Identifying Financial Risk Factors
with a Low-Rank Sparse Decomposition

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Outline

1. A Brief History of Factor Models in Finance
3. Low Rank Plus Sparse Decompositions of Covariance Matrices
4. Simulation Results
5. Empirical Results
6. Summary
A Brief History of Factor Models in Finance

- Market Model and CAPM (1960s)
- Fundamental Models (1970s–today)
Market Model and CAPM

In the 1960s, Jack Treynor (1930 – ) and Bill Sharpe (1934 – ) developed the Capital Asset Pricing Model, which relates security expected returns to market returns.

More than half a century after the appearance of [Treynor, 1962] and [Sharpe, 1964], the market model and CAPM remain central to quantitative finance.
Market Model

Security return $R$ is a sum of a component due to a market factor $M$ and a specific component $\epsilon$

$$R = M \beta + \epsilon$$

- $\beta$ is the sensitivity of security return to market return
- Specific returns $\epsilon$ are uncorrelated across securities

As a consequence of these assumptions and others, the security covariance matrix $\Sigma$ can be decomposed as a sum of a \textit{rank-one} factor component and a \textit{diagonal} security specific return component

$$\Sigma = \sigma_M^2 \beta^T \beta + \Delta$$
Arbitrage Pricing Theory and Multi-Factor Models

In the 1970s, Stephen Ross (1944– ) expanded on ideas in the CAPM to allow for more factors [Ross, 1976] leads to a security covariance matrix that can be decomposed (using PCA) as a sum of a low-rank factor component and a diagonal security specific return component

\[ \Sigma = X^\top FX + \Delta \]
Arbitrage Pricing Theory and Multi-Factor Models

[Chamberlain and Rothschild, 1983] built on [Ross, 1976] by developing approximate factor models. Their construction relies on asymptotic results and it leads to a covariance matrix decomposition as a sum of a low-rank factor component and a sparse security specific return component:

\[ \Sigma = X^\top FX + S \]

but it has not caught on in practice.
Barr Rosenberg founded Barra ("Barr and Associates") in 1975.

[Rosenberg, 1984] and [Rosenberg, 1985] rely on fundamental factors, reversing the roles of the known and unknown variables in a regression to estimate factor returns from pre-specified exposures (factor betas)
Fundamental Factor Models

According to the fundamental models,

\[ \Sigma = \mathcal{L} + \Delta \]

- \( \mathcal{L} = X^\top FX \) has low rank
- \( \Delta \) is a diagonal security specific return covariance matrix

Barra’s fundamental models dominate industry practice today
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Examples of Risk Factors

- Market
- Equity styles
- Country, industry, currency
- Creditworthiness
- Prepayment sensitivity
- Liquidity
- Emerging factors: carbon reserves, cyberterrorism, longevity
<table>
<thead>
<tr>
<th>Factor Type</th>
<th>Broad</th>
<th>Narrow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistent</td>
<td>Easy</td>
<td>Fundamental</td>
</tr>
<tr>
<td>Transient</td>
<td>Traditional PCA</td>
<td></td>
</tr>
</tbody>
</table>
Which Models are Expensive to Run?

<table>
<thead>
<tr>
<th></th>
<th>Human Intensive</th>
<th>Machine Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>X</td>
<td></td>
</tr>
<tr>
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Low Rank Plus Sparse Decompositions of Covariance Matrices

Inspired by sparse and low-rank decompositions developed in [Candès et al., 2011] and elsewhere

as well as graphical lasso decompositions with origins in [Speed and Kiiveri, 1986] and [Yuan and Lin, 2007]

[Chandrasekaran et al., 2012] develop a convex optimization that, under hypotheses, provides a latent factor decomposition:

\[ \hat{\Sigma}^{-1} \approx \hat{S} - \hat{L} \]
Low Rank Plus Sparse Decompositions of Covariance Matrices

The routine maximizes an objective function:

\[ \text{Gaussian likelihood } \left( S - L, \hat{\Sigma} \right) - \lambda ( \gamma \| S \|_1 + \text{tr}(L)) - \text{PDC}, \]

which we solved with an algorithm developed in [Ma et al., 2013]. There is no reliance on asymptotic theory, but there is a normality assumption.
Suppose the inverse of a covariance matrix admits a low-rank plus sparse decomposition:

\[ \Sigma^{-1} = S - L \]

Then the covariance matrix also admits a low-rank plus sparse decomposition:

\[ \Sigma = L + S \]

and the Woodbury formula transforms one decomposition to the other: \( S = S^{-1} \) and \( L = SL\Sigma \).
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Specification

- \( N = 32 \) securities
- \( T = 260 \) observations (one year of daily data)
- \( K = 2 \) broad factors
  - The market, with annualized volatility of 20% (long only factor: all securities have positive exposure)
  - Creditworthiness with annualized volatility of 8% (long/short factor: have the securities are creditworthy, half are close to default)
- \( \kappa = 4 \) narrow factors
  - China
  - Argentina
  - India
  - Saudi Arabia
Input to Algorithm: Sample Covariance Matrix
Low-Rank Component of the Decomposition
Sparse Component of the Decomposition
True and Recovered Eigenvalues

- **True low rank**
- **Recovered low rank**
- **True sparse**
- **Recovered sparse**
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Specification

- $N = 32$ securities
- $T = 260$ observations (one year of daily data)
- $K = ?$ broad factors
- Securities drawn from $\kappa = 4$ countries
  - China
  - Argentina
  - India
  - Saudi Arabia
Input to Algorithm: Sample Covariance Matrix, Oct 2015
Low-Rank and Sparse Decomposition: Covariance Matrices
Low-Rank and Sparse Decomposition: Correlation Matrices
Recovered Eigenvalues

- Recovered low rank
- Recovered sparse
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Summary: What Would Barra Do if It Were Google?

- Fundamental risk models dominate the financial services industry
- PCA-based risk models have not been competitive
- We explore low-rank sparse decompositions of financial data
  - The approach pioneered in [Chandrasekaran et al., 2012] identified and separated broad and narrow factors in simulated data and in empirical data
  - Unlike traditional PCA, this algorithm does not rely on asymptotic results or rank orderings by eigenvalues
Ongoing Research

- Investigate the impact of the normality assumptions and, if appropriate, generalize the algorithm
- Automate the search for optimal calibration parameters
- Benchmark the performance of low-rank sparse models against fundamental and PCA-based models
Acknowledgement

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Thank You
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