0$$

and *therefore the equilibrium price of the portfolio that the representative agent will hold is*

$$p_{1t} + swap_t = \tfrac{b}{1-b}(1 - \overline{\tau})d.$$

PROOF : See Appendix II.

III. Conclusion

Although the current tax rates are definitely the lowest in postwar U.S. history, the current political and economic environment, combined with the borrowing constraints that the U.S. government is potentially facing, make the long-term sustainability of current tax rates highly doubtful, thus creating significant tax rate uncertainty. This paper designs tax rate swaps to insure against this tax rate uncertainty. My major finding is that holding a portfolio consisting of one share of an underlying asset and a tax rate swap contract written on the stochastic tax burden imposed on the dividend stream of this asset is equivalent to holding an asset with the same dividend stream as the underlying asset but subject to the constant tax rate. Therefore, this portfolio is a hedge that completely eliminates tax rate uncertainty.

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Appendix I

Personal Income Tax

The passage of the 16th Amendment to the Constitution in 1913 gave the U.S. Congress permanent authority to lay and collect taxes on income.[3] The top personal income tax rate reached 7%. Then, as the size of government increased, the top personal income tax rate reached 77% by 1918, but dropped to 24% by 1929. From 1929 to 1945, as the size of government started to increase again, the trend for the top personal income tax rate was upward, reaching 94% in 1944. Between 1945 and 1963, the top personal income tax rate dropped a bit but still remained at 86 to 91%. Starting in 1964, the trend was mostly downward. The 1964 Kennedy tax cut reduced the highest income tax rate from 91% to 77%. The 1981 Reagan tax cut reduced the

[3]Contrary to popular belief, the United States government imposed its first personal income tax of 3% on all incomes over $800 on August 5, 1861, as a part of the Revenue Act, to pay for the American Civil War. This tax was abolished and another income tax was introduced in 1862. In 1894, the Wilson-Gorman tariff was introduced, which imposed a largely symbolic income tax rate of 2% on income over $4000. Fewer than 10% of households had this level of income and therefore paid any income tax. From 1895 to 1913, for a number of legal, political, and practical reasons, there were no federal income tax in the United States.

highest personal income tax rate further to 50%. The 1986 Tax Reform Act reduced the highest rate even more to 38.5% in 1987 and to 28% from 1988 to 1990. From 1991, tax rates started to climb up again, with the top rate reaching 38.6% in 2002. However, the 2003 Bush tax cut reduced the highest rate to 35%.[4]

Investment Tax

Investment tax has three components: the dividend tax, the short-term capital gains tax and the long-term capital gains tax. In the past, the tax burden on investors was largely driven by the personal income tax. Indeed, before the 2003 Tax Act, dividends and short-term capital gains were taxed as ordinary income. Starting in 2003, the Tax Act effectively reduced the dividend tax rate to 15% for those in the ordinary income tax brackets above 15% and to 5% for those in the 10% and 15% ordinary income tax brackets, while short-term capital gains are still taxed as ordinary income. Prior to

[4] Historical data on personal income tax rates for 1913-2009 is available at the IRS website http://www.irs.gov/taxstats/article/0,,id=175910,00.html, Table 23. U.S. Individual Income Tax: Personal Exemptions and Lowest and Highest Bracket Tax Rates, and Tax Base for Regular Tax, Tax Years 1913 - 2010. This data allows one to calculate effective, not just statutory tax rates.

the 2003 Tax Act, long-term capital gains were taxed at 20% for those in the ordinary income tax brackets of 25% or above and at 10% for those in the 10% and 15% ordinary income tax brackets. Starting in 2003, the Tax Act reduced the long-term capital gains tax to 15% for those in the ordinary income tax brackets of 25% or above and to 5% for those in the 10% and 15% ordinary income tax brackets.

While it is very straightforward to obtain dividend tax rates, it is a major challenge to measure the average effective short-term and long-term capital gains taxes. The problem is that the capital gains tax is imposed only when the capital gains are actually realized. Fortunately for this research, Sialm (2008) has developed a measure of the aggregate personal tax burden on equity securities. Sialm (2008) uses historical data to estimate average dividend yield and average realized short-term and long-term capital gains as follows:

$$\frac{d_{t+1}}{p_t} = 0.045,$$

$$\frac{SCG_{t+1}}{p_t} = 0.001,$$

$$\frac{LCG_{t+1}}{p_t} = 0.018,$$

where p_t is the price per share,

d_{t+1} is the dividend per share,

SCG_{t+1} are realized short-term capital gains and

LCG_{t+1} are realized long-term capital gains.

He then uses the data to estimate the tax yield as

$$TY_{t+1} = \frac{\tau_{t+1}^{d} d_{t+1} + \tau_{t+1}^{SCG} SCG_{t+1} + \tau_{t+1}^{LCG} LCG_{t+1}}{p_t} =$$

$$\tau_{t+1}^{d} \cdot 0.045 + \tau_{t+1}^{SCG} \cdot 0.001 + \tau_{t+1}^{LCG} \cdot 0.018,$$

where τ_{t+1}^{d} is the dividend tax,

τ_{t+1}^{SCG} is the tax on short-term capital gains,

τ_{t+1}^{LCG} is the tax on long-term capital gains.[5]

Corporate Income Tax

The corporate income tax was first introduced by President Taft in 1909. The Supreme Court had held that a personal income tax was unconstitutional, and so the government found that taxing corporations was a promising alternative source of revenue as a way of redistributing income from the

[5] Historical data on the U.S. federal plus state average marginal dividend tax rates, short-term capital gains tax rates and long-term capital gains tax rates for 1913 through 2009 is available at the NBER website

http://www.nber.org/~taxsim/marginal-tax-rates/plusstate.html.

