Do Steph Curry and Klay Thompson Have Hot Hands?

Alon Daks Nishant Desai Lisa R. Goldberg*

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Abstract

The Splash Brothers, Steph Curry and Klay Thompson, are great shooters but they are not streak shooters. Only rarely do they show signs of a hot hand. This conclusion is based on an empirical analysis of field goal and free throw data from the 82 regular season games played by the Golden State Warriors in 2016–2017. Our analysis is inspired by the iconic 1985 hot-hand study by Thomas Gilovitch, Robert Vallone and Amos Tversky, and it uses a permutation test, which automatically accounts for Josh Miller and Adam Sanjurjo's recent small sample correction.

*All three authors are at University of California, Berkeley. Please contact `lrg@berkeley.edu` for more information.

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The true believer in the law of small numbers commits his multitude of sins against the logic of statistical inference in good faith.

Amos Tversky and Daniel Kahneman, "Belief in the Law of Small Numbers"

1 The original hot-hand study

All basketball fans know about the hot hand: it's a good idea to pass to a teammate on a scoring streak since his chances of making the next basket are higher than usual.

This venerated principle was discredited in 1985 by Amos Tversky and two co-authors, Thomas Gilovich and Robert Vallone.[1] Their statistical study of field goal data from the Philadelphia 76ers, free throw data from the Boston Celtics, and a controlled 100-shot-per-player experiment on Cornell University varsity and junior varsity basketball players seemed to prove that such scoring streaks are not out of the ordinary. Although fans think that their players have hot hands, the streaks can be explained by mere chance.

This game-changing news received a lukewarm reception from professional sports. Red Auerbach, President of the Boston Celtics when the hot-hand study was released, famously gave his views on Tversky: "Who is this guy? So, he makes a study. I couldn't care less."

Academics, however, fully engaged with the finding. The 1985 study launched an avalanche of scholarly literature, and the hot hand question has propelled investigations about the conflict between the instincts of professionals and the cold hard facts of science. In his recent best-selling book, *The Undoing Project,* Michael Lewis tells the story of Amos Tversky and his lifelong collaborator, Nobel Laureate Daniel Kahneman. Their research prompted the study of behavioral economics and it has transformed our understanding of the flaws in human decision making. Lewis writes, "Tversky had the clear idea of how people misperceived randomness. [...] People had incredible ability to see meaning in these patterns where none existed."

Early in their careers, Amos Tversky and Daniel Kahneman considered the human tendency to draw conclusions based on a few observations, which they called "the law of small numbers."[2] This is a playful allusion to the law of large numbers,

[1]Please refer to (Gilovich, Vallone & Tversky 1985).
[2]Please refer to (Tversky & Kahneman 1971).

which provides guidance about when accurate inference can be drawn from a large data set. There is no general rule about how to draw inference from a small data set, and it can be difficult to notice that there is a problem. Even for an expert.

2 A basic statistical formulation of a hot hand

There are many ways to represent the notion of “hot hand” in a statistical experiment. In this study as in the original study, a *hot hand* is an abnormally high probability of making a shot, given a string of hits.

To someone familiar with basketball, this simple formulation might seem silly. Some shots are harder to make than others, and defensive maneuvers by the opposition may put high performing player in a disadvantageous spot. Still, it is interesting to look at the results of this simple experiment before attempting to add realism.

There are also many ways that one might define the term “abnormally high.” Again we follow the original hot-hand study, which relied on a difference of conditional probabilities: the probability of making a shot given a string of hits minus the probability of a hit, given an equally long string of misses. If the observed difference is large relative to the difference averaged over all strings of hits and misses with the same length and the same number of hits, the observation corresponds to a hot hand.

3 A “law of small numbers” error in the original hot-hand study, and a correction

In 2015, statisticians Josh Miller and Adam Sanjurjo documented an error in the original hot-hand study.[3] Remarkably, the error concerns the law of small numbers.

To understand the error, consider Klay Thompson’s shooting record in the December 23, 2016 game against the Detroit Pistons. The record is represented by a string of 1’s (hits) and 0’s (misses).

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[3] Please refer to (Miller & Sanjurjo 2016).

Thompson took 16 shots in this game and as it happened, he made exactly half of them. We can look at our statistical formulation of the hot hand on this string. First, we compute the empirical probability of a hit, given two previous hits. There are four instances of two hits in a row, indicated by the string "11." We know what happened after the first three instances—Thompson hit the first time and missed the second and third times. But we don't know what happened after the fourth instance of "11" because the game ended before Thompson took another shot. So we'll call the final "11" is a *unrealized conditioning set*, and it complicates the estimation of the conditional probabilities used in hot-hand studies. Perhaps the best we can say is that in the game under consideration, we observed Thompson scoring 1/3 of the time following two hits in a row. In the other direction, given that Thompson missed twice in a row, he scored 2/5 of the time. The second calculation is more straightforward because there is no unrealized conditioning set. The difference of the two conditional probabilities is $1/3 - 2/5 = -1/15$.