This data allows one to calculate effective, not just statutory tax rates.

supply side to the demand side—and obtaining money for government operations without ever sending an explicit tax bill to any voting individual. Originally, the corporate income tax was only 1%. Then, as the size of government increased, the average effective corporate income tax reached 50% by 1970. At the moment, the corporate income tax with the average effective rate of roughly 20%, is a less important source of government revenue than it has been since the Great Depression. The trend of corporate taxes has been down since the 1970s, in large part because economists and other tax professionals have strenuously argued that special taxes on the corporate form of organization carry a high excess burden. The fall in the corporate income tax rate continued in the 1980s. It may well be no accident that the government's shift away from reliance on the corporate tax coincided with the deregulation movement begun under Carter and continued under Reagan and subsequent presidents. However, once again, greater and greater commitments of the U.S. government cast a great doubt on the sustainability of low tax rates.[6]

[6]Historical data on corporate income and the corporate income tax for 1960 through 2009 is available at the GPO website

http://www.gpo.gov/fdsys/pkg/ERP-2010/pdf/ERP-2010-table90.pdf.

This data allows one to calculate effective, not just statutory tax rates.

Appendix II

PROOF OF PROPOSITION 1 : Using the CCAPM we obtain

$$p_{1t}= E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'(c_{t+T})}{u'(c_t)}[1-\tau_{t+T}]d_{1t+T}\right] \quad (1)$$

and

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) =$$
$$= E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'(c_{t+T})}{u'(c_t)}\tau_{t+T}d_{1t+T}\right] - E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'(c_{t+T})}{u'(c_t)}\overline{\tau}d_{1t+T}\right] =$$
$$= E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'(c_{t+T})}{u'(c_t)}[\tau_{t+T}-\overline{\tau}]d_{1t+T}\right] \bullet$$

So,

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) = E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'(c_{t+T})}{u'(c_t)}[\tau_{t+T}-\overline{\tau}]d_{1t+T}\right] \bullet \quad (2)$$

But since all agents are the same, there is no trade. Hence, in equilibrium all agents hold the same portfolio. Therefore in equilibrium, agents' consumption is given by the total income generated by the agents' portfolio

$$c_{t+T} = \underbrace{(1-\tau_{t+T})\cdot d_{1t+T}}_{After-tax\ Income\ from\ 1\ Share\ of\ Asset\ 1} + \underbrace{[\tau_{t+T}-\overline{\tau}]\cdot d_{1t+T}}_{Net\ Income\ from\ Tax\ Rate\ Swap} =$$

$$= (1-\overline{\tau})\cdot d_{1t+T} \text{ for all } T = 0, ..., \overline{T}. \ (3)$$

Therefore, substituting (3) into (1) and (2) we obtain the equilibrium asset prices *at period t*

$$p_{1t} = E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'((1-\overline{\tau})d_{1t+T})}{u'((1-\overline{\tau})d_{1t})}[1-\tau_{t+T}]d_{1t+T}\right]$$

and

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) = E\left[\sum_{T=1}^{\overline{T}} \frac{b^T u'((1-\overline{\tau})d_{1t+T})}{u'((1-\overline{\tau})d_{1t})}[\tau_{t+T}-\overline{\tau}]d_{1t+T}\right].\blacksquare$$

PROOF OF PROPOSITION 2 :

$$p_{1t} = E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{(1-\overline{\tau})d_{1t+T}}{(1-\overline{\tau})d_{1t}}\right)^{-\lambda}[1-\tau_{t+T}]d_{1t+T}\right] =$$
$$= E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{d_{1t+T}}{d_{1t}}\right)^{-\lambda}[1-\tau_{t+T}]d_{1t+T}\right].$$

and

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) = E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{(1-\overline{\tau})d_{1t+T}}{(1-\overline{\tau})d_{1t}}\right)^{-\lambda}[\tau_{t+T}-\overline{\tau}]d_{1t+T}\right] =$$
$$= E\left[\sum_{T=1}^{\overline{T}} b^T \left(\frac{d_{1t+T}}{d_{1t}}\right)^{-\lambda}[\tau_{t+T}-\overline{\tau}]d_{1t+T}\right].\blacksquare$$

PROOF OF PROPOSITION 3 : Since dividends are constant, we obtain that in equilibrium

$$c_{t+T} = (1-\overline{\tau})\cdot d \text{ for all } T = 0, ..., \overline{T}.$$

Therefore, using the previous Proposition 1 we obtain

$$p_{1t}= E\left[\sum_{T=1}^{\overline{T}} b^T[1-\tau_{t+T}]\right] d$$

and

$$swap_t(\tau_{t+T},\, \overline{\tau},\, d_{1t+T},\, \overline{T}) =$$

$$= E\left[\sum_{T=1}^{\overline{T}} b^T[\tau_{t+T}-\overline{\tau}]\right] d = \sum_{T=1}^{\overline{T}} b^T E\left[\tau_{t+T}\right] d - \tfrac{b}{1-b}\overline{\tau} d. \blacksquare$$