Is this difference, $-1/15$, abnormally high? Perhaps for this string, which is half hits and half misses, there is a natural benchmark against which to measure "abnormal". Based on the data, perhaps it is reasonable to assume that the probability of making a shot after two hits is the same as the probability of making a shot after two misses: 50 percent. Against this benchmark, the average difference in conditional probabilities is 0, which does not make $-1/15$ look abnormally high. This benchmark is consistent with the original hot-hand study.

However—and this is the where the law of small numbers comes in—the 50-50 benchmark is the wrong choice. It would have been correct had we been dealing with infinite strings, but games don't go on indefinitely. So in practice, we deal with finite strings. Many have unrealized conditioning sets, in which case the natural benchmark requires a small sample adjustment. In a string of length 16 that is half 1's and half 0's, the average probability of a hit following two hits is less than the average probability of a hit following two misses: reversals are more probable than continuations. That is the observation of Miller and Sanjurjo, and it is consistent with the gambler's fallacy, the impression that a reversal in fortunes is "due."

The no-hot-hand conclusion in the original study was based on a statistically insignificant difference between the observed data and the erroneous benchmarks. When the required adjustment was applied to the controlled 100-shot-per-player experiment on Cornell University players, the no-hot-hand finding was reversed in several cases.

4 In search of the Warriors' hot hand

As noted by Miller and Sanjurjo, a permutation test of an observed string of hits and misses automatically implements the small sample correction.[4] Here, we use the permutation test to decide in which games Curry and Thompson had hot hands. We also investigated the hot handedness of the Warriors, quarter by quarter.

4.1 Data

	Curry	Thompson	Warriors
Games	79	78	82
Observations	79	78	328
Season Percentage	55%	52%	55%
Average Game Percentage	55%	51%	55%
StDev Game Percentage	11%	11%	10%
Average Number of Shots	23	20	27
StDev Number of Shots	5	5	4

Table 1: Summary statistics for the 2016–2017 regular season games played by the Warriors. The shot patterns we analyze include field goals and free throws.

For each Splash Brother, we compiled a string of 1's and 0's representing hits and misses for each of the 82 regular season games that they played in 2016–2017. Curry played in 79 games and Thompson played in 78 games. We also compiled the strings of hits and misses for the Warrior team, quarter by quarter, leading to $328 = 82 \times 4$ quarters over the season.

4.2 Experimental design and test statistics

An observation X is a game-long string of hits and misses for Steph Curry or Klay Thompson, or a quarter-long string of hits and misses for the Warriors. The string includes both field goals and free throws. We use a permutation test to determine whether the observation showed evidence of a hot hand.

[4] Please see (Miller & Sanjurjo 2016, Section 3.1).

For an observed string, we computed a test statistic, t_k, the conditional fraction of hits, given k prior hits less the conditional fraction of hits given k prior misses, where k equals 1, 2 or 3. Then we permuted the string of 0's and 1's representing the shot pattern 10,000 times and computed t_k on each permutation.

Mathematically, the test statistic t_k on a string X of length L is defined as follows:

$$\begin{aligned} t_k(X) &= \frac{1}{H_k} \sum_{\tau=k+1}^{L} I\left(X_\tau = 1 \middle| \prod_{u=\tau-k}^{\tau-1} X_u = 1 \right) - \frac{1}{M_k} \sum_{\tau=k+1}^{L} I\left(X_\tau = 1 \middle| \prod_{u=\tau-k}^{\tau-1} (1 - X_u) = 1 \right) \\ &= \hat{P}\left(X_\tau = 1 \middle| \prod_{u=\tau-k}^{\tau-1} X_u = 1 \right) - \hat{P}\left(X_\tau = 1 \middle| \prod_{u=\tau-k}^{\tau-1} (1 - X_u) = 1 \right), \end{aligned}$$

where H_k and M_k are the numbers of substrings of k hits and k misses that are followed by shots, X_τ is the τth entry of X, $\hat{P}$ is the empirical probability. The value k is the depth of the conditioning set.

The fraction of permuted test statistics that exceed the observed test statistic is its p-value.[5] A smaller p-value corresponds to a hotter hand.

4.3 Results

Statistics describing the number of shots taken and hit frequency are in Table 1. Box plots of p-values for the test statistic t_2 calculated for the 2016–2017 regular season games played Curry and Thompson and for the quarters played by the Warriors, are shown in Figure 1. Few observations are significant at the 5% level.

[5] In practice, the value of the test statistic on many of the permuted strings is the same as the value of the test statistic on the observed shot pattern. Therefore, there is some latitude in how to to define the p-value. Our choice makes it as easy as possible to reject the null hypothesis of "no hot hand."

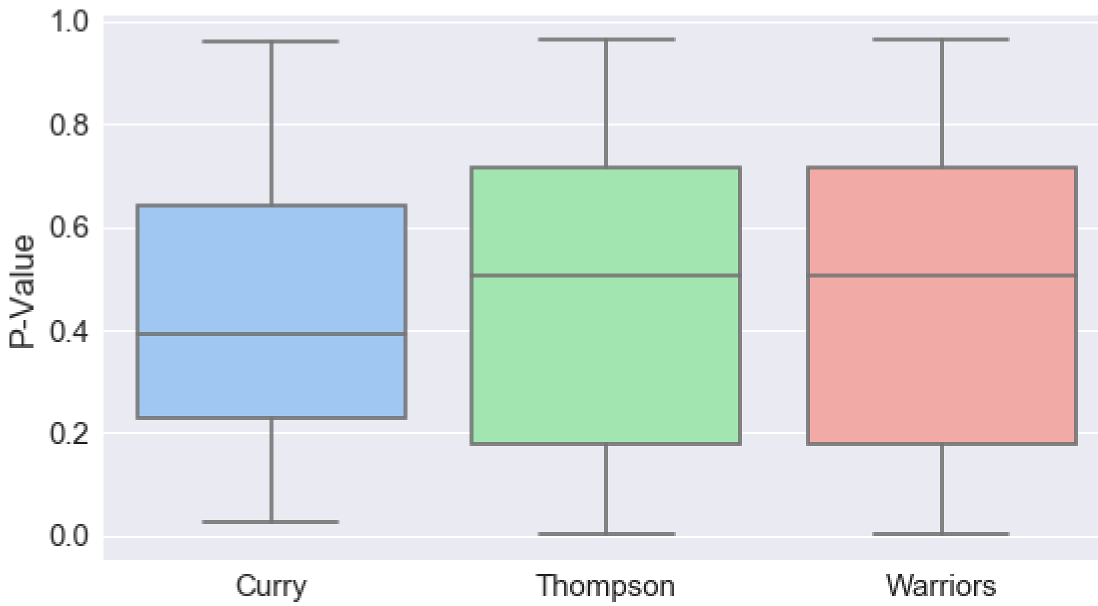

Figure 1: Box-plots of p-values for permutation tests with a depth-2 conditioning set: the conditional number of hits given two prior hits minus the conditional number of hits given two prior misses.

Table 2 displays the number of observations that are significant at the 5% level for conditioning sets of depths 1, 2 and 3.

	Curry	Thompson	Warriors
Games	79	78	82
Observations	79	78	328
Conditioning set depth			
1	10	4	28
2	1	4	18
3	2	3	14

Table 2: Number of hot hand observations at the 5% confidence level with a depth-2 conditioning set.

4.4 An example

On December 5, 2016, Thompson scored 60 points against the Los Angeles Clippers. He made 31 of the 44 shots he took, and his record for the game is shown below.

11011110010111111001110111101110111101010101

Does this indicate a hot hand? There are 19 instances of the string "11," and they are followed by hits in 12 of the 19 cases. There are two instances of the string "00," and they are both followed by hits. So $t_2 = 12/19 - 1 = -7/19$.

This observation is exceptional for its length, and it is exceptional for its percentage, $31/44 = 0.70$. But the difference of conditional probabilities, $-7/19$, had a p-value of 0.84, which is not exceptional at all.

5 Summary

We examined the 2016–2017 regular season shooting records of Splash Brothers Steph Curry and Klay Thompson. It is a magical experience to watch these players on the court. When they are on a roll, they seem to be the essence of hot handedness. But our statistical study tells a different story. It indicates that in most of the 2016–2017 regular season games, they were not streak shooters—they did not have hot hands. So our conclusion, after adjusting for the small sample effect, is similar to the original conclusion, which did not account for the small sample effect.

Of course, this is not the end of the story. Hot hands have long fascinated sports professionals and researchers, and we are not close to consensus on the right way to think about the issue. However, every empirical hot-hand study will rely on a finite data set, so small sample effects are bound to play a role in any correct interpretation of the results.

Amos Tversky was 59 years old when he died in 1996, five years before the Nobel Prize that he would surely have shared with Daniel Kahneman was awarded, and almost two decades before the error in his hot-hand study was found. An awesome researcher and a huge basketball fan, he would no doubt be pleased about the correction of his error if he were with us today, and he would surely be watching the spellbinding Golden State Warriors.

References

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